

# The Vertical Reference System for Europe<sup>1</sup>

J. IHDE<sup>2</sup>, W. AUGATH<sup>3</sup>

## Introduction

Responding to an urgent request of the Comité Européen des Responsables de la Cartographie Officielle (CERCO) for an European Height System a 0.1 m accuracy level the Technical Working Group of the IAG Subcommission on Continental networks for Europe (EUREF) proposed in 1994 a new adjustment and an enlargement of the United European Levelling Network to Eastern Europe (Resolution 3 of the EUREF Symposium in Warsaw, 1994). The decision for the realization of the European Vertical Reference System (EUVN) in 1995 was a big step toward a modern integrated reference system for Europe which combines GPS coordinates, gravity related heights and sea level heights in one data set. It was decided for Europe to derive the gravity related heights as normal heights from geopotential numbers (Resolution 2 of the EUREF Symposium in Ankara, 1996). In 1999 the European Spatial Reference Workshop recommended the European Commission (EC) to adopt a vertical reference system on the basis of the results of the UELN and EUVN projects for the specifications of the products to be delivered to the EC. Furthermore it promoted the wider use within all member states in future.

## Present status

A height reference system is characterized by the vertical datum and the kind of gravity related heights. The vertical datum is in most cases related to the mean sea level, which is estimated at one or more tide gauge stations. The tide gauge stations of the national height systems in Europe are located at various oceans and inland seas: Baltic Sea, North Sea, Mediterranean Sea, Black Sea, Atlantic Ocean. The differences between these sea levels can amount to several decimeters. They are caused by the various separations between the searface and the geoid.

In addition the used height datums often are of historical nature, as well as not all zero levels are referred to the mean sea level. There are also zero levels referred to the low tide (Ostend) or to the high tide. For example the Amsterdam zero point is defined by mean high tide in 1684.

In Europe three different kinds of heights are being used: normal heights, orthometric heights and normal-orthometric heights. Examples for the use of orthometric heights are Belgium, Denmark, Finland, Italy and Switzerland. Today normal heights are being used in France, Germany, Sweden and in the most countries of Eastern Europe.

## United European Levelling Network (UELN)

After a break of ten years, the work on the UELN was resumed in 1994 under the name UELN-95. The objectives of the UELN-95 project were to establish a unified height system for Europe at the one decimeter level with the simultaneous enlargement of UELN as far as possible to include Central and Eastern European countries and the development of a kinematic height network "UELN 2000" step by step. Starting point for the UELN-95 project has been a repetition of the adjustment of the UELN-73/86. In contrast to the weight determination of the 1986 adjustment for UELN-95 the weights were derived from a variance component estimation of the observation material which was delivered by the participating countries and introduced into the adjustment.

The adjustment is performed in geopotential numbers as nodal point adjustment with variance component estimation for the participating countries and as a free adjustment linked to the reference point of UELN-73 (Amsterdam).

The development of the UELN-95 is characterized by two different kinds of enlargements: The substitution of data material of such network blocks (which had been already part of UELN-73) by new measurements with improved network configuration, and on the other hand by adding new national network blocks of Central and Eastern Europe which were not part of UELN-73.

In the year 1998 more than 3000 nodal points were adjusted and linked to the Normaal Amsterdams Peil (the reference point of the UELN-73). The normal heights in the system UELN-95/98 are available for more than 20 participating countries.

<sup>1</sup> This contribution as well as the following paper *European Vertical Reference System (EVRS)* are updated versions from February 2001. The original papers were presented and discussed at the meeting of the EUREF Technical Working Group (TWG) Tromsø, 21.6.2000, and then presented to the following EUREF symposium, 22.-24. 6. 2000. Thanks to imperantly contributing remarks by J. Mäkinen, M. Ekman, B. Heck and R. Rummel (cf. the following papers) to this topic, the paper was rediscussed and updated at the TWG meetings Lisbon (09.-10.10.2000) and Munich (15.-16.02.2001). It was decided to publish this final version in this volume together with the mentioned notes to show to the reader the development of this topic.

<sup>2</sup> Johannes Ihde: Bundesamt für Kartographie und Geodäsie, Aussenstelle Leipzig, Karl-Rothe-Strasse 10-14, D-04015 Leipzig, Germany; Fax: +49 341 5634 415, Tel.: +49 341 5634 424, E-mail: ihde@leipzig.ifag.de

<sup>3</sup> Wolfgang Augath: Geodätisches Institut, Technische Universität Dresden, Mommsenstrasse 13, D-01062 Dresden, Germany; Fax: +49 351 463 7201, Tel.: +49 351 463 4249 (Skr.), - 7503 (selbst); E-mail: wolfgang.augath@mailbox.tu-dresden.de

## European Vertical Reference Network (EUVN)

The initial practical objective of the EUVN project was to unify different national height datums in Europe within few centimeters also in those countries which were not covered by the UELN. Additionally this project was thought as preparation of a geokinematic height reference system for Europe and a way to connect levelling heights with GPS heights for the European geoid determination.

At all EUVN points three-dimensional coordinates in the ETRS89 and geopotential numbers will be derived. Finally the EUVN is representing a geometrical-physical reference frame. In addition to the geopotential numbers the corresponding normal heights will be provided. In the tide gauge stations the connection to the sea level will be realized.

In total the EUVN consists of about 196 sites: 66 EUREF and 13 national permanent sites, 54 UELN and UPLN (United Precise Levelling Network of Central and Eastern Europe) stations and 63 tide gauges.

The final GPS solution was constrained to ITRF96 coordinates (epoch 1997.4) of 37 stations. For many practical purposes it is useful to have the ETRS89 coordinates available. To reach conformity with other projects, the general relations between ITRS and ETRS were used.

In the year 2000 the connection levellings and computations of normal heights in UELN-95/98 were finished.

## European Vertical Reference System (EVRS)

The Spatial Reference Workshop in Marne-la-Vallée in November 1999 recommended the European Commission European reference systems for referencing of geo data. For the height component the workshop recommended that the European Commission:

- *Adopts the results of the EUVN/UELN initiatives when available, as definitions of vertical datum and gravity-related heights;*
- *Includes the EUVN reference system so defined for the specifications of the products to be delivered to the EC, within projects, contracts, etc;*
- *Future promotes the wider use of the European vertical reference system within all member states, by appropriate means (recommendations, official statement, ...).*

The Technical Working Group of the IAG Subcommittee for Europe (EUREF) was asked to define a European Vertical Reference System and to describe its realization. After a discussion at the plenary of the symposium it was decided to specify the definition. Two contributions in this discussion about the treatment of the permanent tidal effect (MÄKINEN, EKMAN) are added to this publication.

The principles of the realization of the EVRS were adopted at the EUREF Symposium 2000 in Tromsø by the resolution no. 5:

*The IAG Subcommittee for Europe (EUREF)*

*noting the recommendation of the spatial referencing workshop, in Marne-la-Vallée 27-30 November 1999, to the European Commission to adopt the results of the EUVN/UELN projects for Europe wide vertical referencing,*

*decides to define an European Vertical Reference System (EVRS) characterized by:*

- *the datum of 'Normaal Amsterdams Peil' (NAP)*
- *gravity potential differences with respect to NAP or equivalent normal heights,*

*endorses UELN95/98 and EUVN as realizations of EVRS using the name EVRF2000,*

*asks the EUREF Technical Working Group to finalize the definition and initial realization of the EVRS and to make available a document describing the system.*

For referencing of geo information in a unique system transformation parameters between the national heights systems and the EVRS frame are also available, see SACHER et al. (1999a).

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## European Vertical Reference System (EVRS)

### Preamble

This document

- defines the European Vertical Reference System (EVRS) including a European Vertical Datum and the European Vertical Reference Frame as its realization and for practical use as a static system under the name EVRF2000;
- is for adoption by the European Commission to promote widespread use as a defacto standard for future pan-European data products and services.

### 1. Definition

The European Vertical Reference System (EVRS) is a gravity-related height reference system. It is defined by the following conventions:

- a) The vertical datum is the zero level for which the Earth gravity field potential  $W_0$  is equal to the normal potential of the mean Earth ellipsoid  $U_0$ :

$$W_0 = U_0.$$

- b) The height components are the differences  $DW_P$  between the potential  $W_P$  of the Earth gravity field through the considered points  $P$  and the potential of the EVRS zero level  $W_0$ . The potential difference  $-DW_P$  is also designated as geopotential number  $c_P$ :

$$-DW_P = W_0 - W_P = c_P.$$

Normal heights are equivalent to geopotential numbers.

- c) The EVRS is a zero tidal system, in agreement with the IAG Resolutions.<sup>4</sup>

### The European Vertical Reference Frame 2000 (EVRF2000)

*The EVRS is realized by the geopotential numbers and normal heights of nodal points of the United European Levelling Network 95/98 (UELN 95/98) extended for Estonia, Latvia, Lithuania and Romania in relation to the Normaal Amsterdams Peils (NAP). The geopotential numbers and normal heights of the nodal points are available for the participating countries under the name UELN 95/98 to which is now given the name EVRF2000.*

#### 2.1 Realization of the datum

*The vertical datum of the EVRS is realized by the zero level through the Normaal Amsterdams Peil (NAP). Following this, the geopotential number in the NAP is zero:*

$$c_{NAP} = 0.$$

For related parameters and constants of the Geodetic Reference System 1980 (GRS80) is used. Following this the Earth gravity field potential through NAP  $W_{NAP}$  is set to be the normal potential of the GRS80

$$W_{NAP}^{REAL} = U_{GRS80}.$$

*The EVRF2000 datum is fixed by the geopotential number and the equivalent normal height of the reference point of the UELN No. 000A2530/13600.*

<sup>4</sup> In a) and b) the potential of the Earth includes the potential of the permanent tidal deformation but excludes the permanent tidal potential itself.

Station name / Country	UELN number	Position in ETRS89	Height in UELN95/98		Gravity in IGSN71
		ellipsoidal latitude ellipsoidal longitude in ° ' "	geopotential number in $m^2 \cdot s^{-2}$	normal height in m in $m \cdot s^{-2}$	
Reference point of EVRS 000A2530 The Netherlands	<b>13600</b>	52° 22 53" 4° 54 34"	<b>70259</b>	<b>71599</b>	981277935

## 2.2 The Adjustment of UELN-95/98

The adjustment of geopotential numbers was performed as an unconstrained adjustment linked to the reference point of UELN-73 (in NAP). Both the geopotential numbers and the normal heights of UELN 95/98 of the adjustment version UELN-95/13 were handed over in January 1999 to the participating countries as the UELN-95/98 solution.

Parameters of the UELN-95/98 adjustment are the following:

- number of fixed points: 1
- number of unknown nodal points: 3063
- number of measurements: 4263
- degrees of freedom: 1200
- a-posteriori standard deviation referred to a levelling distance of 1 km: 1.10 kgal × mm

- mean value of the standard deviation of the adjusted geopotential number differences: 6.62 kgal × mm
- mean value of the standard deviation of the adjusted geopotential numbers ( $\hat{\gamma}$  heights): 19.64 kgal × mm
- average redundancy: 281

The normal heights  $H_n$  were computed by  $H_n = c_p \bar{\gamma}$ , where  $\bar{\gamma}$  is the average value of the normal gravity along the normal plumb line between the ellipsoid and the telluroid. The average value of the normal gravity along the normal plumb line is determined by

$$\bar{\gamma} \approx \gamma_m = \gamma_0 - \frac{0.3086 \text{ mgal/m} \cdot h}{2} + \frac{0.072 \cdot 10^{-6} \text{ mgal/m}^2 \cdot h^2}{2}$$

with the Gravity Formula 1980 and latitude in ETRS89.



Figure 1: United European Levelling Network 1995



Figure 2: UELN 95/98 – Isolines of Precision [kgal × mm] (UELN-95/98 – extended for Estonia, Latvia, Lithuania and Romania)

## Addendum

### A 1 Datum relations

#### A 1.1 Relations between the defined and the realized EVRS datum

The potential of the Earth gravity field in the NAP is processed by

$$W_{NAP} = W_0 + \Delta W_{SST} + \Delta W_{TGO}$$

where  $\Delta W_{SST}$  is the sea surface topography potential difference at the tide gauge Amsterdam in relation to a geoid with  $W_0 = U_0$ .

$\Delta W_{TGO}$  is the potential deviation between the NAP level  $W_{NAP}$  and the level of the mean sea surface at the tide gauge Amsterdam

The relation between the EVRS datum and its realization in EVRF2000 is expressed by

$$\begin{aligned} \Delta W_{EVRS} &= W_{NAP} - W_{NAP}^{REAL} \\ &= W_{NAP} - U_{0GRS80} \\ &= U_0 - U_{0GRS80} + \Delta W_{SST} + \Delta W_{TGO} - \Delta W_{EVRS} \end{aligned}$$

is the offset to a world height system. The relation to a world height system with  $W_0 = U_0$  needs the knowledge of the

sea surface topography and the deviation in the NAP in connection with the normal potential at the mean Earth ellipsoid  $U_0$  (at present  $U_0 \sim 62636856 \text{ m}^2 \cdot \text{s}^{-2}$ ) at a cm-accuracy level.

#### A 1.2 Relations between the EVRS2000 datum and datums of National Height Systems in Europe

In Europe three different kinds of heights (normal heights, orthometric heights and normal-orthometric heights) are used: Examples for the use of orthometric heights are Belgium, Denmark, Finland, Italy and Switzerland. Today normal heights are used in France, Germany, Sweden and in most countries of Eastern Europe. In Norway, Austria and in the countries of the former Yugoslavia normal-orthometric heights are used.

The vertical datum is determined by the mean sea level, which is estimated at one or more tide gauge stations. The reference tide gauge stations to which the zero levels of the national European height systems in Europe are related are located at various oceans and inland seas: Baltic Sea, North Sea, Mediterranean Sea, Black Sea, Atlantic Ocean. The differences between the zero levels can come up to several decimeters. They are caused by the various separations between the ocean surface and the geoid as well as by the definition of the level.

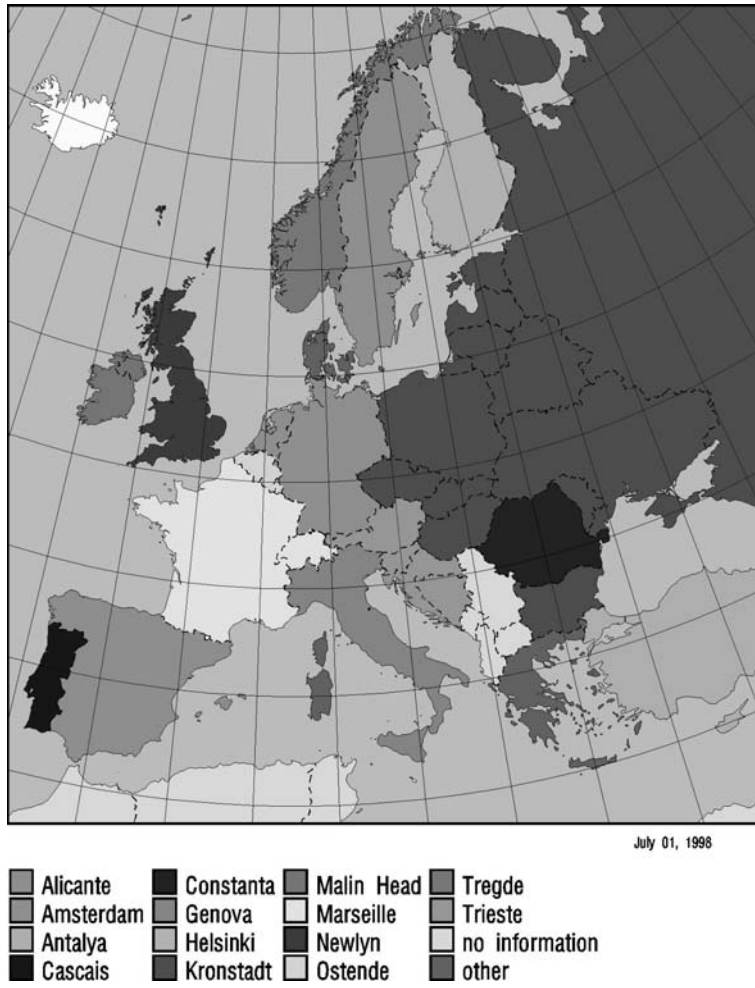


Figure 3: Reference Tide Gauges of National Height Systems in Europe

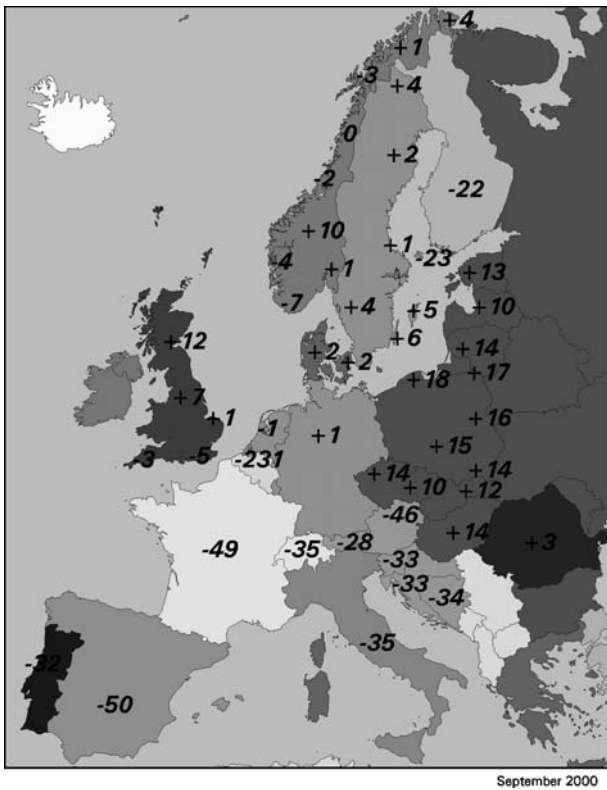


Figure 4: Kind of Heights of National Height Systems in Europe

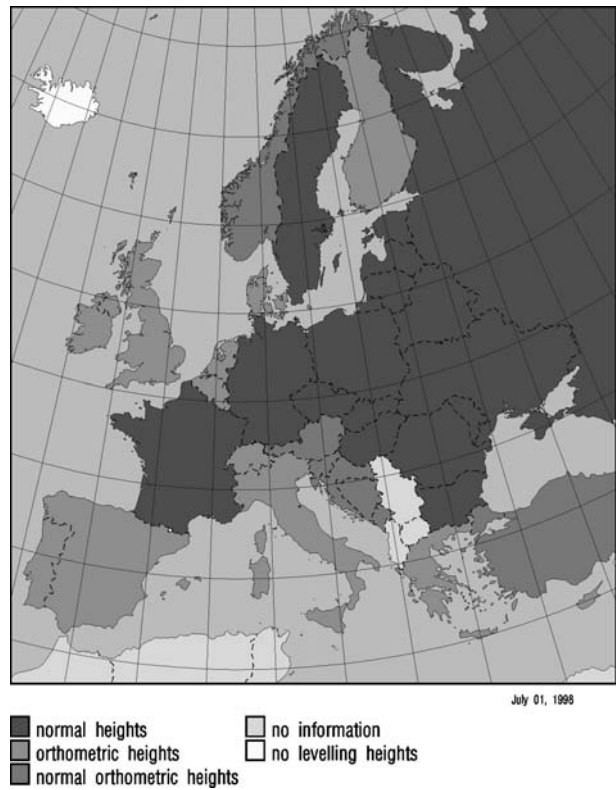


Figure 5: Differences between EVRF2000 zero level and the zero levels of national height systems in Europe (in cm)

The current situation of national height systems in Europe is characterised by Figure 3 and Figure 4. Figure 5 shows the distribution of the mean transformation parameters from the national height systems to the EVRF2000.

The following table summarizes the information about the relations between the EVRF2000 zero level and the zero levels of national height systems in Europe.

**Preliminary Transformation Parameters from National European Height Systems to the EVRF2000**

Country / (country code)	Reference Tide Gauge	Kind of Heights	UELN-Nat. Height in cm	min/max in cm	No. of Identical Points
Albania	Durres				
Austria	Trieste	NOH	-35	- 25/- 48	100 (UELN)
Belgium	Ostend	OH	-231	- 230/- 232	4 (EUVN)
Belorussia	Kronstadt	NO	+ 15		
Bosnia/Herzegovina	Trieste	NOH	-34	- 33/- 34	5 (UELN)
Bulgaria	Kronstadt	NH	+ 15		
Croatia	Trieste	NOH	-33	- 32/- 35	4 (UELN)
Czech Republic	Kronstadt	NH	+ 11	+ 8/+ 16	53 (UELN)
Denmark	10 Danish tide gauges	OH	+ 2	+ 1/+ 3	733 (UELN)
Estonia	Kronstadt	NH	+ 13	+ 12/+ 15	36 (UELN)
Finland	Helsinki	OH	+ 22	+ 22/+ 23	8 (EUVN)
France	Marseille	NH	-49	- 48/- 49	7 (EUVN)
Germany	Amsterdam	NH	+ 1	+ 1/+ 2	431 (UELN)
Greece					
Hungary	Kronstadt	NH	+ 14	+ 13/+ 14	4 (EUVN)
Iceland			no levelling heights		
Ireland	Malin Head	OH			
Italy	Genoa	OH	-35	- 33/- 36	11 (EUVN)
Latvia	Kronstadt	NH	+ 10	+ 8/+ 12	124 (UELN)
Lithuania	Kronstadt	NH	+ 14	+ 13/+ 14	46 (UELN)
FYR of Macedonia	Trieste	OH			
Moldavia	Kronstadt	NH	+ 15		
Netherlands	Amsterdam	OH	-1	0/- 3	758 (UELN)
Norway	Tregde	NOH	0	- 7/+ 10	10 (EUVN)
Poland	Kronstadt	NH	+ 16	+ 14/+ 18	117 (UELN)
Portugal	Cascais	OH	-32	- 29/- 33	5 (EUVN)
Romania	Constanta	NH	+ 3	+ 2/+ 4	64 (UELN)
Russia	Kronstadt	NH	+ 15		
Slovakia	Kronstadt	NH	+ 12	+ 11/+ 13	3 (EUVN)
Slovenia	Trieste	NOH	-33	- 33/- 34	9 (UELN)
Spain	Alicante	OH	-50	- 47/- 52	7 (UELN)
Sweden	Amsterdam	NH	+ 3	0/+ 6	11 (EUVN)
Switzerland	Marseille	OH	- 35	- 16/- 56	7 (EUVN)
		(NH)	- 17	- 15/- 22	
Turkey	Antalya	OH			
Ukraine	Kronstadt	NH	+ 15		
United Kingdom	Newlyn	OH	+ 2	+ 12/- 5	5 (EUVN)
Yugoslavia	Trieste	NOH			

(NH ... normal heights, NOH ... normal-orthometric heights, OH ... orthometric heights)

## A 2 European Spatial Reference System

### A 2.1 European Vertical Reference Network (EUVN)

The initial practical objective of the EUVN project is to unify different European height datums within few centimeters. The EUVN project contributes to the realization of a European vertical datum and to the connection of different sea levels of European oceans with respect to the work of PSMSL (Permanent Service of Mean Sea Level) and of anticipated accelerated sea level rise due to global warming. The project provides a contribution to the determination of an absolute world height system.

At all EUVN points  $P$  three-dimensional coordinates in the ETRS89  $(X_p, Y_p, Z_p)_{ETRS}$  and geopotential numbers  $c_p = W_o_{UELN} - W_p$  will be derived. Finally the EUVN is representing a geometrical-physical reference frame. In addition to the geopotential numbers  $c_p$  normal heights

$$H_n = c_p / \bar{g} \text{ will be provided.}$$

In total the EUVN consists of 196 sites: 66 EUREF and 13 national permanent sites, 54 UELN and UPLN (United Precise Levelling Network of Central and Eastern Europe) stations and 63 tide gauges (Figure 4).

The final GPS solution was constrained to ITRF96 coordinates (epoch 1997.4) of 37 stations with an a-priori standard deviation of 0.01 mm for each coordinate component. As a consequence of these tight constraints the resulting coordinates of the reference points are virtually identical with the ITRF96 values. To get conformity with other projects, the general relations were used to transform the ITRS coordinates to ETRS. The coordinate transform formula from ITRF96 to ETRF96 and the final coordinates are given in INEICHEN et al 1999.



- |                                  |   |
|----------------------------------|---|
| ▲ EUREF sites                    | ⊙ GPS permanent stations - nodal points |
| ▲ GPS permanent stations - EUREF | ◆ Tide gauge sites                      |
| △ GPS permanent stations         | ⊙ GPS permanent stations - tide gauge   |
| ● UELN & UPLN nodal points       | ∕ UELN lines                            |

Figure 6: Distribution of EUVN stations



In order to reach the goal it is necessary to connect the EUVN stations by levellings to nodal points of the UELN 95/98 network. The geopotential numbers are related to the EVRS2000 zero level. As the EUVN is a static height network it is necessary to know the value of the mean sea level in relation to the tide gauge bench mark at the epoch of EUVN GPS campaign 1997.5.

The Permanent Service for Mean Sea Level (PSMSL) as member of the Federation of the Astronomical and Geophysical Data Analysis Service (FAGS) is in principle in charge of the data collection. The information which is sent to the PSMSL databank is available for the EUVN project.

## A 2.2 A Kinematic European Vertical System (EVS)

The European Vertical System is planned as geokinematic height network as combination of the European GPS permanent station network, the UELN with repeated levellings, the European gravimetric geoid and tide gauge measurements along European coast lines as well as repeated gravity measurements. In May 1999 a special working group was formed to determine the direction of future work. At the first working group meeting three first tasks were established:

- analysis of available repeated levelling measurements and storage in the data base of the UELN
- development of software as base for test computation
- testing of the principles in a test area (Netherlands, Denmark, northern part of Germany).

The GPS observations of about 80 European permanent stations are available. The analysis of 10 European GPS permanent stations shows daily repeatabilities between 7 to 9 mm in the height component. This is in good agreement with the special GPS height campaigns in Germany for deriving GPS levelling geoidal heights ( $m_h = \pm 7$  mm).

Furthermore the linear height regression analysis gives for a three year period an accuracy of a GPS height difference of about

$$m_{v_i} = m_h \sqrt{2} / \sqrt{365} / \text{year} = \pm 0.5 \text{ mm/year},$$

that means from a statistical point of view that a vertical movement of  $V_h = 1.0$  mm/year can be significantly determined after a three years GPS observation period

$$(m_{V_h} = \pm 0.3 \text{ mm/year}).$$

Repeated precise levellings ( $1 \text{ mm} \times \text{km}^{-1/2}$ ) with an epoch difference of 20 years give velocities for height differences with an accuracy of about  $\pm 0.07 \text{ mm} \times \text{km}^{-1/2} / \text{year}$ .

From this follows, that GPS permanent stations in a distance of about 300 km can significantly support repeated levellings with above mentioned suppositions. This combination of GPS and levelling is promising for a stable kinematic height reference system (IHDE, 1999).

The observation equation for levelling observations  $Dh_{ij,k}$  between points  $i$  and  $j$  at the epoch  $k$  is:

$$\Delta h_{ij,k} = H_j - H_i + V_j(t_k - t_0) - V_i(t_k - t_0). \quad (1)$$

Two unknowns per point are to be determined: the levelling height  $H$  (gravity related height) at the reference epoch  $t_0$  and the velocity  $V$ .

For datum fixing of the network a height for one point at a determined epoch and a velocity for this or another point shall be given.

The relation between levelling heights  $H$  and GPS heights  $h$  is given by the geoid height  $N$

$$h = H + N. \quad (2)$$

Since the accuracy of the geoid heights resp. geoid height differences is not in the same order like the levelling observations, GPS heights cannot be used as observations. But under the condition of no significant geoid height changes, velocities  $v$  derived from GPS permanent station observations can be used as additional observation type in levelling points  $I$

$$v_i = V_i \quad (3)$$

The unknown velocities  $V$  are to be determined in combination with the repeated levellings. It is necessary, that the variance-covariance matrix of the observed GPS velocities is given.

The EVS project has been started in 1999. It would be useful to integrate

- precise absolute gravity measurements
- sea level monitoring in tide gauge stations.

## A 3 Geodetic Reference System 1980 (GRS80)

Excerpt from H. MORITZ: *Geodetic Reference System 1980* Bulletin Géodésique, The Geodesists Handbook, 1988, International Union of Geodesy and Geophysics

### A 3.1 Definition

The GRS80

a) based on the theory of the geocentric equipotential ellipsoid, defined by the following conventional constants:

- equatorial radius of the Earth:

$$a = 6378\,137 \text{ m},$$

- geocentric gravitational constant of the Earth (including the atmosphere):

$$GM = 3986\,005 \times 10^8 \text{ m}^3 \text{ s}^{-2},$$

- dynamical form factor of the Earth, excluding the permanent tidal deformation:

$$J_2 = 108\,263 \times 10^{-8},$$

- angular velocity of the Earth:

$$\omega = 7292\,115 \times 10^{-11} \text{ rad s}^{-1},$$

a) used the same computational formulas, adopted at the XV General Assembly of IUGG in Moscow 1971 and published by IAG, for the Geodetic Reference System 1967,

b) is orientated in such kind, that the minor axis of the reference ellipsoid, defined above, be parallel to the direction defined by the Conventional International Origin, and that the primary meridian be parallel to the zero meridian of the BIH adopted longitudes.

### A 3.2 Numerical Values

#### Derived Geometrical Constants

$b = 6\,356\,752.3141\text{ m}$  semiminor axis

$e^2 = 0.006\,694\,380\,022\,90$   $e$  = first excentricity

$f = 0.003\,352\,810\,681\,18$  flattening

#### Derived Physical Constants

$U_0 = 6\,263\,686.0850 \times 10^2 \text{ m}^2 \text{ s}^{-2}$  normal potential at ellipsoid

$m = 0.003\,449\,786\,003\,08$   $m = \omega^2 a^2 b / GM$

$g_e = 9.780\,326\,7715 \text{ ms}^{-2}$  normal gravity at equator

$g_p = 9.832\,186\,3685 \text{ m} \cdot \text{s}^{-2}$  normal gravity at pole

$f^* = 0.005\,302\,440\,112$   $f^* = (g_p - g_e) / g_e$

$k = 0.001\,931\,851\,353$   $k = (b g_e - a g_e) / a g_e$

### A 3.3 Gravity Formula 1980

Somigliana's closed formula for normal gravity is

$$\gamma_o = \frac{a \gamma_e \cos^2 \phi + b \gamma_p \sin^2 \phi}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}}$$

For numerical computations, the form

$$\gamma_o = \gamma_e \frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}},$$

with the values of  $g_e$ ,  $k$ , and  $e^2$  shown above, is more convenient.  $f$  denotes the geographical latitude.

The series expansion

$$\gamma_o = \gamma_e \left( 1 + \sum_{n=1}^{\infty} a_{2n} \sin^{2n} \phi \right)$$

with

$$\dots \dots \dots a_2 = \frac{1}{2} e^2 + k \quad a_6 = \frac{5}{16} e^6 + \frac{3}{8} e^4 k$$

$$a_4 = \frac{3}{8} e^4 + \frac{1}{2} e^2 k$$

$$a_8 = \frac{35}{128} e^8 + \frac{5}{16} e^6 k$$

becomes

$$\begin{aligned} \gamma_o = \gamma_e & (1 + 0.0052790414 \sin^2 \phi \\ & + 0.0000232718 \sin^4 \phi \\ & + 0.0000001262 \sin^6 \phi \\ & + 0.0000000007 \sin^8 \phi); \end{aligned}$$

it has a relative error of  $10^{-10}$ , corresponding to  $10^{-3} \mu\text{ms}^{-2} = 10^{-4} \text{ mgal}$ .

The conventional series

$$\gamma_o = \gamma_e \left( 1 + f^* \sin^2 \phi - \frac{1}{4} f_4 \sin^2 2\phi \right)$$

with

$$f_4 = -\frac{1}{2} f^2 + \frac{5}{2} f m$$

becomes

$$\gamma_o = 9.780327 \left( \begin{array}{l} 1 + 0.0053024 \sin^2 \phi \\ - 0.0000058 \sin^2 2\phi \end{array} \right) \text{ms}^{-2}$$

### 3.4 Origin and Orientation of the Reference System

IUGG Resolution No. 7, quoted at the beginning of this paper, specifies that the Geodetic Reference System 1980 be geocentric, that is, that its origin be the center of mass of the earth. Thus, the center of the ellipsoid coincides with the geocenter.

The orientation of the system is specified in the following way. The rotation axis of the reference ellipsoid is to have the direction of the Conventional International Origin for Polar Motion (CIO), and the zero meridian as defined by the Bureau International de l'Heure (BIH) is used.

To this definition there corresponds a rectangular coordinate system XYZ whose origin is the geocenter, whose Z-axis is the rotation axis of the reference ellipsoid, defined by the direction of CIO, and whose X-axis passes through the zero meridian according to the BIH.

### A 4 Related Resolutions

(1) Resolution No. 3 of the EUREF Symposium in Warsaw, 8 -11 June 1994

The IAG Subcommission for the European Reference Frame recognizing the close relationship of vertical datum problems to EUREF activities and

considering the proposal of the EUREF Technical Working Group to respond to an urgent request of CERCO for a European Vertical Datum at the 0.1 m level

recommends

- that the Technical Working Group undertakes action and reports at the next meeting
- an enlargement of UELN to Eastern Europe for this purpose

requests the Eastern European agencies to make their national data available for UELN-CRCM Data Centre at Hanover within 1994.

(1) Resolution No. 2 of the EUREF Symposium in Helsinki, 3 – 6 May 1995

The IAG Subcommittee for the European Reference Frame *noting* the resolution No. 3 of the EUREF Warsaw Symposium in 1994 and

*taking into account* the principals of EPTN and EUVERN proposals presented during this meeting

*recommends* that a European Vertical Reference Network (EUVN) should be defined as part of the EUREF network with stations co-located with the European levelling or tide gauge networks

*asks* the EUREF Technical Working Group to organize the determination of the EUVN:

- by co-ordinating as many EUREF permanent GPS stations as possible
- by implementing a suitable GPS campaign to obtain a first epoch determination of all the EUVN stations as soon as possible.

(1) Resolution No. 2 of the EUREF Symposium in Ankara, 22 – 25 May 1996

The IAG Subcommittee for Europe (EUREF)

*recognized* that this Subcommittee includes the responsibilities of the former UELN Subcommittee

*and noting* the increasing need for a unified European height system at the decimetre level

*decides* to realise such a system through the conversion of the future UELN95 results from geopotential numbers to normal heights.

(2)

(1) Resolution No. 3 of the EUREF Symposium in Ankara, 22 – 25 May 1996

The IAG Subcommittee for Europe (EUREF)

*noting* the efforts of the European Vertical GPS Reference Network (EUVN) Working Group

*endorses* their proposal to have a GPS campaign between the 21 and 29 of May, 1997

*and urges* all EUREF member countries to make their best endeavours in ensuring the success of this campaign.

(2) Resolution No. 4 of the EUREF Symposium in Ankara, 22 – 25 May 1996

The IAG Subcommittee for Europe (EUREF)

*recognizing* the progress of UELN95, the forthcoming EUVN GPS Campaign, and the requirements for a continental vertical reference system at the centimetre level

*decides* to develop a new European geokinematic height reference network with all available kinematic observations (e.g. GPS, levelling, tide gauges, gravity)

*urges* all EUREF member countries to deliver relevant data to the data centre, Institut für Angewandte Geodäsie (IfAG)

*and asks* the Technical Working Group to form a special Working Group to oversee the development of the computation method and methodologies.

(3) Resolution No. 3 of the EUREF Symposium in Bad Neuenahr – Ahrweiler, 10 – 13 June 1998

The IAG Subcommittee for Europe (EUREF)

*recognizing* the outstanding success of the European Vertical Reference Network 97 (EUVN97) GPS Campaign

*thanks* the EUVN working group and all the contributors to the campaign

*accepts* the adjustment presented at the symposium and asks the Technical Working Group to derive the final EUVN 97 GPS co-ordinates from this adjustment and

*urges* all EUREF member countries to submit the requested levelling/gravity and tide gauge data, to the data centre in order to achieve the EUVN objectives.

(4) Resolution No. 4 of the EUREF Symposium in Bad Neuenahr – Ahrweiler, 10 – 13 June 1998

The IAG Subcommittee for Europe (EUREF)

*recognizing* the progress of the UELN95 project work

*asks* the data centre and Technical Working Group, to make the solution presented at the symposium, available as the UELN98 solution and

*urgently requests* the participating countries to make the missing levelling data available, particularly to extend and improve the vertical network to the Black Sea, around the Baltic Sea and including the channel tunnel connection between France and UK.

(5) Resolution No. 1 of the EUREF Symposium in Prague, 2 – 5 June 1999

The IAG Subcommittee for Europe (EUREF)

*noting* resolution 3 of the EUREF Symposium 1998 in Bad Neuenahr – Ahrweiler

*accepts* the GPS frame of the European Vertical Reference Network 1997 (EUVN97) as class B standard (about 1 cm at the epoch of observation), and

*endorses* these results as improvements and extensions to EUREF89.

(6) Resolution No. 5 of the EUREF Symposium in Prague, 2 – 5 June 1999

The IAG Subcommittee for Europe (EUREF)

*recognizing* the progress in the UELN95 and EUVN as static height networks,

*accepts* the concept of an integrated kinematic height network for Europe proposed by the Technical Working Group (e.g. GPS permanent stations, repeated levellings, tide gauge observations, repeated gravity measurements)

*asks* the Technical Working Group to send a circular letter to the EUREF community detailing the proposal and requirements, and seeking participation in all topics (measurements, computing centre, test area).

(7) Resolution No. 3 of the EUREF Symposium in Tromsø, 22 – 24 June 1999

The IAG Subcommission for Europe (EUREF)

noting resolution 3 of the EUREF Symposium 1998 in Bad Neuenahr-Ahrweiler,

recognizing the completion of the EUVN height solution, which includes GPS/levelling geoid heights,

thanks the National Mapping Agencies for their support in supplying data,

recommends that the GPS/levelling geoid heights of the EUVN solution should be used as fiducial control for future European geoid determinations,

asks the relevant authorities to provide the necessary information for tide gauge connections, to densify the network of EUVN GPS/levelling geoid heights and to complete and extend the EUVN project.

(8) Resolution No. 5 of the EUREF Symposium in Tromsø, 22 – 24 June 1999

The IAG Subcommission for Europe (EUREF)

noting the recommendation of the spatial referencing workshop, in Marne-la-Vallée 27-30 November 1999, to the European Commission to adopt the results of the EUVN/UELN projects for Europe wide vertical referencing,

decides to define an European Vertical Reference System (EVRS) characterised by the datum of 'Normaal Amsterdams Peil' (NAP) and gravity potential differences with respect to NAP or equivalent normal heights,

endorses UELN95/98 and EUVN as realisations of EVRS using the name EVRF2000,

asks the EUREF Technical Working Group to finalise the definition and initial realisation of the EVRS and to make available a document describing the system.

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## A Note on the Treatment of the Permanent Tidal Effect in the European Vertical Reference System (EVRS)

J. MÄKINEN<sup>5</sup>

*Submitted to the EUREF Technical Working Group, August 14, 2000*

1. There is abundant literature on the permanent tide in recent years, see for instance POUTANEN et al. (1996), EKMAN (1996), MATHEWS (1999), and their references. This note does not contain any novelties, it is just a compilation of pertinent results.
2. The time average of the tidal forces is not zero, and thus they induce a permanent deformation of the Earth (they increase its flattening).
3. In making the tidal correction to purely geometric coordinates (i.e., quantities that are not defined through gravity, say, Cartesian equatorial coordinates), only two concepts are needed to handle the situation. We either retain the permanent tidal deformation of the Earth (resulting in the *mean crust* or *zero crust* in the terminology of EKMAN (1989) or eliminate it (resulting in the *tide-free* or *non-tidal crust*).
4. With the gravity field, the situation is more complicated, as we then have three alternatives. We may retain both the time average of the tidal forces and the gravity effect of the permanent tidal deformation of the Earth. This is called mean gravity. We may eliminate the time average of the tidal forces, but keep the gravity effect of the permanent tidal deformation of the Earth. This is called zero gravity. Finally, we may eliminate both the time average of the tidal forces, and the gravity effect of the permanent tidal deformation. This is tide-free gravity. The corresponding equipotential surfaces are the mean geoid, zero geoid, and the tide-free geoid.
5. For gravity and related quantities, the *mean* and *zero* concepts are fundamentally different. For the crust, the two names *mean crust* and *zero crust* point at the same concept and the two names are used because this single concept is the logical partner of both mean gravity and zero gravity. Note that in (MCCARTHY, 1996), p. 11, the *mean* concepts are interpreted erroneously.
6. The Standard Earth Tide Committee (RAPP 1983) recommended that *The direct constant part of the tidal effect should be removed from the observed gravity but the indirect part due to the permanent yielding of the earth should not be.* They continued: *For other observations, needing precise tidal reductions, such as satellite altimeter data, laser tracking data, VLBI measurements, levelling, etc., we recommend that analogous procedures be adopted....*
7. Two resolutions of the IAG at the XVIII General Assembly in 1983 took up the question. Resolution No. 9 takes up gravity, and Resolution No. 16 tidal corrections in general. It is perhaps useful to reproduce here their relevant parts. Resolution No 9 states (TSCHERNING, 1984, p. 303 ff.)
 

*The International Association of Geodesy*  
*recognizing* the high level of accuracy of both absolute and relative gravity measurements recently attained  
*considering* the necessity to adopt standard corrections to gravity observation in order to allow intercomparisons between measurements at different epochs of time  
*recommends*

  1. that the tidal correction applied to the gravity observations follow the final recommendations of the Standard Earth Tide Committee as presented at the XVIII IUGG General Assembly
  2. (concerns the atmospheric pressure correction)
  3. (concerns the gravity gradient correction)

Resolution No. 16  
*The International Association of Geodesy*  
*recognizing* the need for the uniform treatment of tidal corrections for various geodetic quantities such as gravity and station positions, and  
*considering* the reports of the Standard Earth Tide Committee and S.S.G. 2.55, Predictive Methods for Space Techniques, presented at the XVIII General Assembly,  
*recommends* that:

  3. the indirect effect due to the permanent yielding of the Earth be not removed
8. Thus the IAG Resolutions recommend mean (= zero) crust and zero gravity. Splitting hairs, one might claim that the mean geoid is not excluded by them, as Resolution No. 16 only excludes the non-tidal quantities, and Resolution No. 9 which excludes mean gravity is drafted in the context of gravity observations and does not

<sup>5</sup> Jaakko Mäkinen: Finnish Geodetic Institute, Geodeetinrinne, FIN-02430 Masala, Finland; E-mail: Jaakko.Makinen@fgi.fi

mention other quantities of the gravity field, the potential for instance. Most geodesists however seem to assume that by logical extension it applies to them as well.

9. The gravimetric community has observed the resolution very well: It seems that all new gravity networks are in the zero tidal system. Of course, old systems like the IGSN71 (with mean gravity) still exist.
10. Despite some contradictions noted by POUTANEN et al (1996), the IERS 1992 standards (MCCARTHY, 1992) recommend the zero crust. The 1996 standards unequivocally recommend zero crust and zero potential (MCCARTHY, 1996).
11. However, the recommendations have not always been followed in practice. Thus ITRF coordinates and the ETRS89 refer to the non-tidal crust. Similarly, many spherical harmonic expansions of the geopotential have a non-tidal second degree zonal coefficient ( $J_2$  or  $C_{20}$ ). This is then inherited by regional geoids which normally derive their long-wavelength part from such expansions.
12. Most regional geoids, however, are post-fitted to the separation between levelled heights and ellipsoidal heights derived from space geodetic methods (mostly GPS). If the permanent tide has not been consistently handled in them, the situation may become quite opaque.
13. It appears that no exhaustive information on the treatment of the permanent tide in the levelling data input to UELN is available. A likely guess would be that in most data, no tidal correction at all was applied. The elevation differences then refer approximately to the height of the mean crust above the mean geoid. This is the case, e.g., for Finland and Norway. For some countries like Sweden and Denmark we know that the non-tidal quantities were used. The tidal system of UELN-95/98 is therefore vague.
14. The European Gravimetric Quasigeoid EGG97 (DENKER and TORGE, 1997) derives its low-frequency part from the EGM96, which is non-tidal (LEMOINE et al., 1998). However, in the computation the geoid was corrected to the zero tidal system (H. DENKER, personal information).
15. Thus the positions in ETRS89 are in the non-tidal system, the EGG97 geoid heights are in the zero tidal system, while the UELN-95/98 is a mixture. From the documents distributed at the EUREF2000 meeting at Tromsø it is not obvious whether these differences were considered when the EUVN GPS/levelling quasigeoid is compared with the EGG97 in Table 2 of IHDE et al. (2000).
16. Let us for the sake of argument assume that they were not considered. Formulas for the differences between the tidal concepts are given by EKMAN (1989). Calculating with them one finds that the quantity  $(EGG97-(h-H))$  in the last column of Table 2 of Ihde et al. (2000) then contains an N-S tilt bias of

$$B_m(\varphi_1, \varphi_2) = 29.6 \times (1-h)(\sin^2 \varphi_2 - \sin^2 \varphi_1) \text{ cm}$$

if the levelling data can be assumed to be in the mean tidal system, and a tilt of

$$B_f(\varphi_1, \varphi_2) = 29.6 \times (1-h-\gamma) \sin^2 \varphi_2 - \sin^2 \varphi_1 \text{ cm}$$

$$= 29.6 \times (-h+h'-k')(\sin^2 \varphi_2 - \sin^2 \varphi_1) \text{ cm}$$

if the levelling data is in the tide-free system. Here  $\varphi_1, \varphi_2$  are the latitudes of two bench marks,  $h$  is the Love number used for the permanent tide in the processing

which produced ETRF89, and  $\gamma = 1+k'-h'$  is the *gamma-factor* used for the permanent tide in the

levelling. The  $B_m(\varphi_1, \varphi_2)$ ,  $B_f(\varphi_1, \varphi_2)$  should be subtracted from the difference  $(EGG97-(h-H))_2 - (EGG97-(h-H))_1$ .

Using nominal values  $h = h' = 0.6$   $k' = 0.3$  we find between Catania ( $\varphi_1 = 37^\circ 30'$ ) and Vardoe ( $\varphi_2 = 70^\circ 23'$ )

$$B_m(\varphi_1, \varphi_2) = +6.1 \text{ cm}$$

$$B_f(\varphi_1, \varphi_2) = -4.6 \text{ cm}$$

17. What kind of heights should we choose for the EVRS (European Vertical Reference System)? This is implemented through the interpretation of the words *gravity potential differences with respect to NAP* in the Resolution No. 5 of the EUREF2000 Symposium in Tromsø. By choosing what is included in the gravity potential, we can get the non-tidal crust over the non-tidal geoid, the mean/zero crust over the zero geoid, or the mean/zero crust over the mean geoid.
18. One desirable goal is the consistency of all geodetic quantities (EKMAN, 1996). However, since we have a non-tidal ETRS89, zero geoid and zero gravity, this cannot be achieved by choosing the system for heights. It seems unlikely that ETRS users would agree to change to the zero system. Going to a non-tidal geoid and gravity would be a step backwards and find few supporters.
19. The simple choice would be to heed the IAG resolutions of 1983 and interpret accordingly the *gravity potential* to exclude the time average of the direct tidal potential but include the potential of the permanent tidal deformation. This would give us the mean/zero crust over the zero geoid, and a consistent system to calculate the geoid (but the comparison with ETRS would still be biased).
20. The drawback would be that in this system the mean sea level (MSL), even in the absence of any sea surface topography (SST) would be tilted in the N-S direction (EKMAN, 1989). The tilt would be
 
$$MSL(\varphi_2) - MSL(\varphi_1) = -29.6 \times (\sin^2 \varphi_2 - \sin^2 \varphi_1) \text{ cm}$$
 or -15.4 cm from Catania to Vardoe. A similar N-S-tilt would appear in the national systems of mean heights

as seen from the UELN-95/98.

21. The question is, would anybody except the oceanographers care much, and they can always make the correction.
22. On the other hand, if we adopt the more *realistic* mean crust over mean geoid, then one consequence is that such heights cannot be used to calculate (rigorously) gravity anomalies as an input to the computation of the zero geoid.
23. At this stage, we are formally picking a solution for the GIS community with limited accuracy requirements. But the decision will probably have much farther-reaching consequences, and should be thoroughly debated in advance, to gain the widest possible acceptance.

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# On the Permanent Tide in the European Vertical Reference System

M. EKMAN<sup>6</sup>

*Submitted to the EUREF Technical Working Group, August 18, 2000*

I have received a copy of a clearly written summary note on the permanent tide in the European Vertical Reference System (EVRS), sent to the EUREF Technical Working Group by Jaakko Mäkinen (MÄKINEN, 2000). Based on his information there as well as my own and other scientific papers on the subject, I would like to give my personal view on how to handle the problem. This view is governed by my wish to find a solution that is as consistent and useful as possible in all kinds of geodetic reference systems and Earth parameters. Although it is not possible to achieve such a consistency immediately because of the strange mixture of solutions already in use, one should aim at it, and avoid

selecting a European solution that runs the risk of becoming out of phase with a future global one.

Very briefly, the main problems on a global scale with the three different tidal solutions are the following.

1. The tide-free (non-tidal) solution is clearly unsuitable for fundamental physical reasons, like inconsistency with the length of the day and impossibility to determine its true deformation of the Earth.
2. The mean solution requires a revised theory of the equipotential ellipsoid and would still not work outside

<sup>1</sup> Martin Ekman: Winter Office for Geodynamics, Sweden and Summer Institute for Historical Geophysics, Åland; E-mail: martin.ekman@zeta.telenordia.se

the Earth, e.g. in satellite orbit analysis (where the zero solution would be the proper one).

3. The zero solution needs just a simple transformation (to the mean one) when making oceanographic interpretations of the sea surface topography.

I find that the zero solution definitely is the one that minimizes the complications. In total, therefore, I prefer the zero solution (as already stated in EKMAN, 1996) and would strongly recommend it for use also in the EVRS.

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## Some Critical Remarks on the Definition and Realization of the EVRS

R. RUMMEL<sup>7</sup>, B. HECK<sup>8</sup>

*Submitted to the EUREF Technical Working Group, January 2001*

In the past, the definition of a vertical reference system has been synonymous with the definition of a vertical datum. The latter, in turn, is equivalent to defining (or fixing) the minimum number of parameters required for a unique determination of "co-ordinates". In the case of a vertical datum it is the definition of a reference height (geopotential number) at a chosen reference bench mark, in order to be able to derive heights (geopotential numbers) for any other point based on the measured potential differences. The realization of a vertical datum would be equivalent to the adoption of height values (geopotential numbers) of a selected set of well-defined and documented bench marks. The height datum definition and realisation (= frame) would have to be accompanied by error measures, i.e. standard deviations of height values (geopotential numbers). Their value would steadily increase with increasing distance from the datum point, being zero at the selected reference bench mark. By S-transformation the height values (geopotential numbers) as well as the error measures could be transformed to any other reference point of choice. Thus, there is no "intrinsic" priority of e.g. the adopted reference point Amsterdam. It can be transformed to any other choice, whether for example Marseille or Bern. A description is given in (RUMMEL and TEUNISSEN, 1988).

For this traditional selection of a vertical reference system neither an absolute value  $W_0$  of the potential nor sea surface topography would play any role (Permanent tide must be considered only in the context of the so-called "astronomical correction" of levelling providing the tidal reduction of the

gravity potential to a quasi-stationary state, see e.g. HECK, 1993). Also in the future a vertical reference system of this type, based on levelling and gravimetry, will maintain its prominent role. Therefore a datum definition and realization in the above sense should be given the necessary attention in the definition of the EVRS.

In recent years new measurement techniques entered the picture, in particular the precise determination of co-ordinate differences by GPS. Also, in view of the running space mission CHAMP and the up-coming dedicated gravity field satellite missions GRACE and GOCE, we should be aware that very accurate and detailed global gravity field models will become available. Thus, it is worthwhile to assess how one can benefit from these developments for the determination of heights and height changes.

It is well known that (1) levelling over large distances is subject to systematic errors and (2) levelling is not really able to provide a reliable picture of temporal changes in height, e.g. caused by vertical crustal motion or coastal subsidence. In particular, levelling over larger distances is so time consuming that it is difficult to assign unique epochs to them. Finally, (3) geometrical levelling is principally restricted to continental areas, excluding any connections between bench marks situated on different continents or islands.

Thus one will conclude that for **the monitoring of temporal height changes GPS is a viable alternative**. The amounts of height changes are so small that a distinction between

<sup>7</sup> Reiner Rummel: Institut für Astronomische und Physikalische Geodäsie, TU München, Arcisstr. 21, 80333 München; E-mail: rummel@bv.tum.de

<sup>8</sup> Bernhard Heck: Geodätisches Institut, Universität Karlsruhe, Englerstr. 7, 76128 Karlsruhe; E-mail: heck@gik.uni-karlsruhe.de



a purely geometric height change (from GPS) and a physical one (from levelling & gravimetry) is irrelevant. It has been shown by HECK AND MÄLZER (1983) in a case study that the differences between both types of height changes are less than 10%. All that is needed for a definition of a vertical reference system for the monitoring of temporal changes is the definition (choice) of the datum and epoch in which the GPS co-ordinate differences are expressed. The realisation (= frame) requires the adoption of well defined and reproducible bench marks for which geometric height differences are listed as well as their epoch. All differences could again be expressed with respect to the reference height bench mark at Dam, Amsterdam, if this point is accessible to GPS and has been included in the previous *GPS-campaigns*. *But any other choice would be equally valid.*

The availability of points with precise 3-D GPS co-ordinates as well as - in the near future - precise and high resolution gravity models opens the possibility to compute directly potential differences between these points. **One should keep in mind - however - that so far this approach cannot provide potential or height differences inside Europe comparable in accuracy with those obtained from levelling and gravimetry.** Nevertheless, if it is the intention of the EVRS to prepare already now for this future possibility one should do it with greatest care so as to avoid confusion and mis-interpretation.

In this case the formula of the series expansion of spherical harmonic coefficients, to be applied, should be given together with the needed constants and explanations on how to use it, to ensure that all European users apply the same model. Apart from the formula itself, this implies that

- the series coefficients are referring to the same terrestrial reference system as the GPS-coordinates,
- the same normal gravity field, e.g. GRS80, is used consistently,
- sea surface topography plays no role, if the reference system is to be confined to the European continent.

**Absolute potential value  $W_0$ :** In practise only potential differences are measurable. This is just a fact, like in electricity. Thus there cannot be any practically relevant situation thought up for which an absolute potential value is needed (you may try to counter-proof this statement!).

Nevertheless, from the solution of the geodetic boundary value problem an absolute  $W_0$  can be determined for the Earth, under the assumption that the gravitational potential converges towards zero for a point running towards infinity

(regularity condition). This means, in fact, that the difference on the gravitational potential between the Earth's surface and a point at infinity is determined (wherever infinity may be and assuming a completely empty universe). But, to repeat it again, this value  $W_0$  is needed nowhere in practise (For a deeper discussion of the difficulties concerning an absolute vertical datum see e.g. HECK and RUMMEL, 1990). Thus, it should not take any prominent place in a definition of an European Vertical Reference System, for it would confuse people.

In detail, it has been proposed in IAG Comm. X (2000) to identify the constant  $W_0$  with the normal potential  $U_0$  on the surface of a level ellipsoid. In this way a specific level surface is selected which depends on the choice of the four parameters of a Somigliana-Pizzetti normal field; any change of these parameters due to an improved knowledge in future will produce changes in  $U_0$ , thus shifting the reference surface  $W = W_0$ . Furthermore, there exists a **discrepancy between the definition  $W_0 = U_0$  and its realization**

$$W_{NAP}^{REAL} = U_{0GRS80}$$

since it cannot be guaranteed that the respective level surface  $W_0 = U_{0GRS80}$  runs through NAP. This inconsistency may give rise to further confusion and provides another argument against fixing any absolute potential value  $W_0$ , which is completely irrelevant in practise.

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