European Velocity Field and EPN ETRS89 positions and velocities

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Dense European Velocity Field

• Long term maintenance of the ETRS89

• Precise ETRS89 station positions & velocities of the EPN (Basis of the Velocity Model)

• Accurate frame definition using minimum constraints approach
Datum Definition / Minimum Constraints (1/2)

\( X_R : \text{Reference TRF Solution} \quad X_C : \text{Estimated TRF Solution} \)

\[ X_R = X_C + A\theta \]

\[ \theta = (T_1, T_2, T_3, D, R_1, R_2, R_3, \dot{T}_1, \dot{T}_2, \dot{T}_3, \dot{D}, \dot{R}_1, \dot{R}_2, \dot{R}_3)^T \]

L.S. gives

\[ \theta = (A^T A)^{-1} A^T (X_R - X_C) \]

To have \( X_C \) expressed in the same frame as \( X_R \) (i.e. \( \theta = 0 \)), we can write

\[ B(X_R - X_C) = 0 \quad (\Sigma_\theta) \]

In terms of NEQ

\[ B^T \Sigma_\theta^{-1} B(X_R - X_C) = 0 \]
Datum Definition / Minimum Constraints (2/2)

The initial NEQ system of space geodesy observations could be written as:

\[ N_{unc}(\Delta X) = K \]  \hspace{1cm} (3)

where \( \Delta X = X - X_{apr} \) \hspace{1cm} (Linearized Unknowns)

Selecting a Reference TRF \( (X_R) \), MC equation is:

\[ B^T \Sigma_\theta^{-1} B(\Delta X) = B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \]  \hspace{1cm} (4)

Cumulating (3) and (4) yields:

\[(N_{unc} + B^T \Sigma_\theta^{-1} B)(\Delta X) = K + B^T \Sigma_\theta^{-1} B(X_R - X_{apr})\]
European Velocity Field
Start with EPN Wkly Combined Solutions

- Remove Constraints
- Add Minimum constraints
- Reject outliers and properly handle discontinuities

First results
Combination of weeks from 837 to 1263
EPN and Selected set of Reference Stations
TRF & EOP time series Combination

**C\textunderscore TREF Software**

**INPUT:** \( X(t), \text{EOP}(t) \) in daily/weekly/monthly SINEX files

**OUTPUT:** \( X(t_0), \dot{X}, \text{EOP}(t), (T_x, T_y, T_z, D, R_x, R_y, R_z) \)

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**Datum Definition**

\[
(A^T A)^{-1} A^T (X_{RS} - X_c) = 0
\]

\[
\begin{align*}
X^i_s &= X^i_{itr} + (t^i_s - t_0) \dot{X}^i_{itr} + T^i_k + D^i_k X^i_{itr} + R^i_k X^i_{itr} \\
X^i_s &= X^i_{itr} + \dot{T}^i_k + \dot{D}^i_k X^i_{itr} + \dot{R}^i_k X^i_{itr}
\end{align*}
\]

\[
\begin{align*}
x^p_s &= x^p + R^2_k \\
y^p_s &= y^p + R^1_k \\
UT_s &= UT - \frac{1}{f} R^3_k \\
\dot{x}^p_s &= \dot{x}^p + \dot{R}^2_k \\
\dot{y}^p_s &= \dot{y}^p + \dot{R}^1_k \\
LOD_s &= LOD + \frac{\Lambda}{f} \dot{R}^3_k
\end{align*}
\]

- Matching common EOP parameters at UT noon
- Propagate at UT noon if rates are available
Differences btw Contsr’d and MC (W-1263)

Using all stations as RS

12 ITRF2000 Ref. Stations

16 ITRF2000 Ref. Stations
Handling discontinuities
Handling discontinuities
EPN ETRS89 Vertical Velocities
Vertical Velocities (?)

ZIMM_14001M004

ZIMM_14001M004

Graphs showing vertical velocities in different directions (North, East, Up) over the years from 1996 to 2004.
Vertical Velocities (?)

ZIMM_14001M004 Velocity (mm)

NORTH

EAST

UP

ZIMM_14001M004 Velocity (mm)

NORTH

EAST

UP
Quality Evaluation

EUREF Weekly WRMS

2D-WRMS (mm) vs. UP-WRMS (mm)

GPS Week
Conclusions

- EPN-ETRS89 station positions & velocities will be available for users
- Updated regularly (annually)
- Dense velocity field will include other networks (national, local)
- Minimum constraints approach insure internal consistency
- EUREF Weekly WRMS:
  - 2 mm in horizontal
  - 4-5 mm in vertical
- Re-computing EUREF weekly solutions will certainly improve their quality