A Decimetre Height Reference Surface (HRS) for the European Vertical Reference System (EVRS) based on the DFHRS Concept

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INTRODUCTION

European GNSS- Services
RTCM-Phase Corrections
GNSS cm-Positioning

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INTRODUCTION

1st Present Transformation Problem

Classical National Datum Systems obsolete
Modern Geo-Referencing is related to ETRS89 / ITRF
Economical Online Positioning in homogeneous Datum
political task of State Land Services of all Nations

SAPOS® - Referenzstationen

ETRS89-Datum

Old Datum (e.g. „DHDN“ in Germany)

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INTRODUCTION

**Strict 3D-Trafo in (B,L,(h))**

Residuals Germany without Patching.

Mean Residual: 1.49 m
Max. Residual: 2.43 m

Longwaved Quasi-Systematic Errors („Weak Shapes“)
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2nd Present Transformation Problem

\[ H = h - N(B,L,h) \]

"Geoid" or better: HRS

"Geoid" = HRS Ellipsoid

\[ \text{Geoid} = \text{HRS} \]

\[ +/\- 100 \text{ m} \]

GNSS Heighting

"H from h- GNNS"
European Vertical Reference System (EVRS)

Discrete Points of the HRSurface \( (B,L,h; H) \)

UELN 95/98 Adjustment by Geopotential Numbers

Datum \( W_0 \) and \( C_0 \)

Amsterdam „NAP“

Mean Accuracy +/- 3.0 cm
Error Budgets

European Vertical Reference System (EVRS)

Analysis of „Weak Shapes“

Jäger, 1989
IAG-Symposium Edinburgh

Mean Accuracy: 3.0 cm

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**Spectral Analysis „Weak Shapes‘ Analysis“**

\[ r = \sum_{i=1}^{n} r_i = \sum_{i=1}^{n} c_i \cdot m_i \]

\[ C_x = \sum_{i=1}^{n} \sum_{j=1}^{n} \Lambda_{ij} \cdot m_i \cdot m_j^T = (A^T \cdot C_1^{-1} A)^{-1} \]

**Idea:**

\[ E\left( \frac{r_i^T \cdot r_i}{r^T r - r_i^T r_i} \right) = \text{Max} \]

(Jäger, 1988)

**Solution:**

\[ m_i \quad - \text{Eigenvectors of } C_x \]

\[ E(c_i c_i) = \lambda_i \quad \Lambda \quad - \text{Diagonal Matrix of Eigenvalues of } C_x \]

\[ \sqrt{\lambda_i} \cdot m_i = „\text{Weak Shapes“} \]
Additionally: Latent "Weak Shapes" due to neglected Physical Correlations

\[ \Delta C(i, j) = k_i \cdot [\sigma_a^2 \cdot c_a (t_i - t_j)] \cdot k_j = k_i \cdot C_{\delta_2 \delta_2} \cdot k_j \]

Fig. 4
Comparative study by leveling–line (fig. 3): Differences in shape and amount of principal bias’ and weak–forms \( r_{\text{max}} \) for DESIGN 1 and DESIGN 2. Comparison to the time–invariant reference of pure random errors \( \varepsilon \).

\[ [(C_{\hat{\mu}_i} - C_{\hat{\mu}_i}) - \mu_i \cdot C_{\hat{\mu}_i}] \cdot m_i = 0 \]
Error Budgets

European Vertical Reference System (EVRS)

Planned:

Increasing the final EVRS Accuracy on introducing Tide Gauge Points as further Fix Points

Mean Accuracy: 1.5 cm
Vertical Reference Surface for Europe

Up to now
EGG97
European Gravimetric Geoid97

1_cm geoid (short-wave)

Quasi-Systematic Errors
0.1 – 1.5 m!

„Weak Shapes“
Long-Waved Domain

EGG97 = Non Fitted QGeoid
Height Reference Surface for the European VRS

EUREF Resolution No. 4, Dubrovnik 2001
The IAG Subcommission for Europe (EUREF) recognising the European Vertical GPS Reference Network (EUVN) with its GPS-derived ellipsoidal heights and levelled connections to UELN, – the definition of the European Vertical Reference System EVRS with its first realisation UELN 95/98, called EVRF2000, considering – this implicit pointwise realisation of a European geoid consistent with both ETRS89 and EVRS, – the existence of a large number of regional and local geoids in Europe, – the urgent need by the navigation community for a height reference surface, asks its Technical Working Group and the European Sub-commission of the IAG IGGC (International Gravity and Geoid Commission) to take all necessary steps to generate a European “geoid model” of decimetre accuracy consistent with ETRS89 and EVRS.

GNSS-Services in Europe

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Digital-FEM-HRS (DFHRS) Approach

Idea: “All observations / data types related to the parameters $p$ of a unique continuous one-layer FEM representation $NFEM(p)$ of the HRS”

$$NFEM(p) = \left\{ N(p_k) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} p_{ij,k} \cdot y^i \cdot x^j \right\}$$

$N(p_k)$ = Local Taylor-Series of HRS

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Continuous FEM Representation of HRS

\[
\begin{bmatrix}
\Delta x_{m,n} \\
\Delta y_{m,n} \\
\Delta N_{m,n}(y, x)
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
N(p_n, y, x) - N(p_m, y, x)
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
\sum_{i=0}^{l} \sum_{j=0}^{l-i} (a_{ij,n} - a_{ij,m}) \cdot y^i \cdot x^j
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
\Delta N_{n,m}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
\sum_{i=0}^{1-i} \sum_{j=0}^{1-i} (a_{ij,n} - a_{ij,m}) \cdot y^i \cdot x^j
\end{bmatrix}
\]

\[
\frac{1}{l} \sum_{i=0}^{l} \sum_{j=0}^{l-i} (a_{ij,n} - a_{ij,m}) \cdot (y_{sa} + t \cdot (y_{se} - y_{sa}))^i \cdot (x_{sa} + t \cdot (x_{se} - x_{sa}))^j = \Delta N_{m,n}(t) \equiv 0!
\]

\[
\Delta N_{m,n}(t) = c_0 + c_1 \cdot t + c_2 \cdot t^2 + \ldots + c_k \cdot t^k = \sum_{k=0}^{l} c_k \cdot t^k \equiv 0!
\]

\[
c_k(p_m; p_n; y_{sa}, x_{sa}, y_{se}, x_{se}) = 0, \quad k = 0, l
\]
Digital FEM Height Reference Surface (DFHRS)- Concept

DFHRS – Approach
Complete New Computation of continuous HRS (repres. by $p$ and $\Delta m$)!

$$h_{\text{GNSS}} + v = H + \text{NFEM}(p) - h_{\text{GPS}} \cdot \Delta m$$

$$N_G^j + v^j = \text{NFEM}(p) + \partial N_G(d^j)$$

$$\Delta g + v = \Delta g(p)$$

$$H + v = H$$

$$\xi^j + v = -F_B / M(B) \cdot p + \partial \xi(d_{\xi,\eta})^j$$

$$\xi\eta^j + v = -F_L/(N(B) \cdot \cos(B)) \cdot p + \partial \eta(d_{\xi,\eta})^j$$

$$\frac{a}{4\pi \gamma(B)} \int \Delta g \cdot S(\psi) \, d\sigma + v = \text{NFEM}(p)$$

$\leq$ Any number Geoidmodels/Patches
(Existing Regional Geoids; EGG97)

$\leq$ Gravity

$\leq$ GPS/Levelling Fitting Points

$\leq$ Sets of Deflections from Vertical
(Modern Astro-Geod. Zenith Cameras)

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DFHRS Approach - „Patching“

\[ \text{Residuals of a Geoid Model } \text{NG}' \]

One Datum

\[ \text{NFEM}(p) + \partial \text{NG}(d_j) \]

7 „Patches“
**Height Reference Surfaces and „Geoidfitting“**

<table>
<thead>
<tr>
<th>Name</th>
<th>Principle, Formulas</th>
<th>Basic Representation</th>
<th>Geoid-Fitting Standard</th>
<th>Fitted HRS Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical Harmonics Series</td>
<td>( N = \frac{GM}{\gamma(\varphi)} \cdot \sum_{n=2}^{N} \left( \frac{a_{GRSS80}}{r} \right)^n \sum_{m=0}^{n} (\delta C_{nm} \cdot \cos m\lambda + \delta S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\sin \varphi) )</td>
<td>Continuous Surface</td>
<td>( N_{\text{fitted}} = N + N_{\text{trend}} + N_{\text{colloc}} )</td>
<td>Grid</td>
</tr>
<tr>
<td>Gravimetric Geoid / HRS</td>
<td>( N = \frac{a_{GRSS80}}{\gamma(\varphi)} \cdot \frac{4\pi}{\sigma} \int_{\sigma} S(\psi) \cdot (\Delta g - \Delta g_{Re}) \cdot d\sigma )</td>
<td>Grid</td>
<td>( N_{\text{fitted}} = N + N_{\text{trend}} + N_{\text{colloc}} )</td>
<td>Grid</td>
</tr>
<tr>
<td>Point Mass Modelling</td>
<td>( N = \frac{1}{\gamma(\varphi)} \cdot (T(\delta C_{mn}, \delta S_{nm}; M_i(x, y, z))) )</td>
<td>Continuous Surface</td>
<td>( N_{\text{fitted}} = h - N )</td>
<td>Grid</td>
</tr>
<tr>
<td>DFHRS Local mesh-wise Taylor Series. Any Area Size</td>
<td>( N = \frac{T}{\gamma(\varphi)} = )</td>
<td>Continuous Surface</td>
<td>( N_{\text{fitted}} = h - N )</td>
<td>Continuous Surface (Grid)</td>
</tr>
</tbody>
</table>

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DFHRS_DB - QUALITY Proof

„Accuracy Surface“ based on Covariance matrix of DFHRS parameters \((p, \Delta m)\)

_\_< 3_cm DFHRS_DB
Windhuk, Namibia
EGM96

+ Statistical Testing
+ Variance Component Estimation

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Any number of different geoid models N may be introduced Patch-wise

\[ \partial N_G(d_j) \]
Any number of observation groups of

• Geoid models

and

• Deflections of the Vertical

may be introduced simultaneously. The groups may refer to different datum systems. Datum parameters may be introduced a priori information.

DFHRS Software

DFHRS Software

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**DFHRS_DB Design Parameters**

- **Meshsize** (p=3)
  - 20-30 km: HRS approximation error < (5-10) cm
  - 10 km: HRS approximation error < 1 cm
  - 5 km: HRS approximation error < 0.5 cm

- **Fitting Point Density** (< 10 mm points, EGG97)
  - 50 points per (100 km x 100 km): <_1_cm DFHRS_DB
  - 10 points per (100 km x 100 km): < 3_cm DFHRS_DB
  - 3-4 points per (100 km x 100 km): < 5-10_cm DFHRS_DB
DFHRS_DB  Design Parameters

Design Studies < 5 - 10 cm DFHRS Germany

Patch-Size (EGG97)

- 30 - 40 km for a < 1 cm DFHRS_DB
- 50 – 60 km for a < 3 cm DFHRS_DB
- 300 km for a < 10 cm DFHRS_DB

(3-5) points per patch
< 10cm DFHRS Europe - Design of Fitting Points

ETRS89/EVRS

„GPS/Levelling Points of EUVN“

Fitting Points

\[ \text{NFEM}(p) = h - H \]

used for the 1st version

< 10 cm DFHRS Europe
<_10_cm DFHRS_DB Europe - Patch Design

HRS Polynomial
Degree N=3

30 km Meshsize

34 Geoid-Patches
Patch Size 100 – 800 km
At least (3-4) Fitting points per patch
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**<10_cm DFHRS_DB - Independent Quality Control**

**<1_dm EVRF2004**  
(Present Version, 35 km meshes, 34 Patches)

<table>
<thead>
<tr>
<th></th>
<th>Austria</th>
<th>Germany</th>
<th>Estonia</th>
<th>Latvia</th>
<th>Lithuania</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of unused control points</td>
<td>9</td>
<td>95</td>
<td>21</td>
<td>25</td>
<td>46</td>
<td>13</td>
</tr>
<tr>
<td>RMS [cm]</td>
<td>7.5</td>
<td>4.2</td>
<td>8.8</td>
<td>9.2</td>
<td>6.8</td>
<td>7.0</td>
</tr>
</tbody>
</table>
< 10 cm   DFHRS_DB   USA
Present Data

ETRS89/EVRS
Fitting Points
NFEM(\(p\)) =: h - H

- EUVN2000 points

and

- National EUVN. Some densification points
  Germany, Estonia, Latvia, Lithuania and Switzerland
Overview on European DFHRS_DB

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European HRS .... including
<(1-3)cm_{DFHRS} Baltics (Latvia, Estonia, Lithuania)
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5 km FEM Meshes

... including < 1 cm DFHRS_DB Germany

www.sapos.de

Abb.: Grobstruktur der DFHBF RP

Herausgeber:

L VermGeo

Landesamt für Vermessung und Geobasisinformation
Rheinland-Pfalz

Ferdinand-Sauerbruch-Straße 15
56073 Koblenz
http://www.lvermgeo.lfp.de
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GEO-Samos
Software for Aktiv Mobile Objectoriented Surveying

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Summary

**Concept (Height)**

- **Fundamental Solution Concept for HRS**
  - Strict mathematical base for a continuous FEM-based HRS
  - New concept for an overdetermined parametric HRS
  - Mesh and patch-design - Any accuracy and any area size
  - Open for all geometrical and physical observations!
  - DFHRS = Leading Geoidfitting Concept
  - Ready for the < 1 cm EVRS using all existing data! and EPN densifications!
  - High Practical Relevance for GNSS services and GIS
  - Industrial Standard GNSS-Equipment and GIS (+ RTCM 3.0)
  - High Capacities for International Co-operations

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