The need of a Local Reference Frame in Greece: The deficiency of ETRS89 and a new proposed strategy

D. Ampatzidis, C. Kotsakis and K. Katsambalos
Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Greece

1. Introduction

ETRS89 is a conventional terrestrial reference system, in alignment with the Eulerian rotational motion of the Eurasia plate, for spatial positioning throughout the European continent. Its primary objective is to provide a stable coordinate system with minimal horizontal velocities, in support of geodetic, surveying and mapping applications over the various European countries. The key aspect of its realization is based on the ITRF/ETRF velocity reduction to the so-called stable part of Europe, using the following formula (Boucher and Altamimi, Memo 2008):

\[ \mathbf{v}_L = \mathbf{v}_T + \mathbf{T} \mathbf{D}_L \mathbf{v}_T + \mathbf{R}_L \]  

where the matrix \( \mathbf{R}_L \) contains the Cartesian components of the angular velocity vector of the Eurasia plate with respect to ITRF and \( \mathbf{T} \), for each point in the network:

\[ \mathbf{v}_T = \mathbf{v}_{\text{ETRF}}^* + \mathbf{E}^* \]  

For the actual realization of the European Terrestrial Reference Frames (ETRFs/y), the angular velocity of the Eurasia plate is deduced either from a global geophysical model such as AMI-2 or NNR-NVEL-1/1A (ETRF89 up to ETRF97) or from a geodetically derived velocity field in a network of high-quality ITRF stations within the stable part of Europe (ETRF00).

2. Contrast between ETRF velocities in North and S/E Europe

In general, the magnitude of the ETRF horizontal velocities over the central and northern parts of Europe is at the mm/yr level, providing a rather stable framework for geodetic positioning applications. On the other hand, at the southern part of Europe (and particularly in Greece) the situation is completely different. In fact, the horizontal velocities with respect to the stable part of Eurasia can reach up to several cm/yr (> 3 cm/yr at TUG in Crete), showing also a spatially inhomogeneous behavior. This is caused by the unique geophysical setting of the Hellenic area, which directly affects the behavior of the ETRS89/ETRF-based velocity field in Greece.

3. Motivation of our study

An ETRF-based implementation of a national TRF in Greece (as described, for example, in the Boucher & Altamimi Memo) will create a strong velocity field over most of the country; thus canceling out one of the major reasons for adopting ETRS89 as a standard geodetic reference system by the European NMAA! Our aim is to present an alternative optimal scheme for implementing a national TRF in Greece (based on an initial ETRF or ITRF realization consisting of an estimated set of coordinates & velocities over a national network) which will ensure the ‘weakest’ possible velocity field throughout the entire part of the country.

4. Mathematical formulation

The key element of our optimization procedure is the implementation of a Helmert-type velocity transformation (over a national network of fiducial stations) from a given ITRF or ETRF-based regional realization (\( \mathbf{v}_{\text{ETRF}}^* \) to a new local reference frame realization (\( \mathbf{v}_{\text{LRF}}^* \)).

Following (Altamimi et al. 2002), we have the general equations:

\[ \mathbf{v}_L = \mathbf{v}_{\text{ETRF}}^* + \mathbf{T} \mathbf{D}_L \mathbf{v}_{\text{ETRF}}^* + \mathbf{R}_L \]  

where \( \mathbf{E}^* \) is the well-known ‘inner-constraint’ matrix, and \( \mathbf{R}_L \) is the vector of the adopted transformation parameters (including, for example, 3 shift rates, 3 rotation rates & 6 scale rates; for more details, see below)

The unknown parameters of the above transformation will be determined according to the following optimal criterion for the LRF velocities:

\[ \phi = \left( \mathbf{v}_{\text{LRF}}^* \right)^T \left( \mathbf{v}_{\text{LRF}}^* \right) + \min \]  

which yields the following solution:

\[ \mathbf{D}_L = 0 \rightarrow \theta = - \left( \mathbf{E}^* \right)^{-1} \mathbf{E}^* \mathbf{v}_{\text{ETRF}}^* \]  

Hence, in the new LRF, the transformed velocities will be given by the ‘projetive’ formula:

\[ \mathbf{v}_{\text{LRF}}^* = \left( 1 - \mathbf{E}^* \left( \mathbf{E}^* \right)^{-1} \right) \mathbf{E}^* \mathbf{v}_{\text{ETRF}}^* \]

5. GPS test network for LRF optimization in Greece

- 16 stations (11 NOANET stations in Greece & 5 EPN stations in central Europe), three years of GPS observations (2007-2010)
- Initial reference frame (TRF: ETRF00 (2009.0))
- Tight constraints at the 5 EPN stations
- Bernese software (precise IGS orbits, QIF ambiguities, Dry Neill tropospheric model, global ionospheric model)
- Estimated coordinate errors 0.5 mm, estimated velocity errors 0.5 mm/yr (1 level)

6. Numerical results

The implementation of the previous optimization procedure was based on the sole use of three rotation-rate parameters between ETRF00 and the new LRF. Both 3D and 2D scenarios for the formulation of the ‘minimum kinetic energy’ criterion were tested.

<table>
<thead>
<tr>
<th>Estimated rotation-rate parameters between ETRF00 and the optimal LRF</th>
<th>3D scenario</th>
<th>2D scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_x )</td>
<td>-3.264 mas/yr</td>
<td>-3.265 mas/yr</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>-0.982 mas/yr</td>
<td>-0.983 mas/yr</td>
</tr>
<tr>
<td>( \omega_z )</td>
<td>-3.101 mas/yr</td>
<td>-3.103 mas/yr</td>
</tr>
</tbody>
</table>

Horizontal velocities with respect to ETRF00 and the optimal LRF

7. Conclusions

(i) We have presented a Helmert transformation scheme for implementing an optimal LRF in Greece. (In the sense that its associated velocity field \( \mathbf{V} \) becomes as small as possible)
(ii) Such an approach is a useful tool towards the establishment of a national spatial reference framework with maximum temporal stability. Its realization, however, requires a sufficiently dense and well-modeled \( \mathbf{V} \) with respect to an ITRF or ETRF based frame.
(iii) Our numerical tests showed that the optimal LRF has an average VF magnitude of < 1 cm/yr. Also, the dispersion of the VF in the optimal LRF drops to ~3 mm/yr, compared to the 10.5 mm/yr dispersion level which is induced by the ETRF00 frame.