Unification of height systems using GNSS-leveling data and global satellite gravity models

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The context

- The realization of the International Height Reference System has been recognized as one important topic for IAG (a resolution on that will be presented at the next IAG/IUGG in Prague)

- In GGOS we have Theme 1 – Unified Height System

- We have new detailed global geopotential models based on satellite observations only (e.g. GOCE)

- GNSS techniques can provide highly accurate estimate of ellipsoidal heights

Merging different height systems at regional/global scale
The normal height

\[ dh = \Delta U = U_0 \quad U_Q = W_0 \quad W_P \]

\[ h^* = \frac{1}{Q} \quad dh = \frac{W_0}{Q} \quad W_P \]

\[ - \frac{1}{Q} = \frac{1}{h^*} \quad dh \]
The normal height at a tide gauge point

\[ h^*_p = \frac{W_0}{p^j_0} - \frac{W_{p^j}}{p^j_0} = \frac{W_0}{p^j_0} \left( \frac{W_0 + W_j}{p^j_0} \right) = \frac{W_j}{p^j_0} = b_{p^j_0} \]

\[ W = W_{p^j} \]

\[ W = W_0 \]

\[ P^j_0 = \text{tide gauge point for the } j\text{th domain (patch)} \]
The biased normal height

In the jth patch normal heights are biased and one can write the equation

\[ h \tilde{h}^j = h \quad h^j + b^j = \quad + b^j = T + b^j \]

\[ h = \text{ellipsoidal height (GNSS)} \]

\[ \tilde{h}^j = \text{biased normal height in the jth patch (spirit leveling)} \]

\[ = T \]

\[ b^j = \text{bias of the jth patch} \]
The observation equation for estimating $b^j$

$$\tilde{\eta} = h \quad \tilde{h}^j = \frac{T}{\rho} + b^j$$

$$b^j = \tilde{\eta} (P^j_k) \quad \frac{T(P^j_k)}{(P^j_k)}$$

$P^j_k = \text{observation point } k \text{ in the } j\text{th patch}$
The observation equation: the anomalous potential problem

\[ T(P) \text{ cannot be estimated using ground gravity data (they are biased too)} \]

EGM2008 cannot be used for the same reason

**Global geopotential models based on satellite data are not affected by the biases and can be used in the estimation equation (GOCE models)**

However, to reduce the omission error, EGM2008 can be used at high degree

\[ T_{\text{EGM2008}} \text{ is biased but e.g. in the range } 201<n<2160 \]

it can be proved that the induced error in terms of geoid is \( \varepsilon(N)<0.5 \text{ cm} \)

\[ T(P^j) = T^L(P^j) + T^H(P^j) \]

\[ T^L(P^j) = \text{GOCE global model to d/o } \approx 200 \]

\[ T^H(P^j) = \text{EGM2008 global model from d/o } \approx 201 \text{ to higher degree} \]
The final observation equation for estimating $b^j$ in the jth patch

$$b^j = \sim_{\text{GNSS/lev}}(P^j_k) \frac{T^L(P^j_k)}{\overline{T^L(P^j_k)}} \frac{T^H(P^j_k)}{\overline{T^H(P^j_k)}}$$

$k = 1, \ldots, N$

In the l.s. solving equation the proper covariance structure must be considered

$$C = C_\sim + C_{T^L} + C_{T^H}$$

$$C_\sim = \sigma^2 I$$

$C_{T^L}$ and $C_{T^H}$ from global model covariance information
The covariance structure of the $T^L$ and $T^H$

- Covariance matrix of GOCE model ($T^L$) is computed following the GOCO model by propagating the block diagonal structure (Pail and Gerlach)

- The covariance matrix of $T^H$ is computed using EGM2008 errors geographically rescaled
A test computation for Italy

- Three main tide gauges

1) Genova (Peninsular Italy)
1) Catania (Sicily Island)
1) Cagliari (Sardinia Island)

3 biases to be estimated
The estimation problem setting

\[ b^j = \sum_{\text{GNSS/lev}} (P_k^j) \frac{T^L(P_k^j)}{T^H(P_k^j)} \quad k = 1, \ldots, N \quad j = 1, 2, 3 \]

- GPS/lev observations supplied by IGM: 977 in the mainland; 43 in Sicily Island; 48 in Sardinia Island

- \( T^L \) component estimated using the GOCO-03 based on ITG-Bonn GRACE and GOCE (TIM-R3) solutions

- \( T^H \) component estimated EGM2008 in the domain \((L+1; 900)\) (error budget has been performed for different \( L \) values)
The bias estimation (L=250)

Computation has been done by setting to zero the mainland bias

<table>
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<th>( )= 1 cm</th>
<th>Bias estimate [cm]</th>
<th>Bias st.dev. [cm]</th>
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<tbody>
<tr>
<td>Mainland</td>
<td>0</td>
<td>0.59</td>
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<tr>
<td>Sicily</td>
<td>9.48</td>
<td>2.75</td>
</tr>
<tr>
<td>Sardinia</td>
<td>-12.81</td>
<td>4.92</td>
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H₀: σ₀² = 1 rejected

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<tr>
<th>( )= 12 cm</th>
<th>Bias estimate [cm]</th>
<th>Bias st.dev. [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mainland</td>
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<td>0.52</td>
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<tr>
<td>Sicily</td>
<td>9.82</td>
<td>2.57</td>
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<tr>
<td>Sardinia</td>
<td>-20.68</td>
<td>2.72</td>
</tr>
</tbody>
</table>

H₀: σ₀² = 1 accepted

\[ t_{emp} = 4.59, \ t_{0.975,1065}=1.96 \rightarrow H₀: b_{sicily} =0 \] rejected

\[ t_{emp} = -4.73, \ t_{0.975,1065}=1.96 \rightarrow H₀: b_{sardinia} =0 \] rejected

Remark: the bias Mainland-Sicily has been directly estimated equal to 14.1 cm by IGM
Least-squares residuals

Latitude

Longitude

(m)
Remarks and conclusions

- The devised method can give proper results based on the error propagation for global geopotential model that has been set up.

- The estimated biases are in a reasonable agreement with the estimated DOT in the Mediterranean Sea and with the direct estimate of the bias Mainland-Sicily by IGM.

- Further investigations are needed for explaining the existing tilt (?) that is present in the least squares residuals (improvements in the levelling data).