Discussion on How to Express a Regional GPS Solution in the ITRF

Z. ALTAMIMI[1]

Abstract

The usefulness of the densification of the International Terrestrial Reference Frame (ITRF) is to facilitate its access as a global frame by users interested on small (regional or local) networks. Although a regional GPS solution of station positions is usually derived using IGS products (orbits, clocks, ...), which are nominally expressed in the ITRF, its corresponding datum definition could be far from that of ITRF due to mainly the network configuration. The main question to be answered here is how to optimally express station positions of a regional network in the global frame of ITRF? This could be achieved by mainly (1) constraining coordinates of a subset of stations to their ITRF values or (2) aligning the regional solution to ITRF using a transformation formula. This paper will focus on the second method, based on minimum constraints approach, yielding an optimal datum definition together with preserving the original characteristic of the regional solution.

1. Introduction

The ITRF is a result of combination of global terrestrial reference frames (stations positions and velocities) provided by 5 space geodesy techniques: Very Long Baseline Interferometry (VLBI), Lunar and Satellite Laser Ranging (LLR and SLR), Global Positioning System (GPS) and Doppler Orbitography Radio-positioning Integrated by Satellite (DORIS). From the geodetic point of view, densification of the ITRF is meant the expression of station positions (and velocities) of a regional or local network in the ITRF. The GPS, compared to the other techniques, has the advantage of being the most efficient one for the ITRF densification purpose, given its ease use, low cost and the availability of the IGS products for all users.

A densification part has been achieved recently in the latest ITRF version, namely the ITRF2000, by including in the global combination some regional GPS solutions (ALTAMIMI et al., 2002a).

In terms of Terrestrial Reference Frame (TRF) definition, all the IGS products are expressed in the ITRF: ITRF91 from the beginning of IGS activities until the end of 1993; ITRF92 during 1994; ITRF93 during 1995 until mid-1996; ITRF94 since mid-1996 until the end of April 1998; ITRF96 starting on March 1, 1998; ITRF97 starting on August 1, 1999 and ITRF2000 since December 20, 2001. Starting with ITRF96, the expression of IGS products in the ITRF is ensured by aligning the global IGS TRF combined solution of station positions and velocities to ITRF. This alignment is performed using 14 transformation parameters between the IGS TRF and the ITRF, estimated over about 50 stations globally distributed (FERLAND et al., 2001).

Station coordinates of a regional network estimated using IGS products are theoretically expressed in the ITRF. While this statement could be valid for the underlying TRF orientation (through the orbit fixing), the TRF origin and scale are generally far from those of the ITRF. Consequently, for various Earth Science applications, GPS solutions of station positions of a regional or local network need to be optimally expressed in the same frame as (and to be consistent with) the global ITRF.

In the following we discuss some technical issues related to the various geodetic methods allowing to integrate regional network into the ITRF.

Disregarding the selected method, and to achieve optimal estimate, it is recommended to ensure the link between the ITRF and the regional solutions through a selection of ITRF stations of high quality. Among the criteria selection, it is advised to select stations having:

- an optimal distribution over the regional network. In case that none of the network stations is already available in the ITRF, a certain number of ITRF stations of high quality, surrounding the implied network, should be included in the GPS processing
- a long observing history (at least 3 years)
- the ITRF residuals should be less than 5 (eventually 10) mm for positions and 3 mm/y for velocities for at least 3 different solutions contributed to ITRF generation.

The are mainly two major methods allowing the expression of the regional network solution in the ITRF:

1. constraining the coordinates of the selected ITRF subset of stations to ITRF values at the central epoch of the implied observations used to generate the regional solution. The constraints should be easily removable (*F* . 10^{-5} m). This is for example the current procedure applied by EUREF for their weekly solution, where about 12 ITRF stations are constrained to ITRF2000 values.
2. aligning the regional solution to the ITRF using transformation parameters which should be estimated using the selected subset of ITRF stations.

In either case, the following points should be observed:

- the selected subset of stations should be under constant surveillance to detect/identify possible discrepancy between ITRF and the regional solution. If significant discrepancy occurs (which is sometimes the case when some station equipment changes), the corresponding station should be excluded from the constraint/alignment process.
- the advantage of method (1) is that the regional solution is well expressed in the ITRF frame, while its disadvan-

[1] Zuheir Altamimi, Géographique National (IGN), Ecole Nationale des Sciences Géographiques, 6 et 8 avenue Blaise Pascal, F - 77455 Marne la Vallée Cedex, France, e-mail altamimi@ensg.ign.fr

tage is that the selected stations will have their coordinates entirely determined by the ITRF selected values. That is why it is recommended to apply removable constraints for possibly later applications of unconstrained solutions by some users.

– given the nature of a regional network and its effect on the estimation of the transformation parameters, its extremely important to be very careful when using method (2). In fact, the most efficient way to use is the transformation parameter alignment using minimum constraints approach as detailed hereafter.

2. Aligning a Regional Solution to ITRF Using Minimum Constraints Approach

In the following we propose a method allowing to efficiently express a regional solution of station positions in the ITRF. This method, based on the equations of minimum constraints, could of course be applied to any kind of network not only for positions, but also for velocities, for more details, see for instance (ALTAMIMI et al, 2002b).

The relation between a regional solution (X_R) and ITRF (X_I), over selected stations, could be written as:

$$X_I = X_R + A\ 1 \qquad (1)$$

where A and 1 are respectively the design matrix of partial derivatives and the vector of 7 transformation parameters:

$$A = \begin{pmatrix} . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_a^i & 0 & z_a^i & -y_a^i \\ 0 & 1 & 0 & y_a^i & -z_a^i & 0 & x_a^i \\ 0 & 0 & 1 & z_a^i & y_a^i & -x_a^i & 0 \\ . & . & . & . & . & . & . \end{pmatrix}$$

and $1 = (T_x, T_y, T_z, D, R_x, R_y, R_z)^T$

Un-weighted least squares adjustment yields a solution for 1 as:

$$\theta = \overbrace{\left(A^T A\right)^{-1} A^T}^{B}\left(X_I - X_R\right) \qquad (2)$$

The approach of minimum constraints consists in using the matrix $B = (A^T A)^{-1} A^T$ in such a way that X_R will be expressed in the same frame as the ITRF solution X_I. Therefore to have X_R be expressed in the ITRF at a certain E_1 level, a "datum definition" equation could be written as:

$$B\ (X_I - X_R) = 0\ (E_1) \qquad (3)$$

where E_1 is the variance matrix at which equation (3) is satisfied. E_1 is a diagonal matrix containing small variances (to be selected at the user level) for each one of the 7 transformation parameters. It is suggested to use 1 mm for translation parameters and an equivalent amount for the scale and orientation parameters.

In terms of normal equation, we then can write:

$$B^T E_1^{-1} B(X_I - X_R) = 0 \qquad (4)$$

Using IGS products (orbits, clocks, etc.), the initial normal equation system of a regional GPS solution before adding any kind of constraints could be written as:

$$N_{unc}\ (DX) = K \qquad (5)$$

where $DX = X - X_{apr}$, with X being the unknown vector, X_{apr} is the vector of a priori values, N_{unc} is the unconstrained normal matrix and K is the righthand side vector.

The normal equation system (\refeq-n) is invertible, but the underlying TRF could be far from that of ITRF, i.e. defined at the level of the orbit precision (a few cm). The same normal equation system could be obtained also after removing classical constraints applied to a given regional solution.

Selecting a subset of ITRF stations (X_I), the equation of minimum constraints (or datum definition) is:

$$B^T E_1^{-1} B(DX) = B^T E_1^{-1} B\ (X_I - X_{apr}) \qquad (6)$$

Note that the righthand side of equation (6) vanishes if the a priori values are those of ITRF selected solution.

Cumulating (5) and (6) yields:

$$(N_{unc} + B^T E_1^{-1} B)\ (DX) = K + B^T E_1^{-}B\ (X_I - X_{apr}) \qquad (7)$$

The minimally constrained solution, expressed in the ITRF upon the selected stations is then:

$$X = (N_{unc} + B^T E_1^{-1} B)^{-1}(K + B^T E_1^{-1} B\ (X_I - X_{apr})) + X_{apr} \qquad (8)$$

3. Numerical Applications

For the purpose of numerical applications of the method proposed above, we selected, as an example, the EUREF combined solution for GPS week 1149. In this solution, the coordinates of 12 ITRF stations (illustrated in Figure 1) were constrained to ITRF2000 values. After removing the constraints, 7 transformation parameters were first estimated between the unconstrained EUREF solution and the ITRF2000, upon the subset of 12 stations. The adjusted values of these 7 parameters are listed in Table 1, distinguishing the weighted and un-weighted estimations.

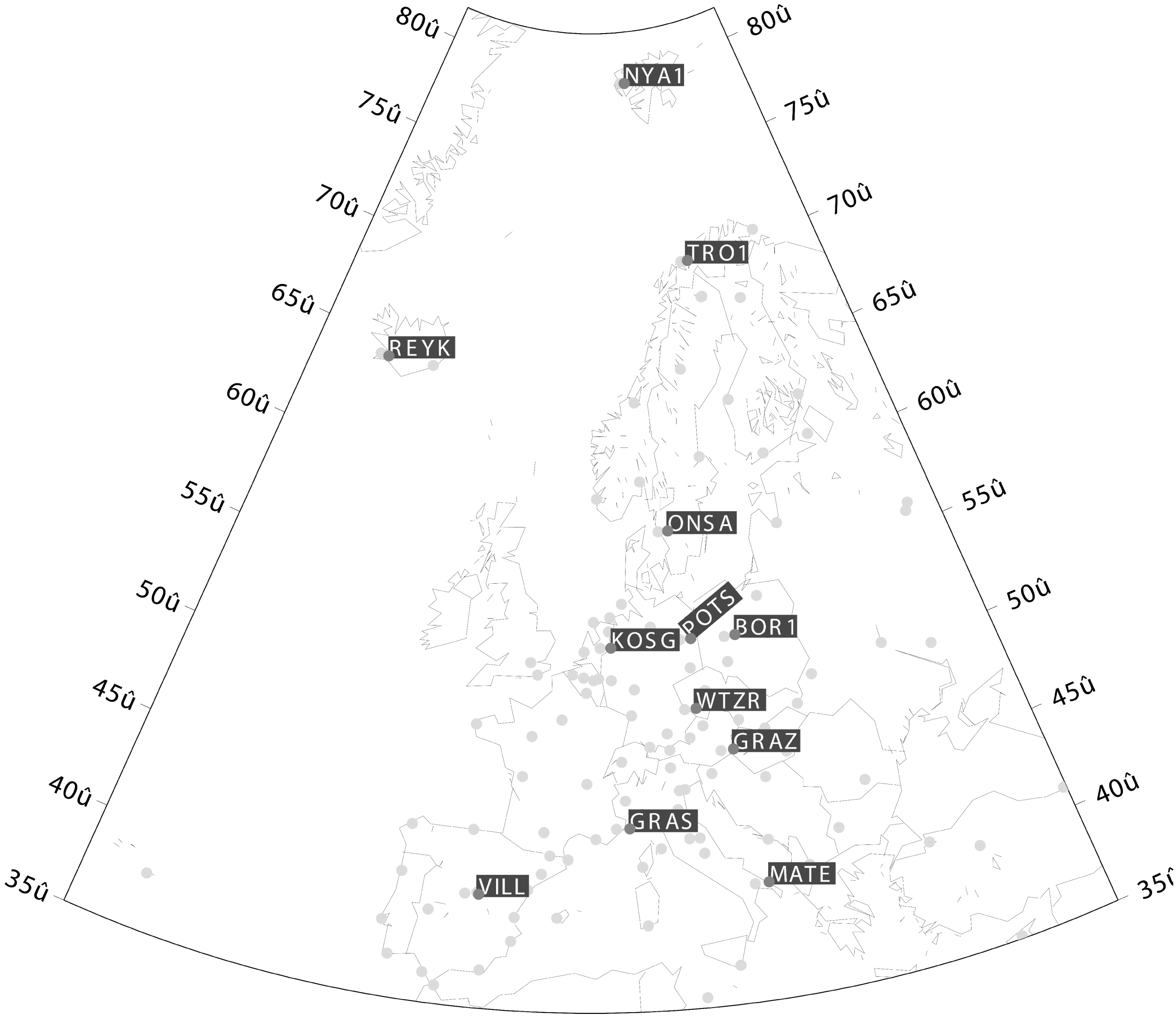

Figure 1. EUREF network underlying the 12 stations whose coordinates are constrained to ITRF2000 values in the combined solution for GPS week 1149.

Table 1. Transformation parameters from ITRF2000 to the Unconstrained EUREF Solution for GPS Week 1149.

T1 cm	*T2* cm	*T3* cm	*D* 10^{-8}	*R1* 0.001"	*R2* 0.001"	*R3* 0.001"
Un-weighted L.S. adjustment						
-16.60 ±.44	-4.77 ± 0.83	-23.00 ±.40	.948 ±.061	.117 ±.244	-.149 ±.148	-.027 ±.179
Weighted L.S. adjustment						
-16.36 ±.66	-3.75 ±1.08	-23.11 ±.42	.922 ±.045	.412 ±.306	-.225 ±.236	-.186 ±.201

As seen in Table 1, the two sets of the 7 parameters are not the same (although they would be equivalent, see below) since these parameters are correlated due to the network geometry. The unconstrained solution was then transformed using the two sets of transformation parameters. The coordinate differences between the two unconstrained \& transformed solutions and the originally constrained one are illustrated in Figure 2. As shown in this figure, the vertical residuals do not have zero mean, reflecting the network effect on the scale factor of this example of network.

The alignment to ITRF2000 was then applied to the EUREF unconstrained solution using the minimum constraints approach discussed above, upon the 12 selected stations. The coordinates differences between the minimally constrained and the constrained solutions are illustrated in Figure 3. Comparing residuals of this figure with those of figure 2, demonstrate that the proposed method of minimum constraint alignment is more efficient than the classical one.

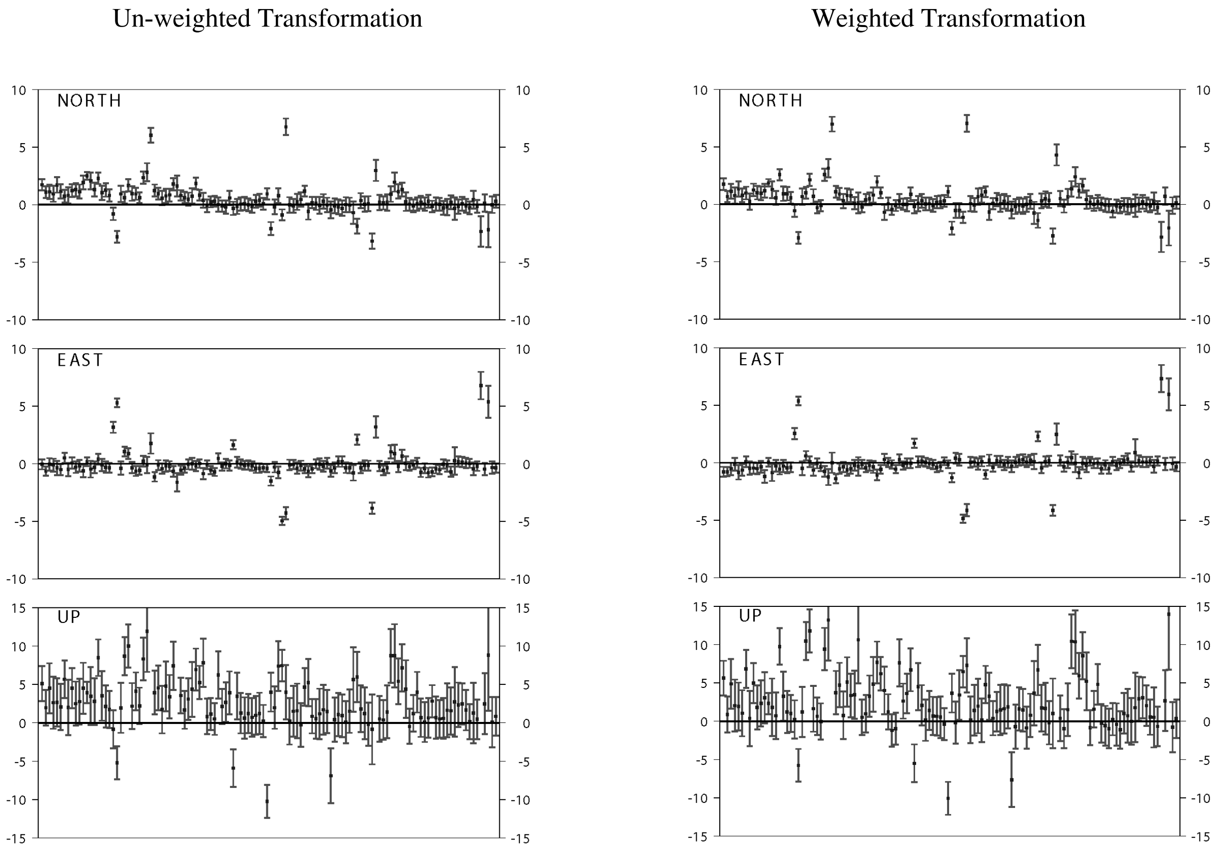

Figure 2. Coordinate differences (mm) between the (unconstrained\& transformed) and the originally constrained EUREF solution.

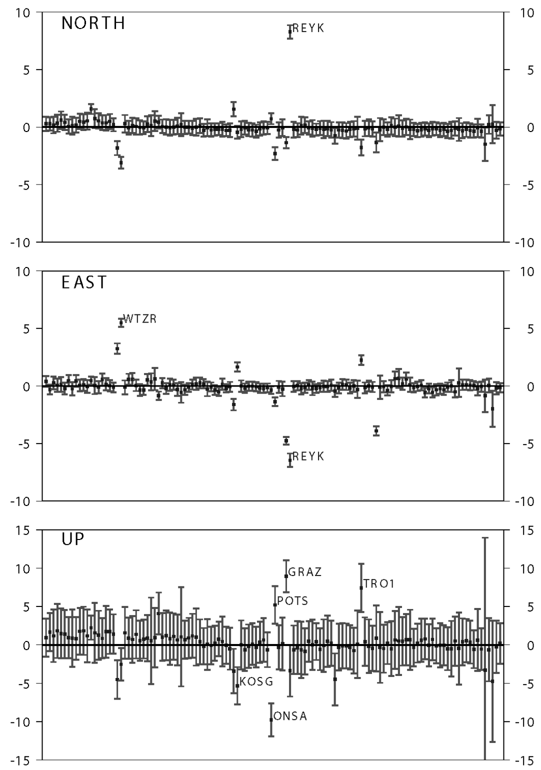

Figure 3. Coordinate differences (mm) between the minimally constrained and the originally constrained EUREF solution.

In order to investigate whether it is equivalent to use station position values directly from ITRF2000 or from the IGS realization of ITRF2000, figure 4 plots the 12 coordinate differences between respectively, IGS weekly and cumulative solutions, IGS weekly and ITRF2000 and between IGS cumulative and ITRF2000. This figure is a "perfect" illustration of the network effect on the datum definition. While the IGS weekly and cumulative solutions are aligned to the ITRF2000 over 54 stations globally distributed and having, by construction, a zero residual mean, picking out 12 stations produces a TRF shift at the regional level of a few millimeters: It could be very easily observed from the 3 plots shown in Figure 4 that the regional residuals do not have zero mean. In order to assess this effect on the EUREF regional solution, we applied the procedure of minimum constraints over IGS weekly (week 1149) and cumulative solutions. Figure 5 shows the position differences between solutions derived using IGS weekly (resp. cumulative) and ITRF2000. The TRF shift due the network effect predicted from Figure 4 is well transferred to Figure 5.

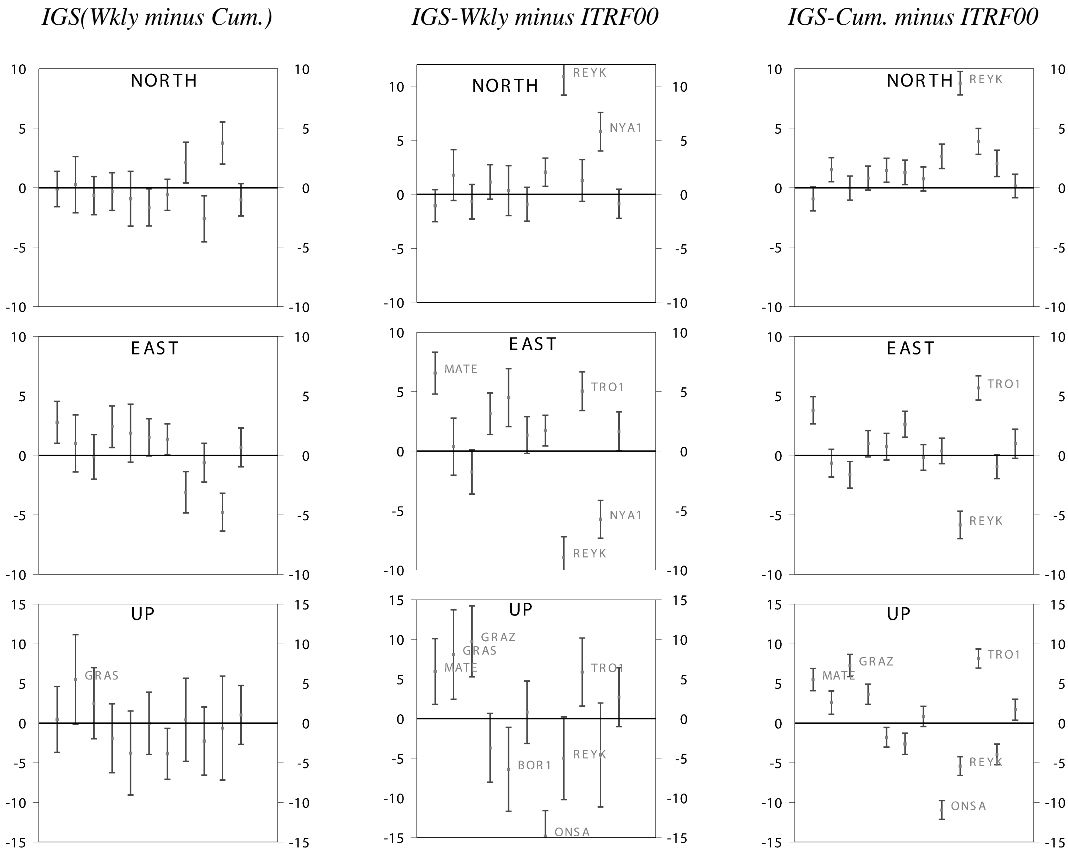

Figure 4. 12 station Coordinate differences (mm) between IGS weekly and cumulative, IGS weekly and ITRF2000 and between IGS cumulative and ITRF2000.

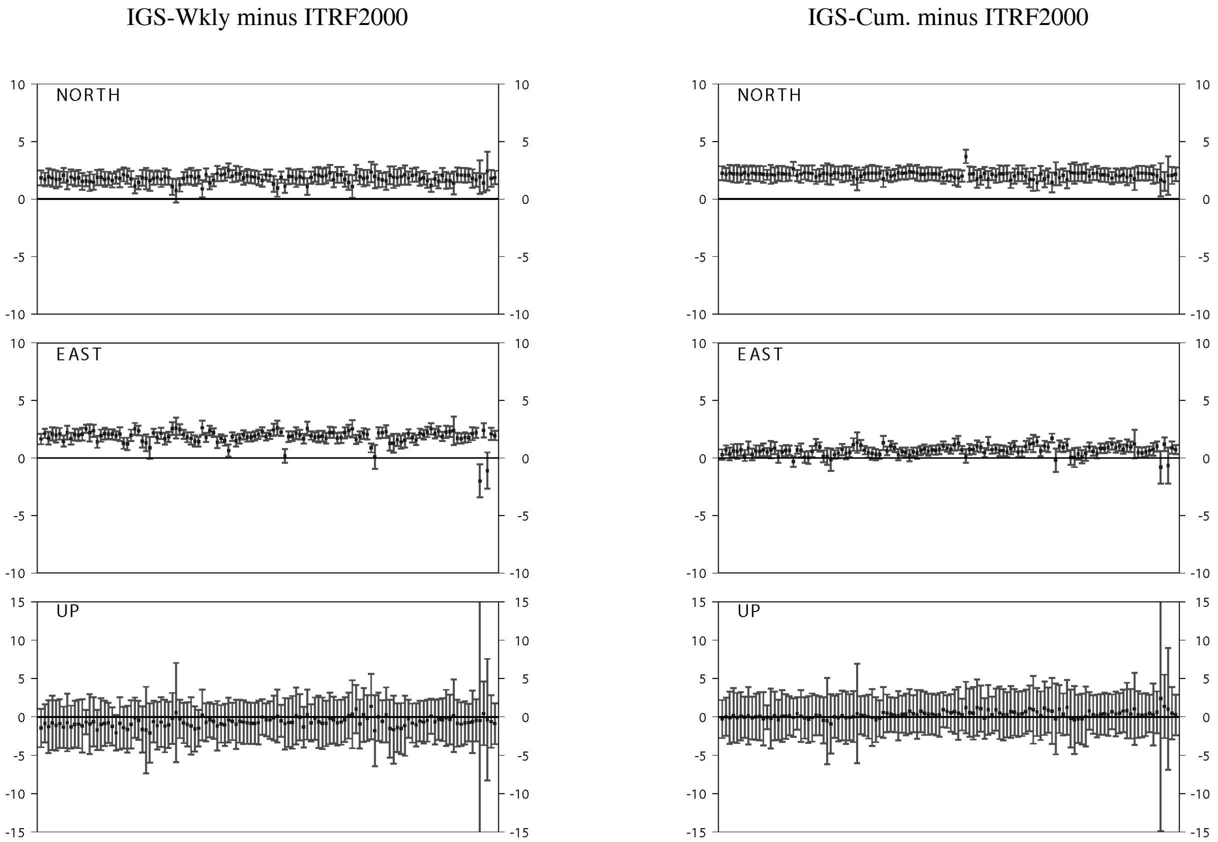

Figure 5. Coordinate differences (mm) between the EUREF minimally constrained solutions using ITRF2000 and IGS weekly or cumulative solutions.

Conclusion

This paper demonstrates that minimum constraints approach is an efficient method to optimally express a regional solution in a global frame such as the ITRF, minimizing so the well known network effect. It is therefore suggested to use this method instead of the classical constraints which hide the original characteristic of the regional solution. All station discrepancies (which very often occur after some station equipment changes) between the ITRF and the regional solution are then identified and, in the same time, the solution itself is fully expressed in the global frame.

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