The Abruzzo earthquake: temporal and spatial analysis of the first geodetic results

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Outline

Introduction: the Abruzzo earthquake
The network and the processing strategies
  Time modelling of daily results: displacements estimation at earthquake epoch
    Spatial interpretation of the horizontal displacements
  The vertical displacements
Future works
Abruzzo earthquake

Main event: 6th april, 1:33 UTC
Location: 42.33° N, 13.33° E,
Depth: 8.8 km
Magnitude: 5.8 Richter
Abruzzo earthquake

Main event: 6th April, 1:33 UTC
Location: 42.33° N, 13.33° E,
Depth: 8.8 km
Magnitude: 5.8 Richter

Before and after the main event: many other pre-seismic and after-seismic events.
The geodetic network

3 Italian IGS stations
32 stations in Abruzzo region
17 other stations within a distance of ~ 50 km from Abruzzo boundaries
The geodetic network

3 Italian IGS stations
32 stations in Abruzzo region
17 other stations within a distance of ~ 50 km from Abruzzo boundaries

Data from
1st February (DOY 32) to 2nd, May (DOY 122)
have been adjusted up to now
from 32 to 95 (64 days): before earthquake
from 96 to 122 (27 days): after earthquake
The geodetic network

ASI-Geodaf, INGV-RING, Leica-ItalPos, TopCon-Geotop, GPSAbruzzo, GPSUmbria, ResNap
The processing strategies 1/2

IGS stations stochastically constrained:

coordinates:
interpolation of last 52 IGS05 weekly solutions,

constraints:
2 mm horizontally, 4 mm in height
The processing strategies 1/2

IGS stations stochastically constrained:

coordinates:
interpolation of last 52 IGS05 weekly solutions,

constraints:
2 mm horizontally, 4 mm in height

Final IGS EOP, EPH and PCV’s
Adoption of the international standards
in the raw data processing
by BSW 5.0 software
The processing strategies 2/2

Outlier rejection
The processing strategies 2/2

Outlier rejection

Modelling the time series to estimate discontinuities
The processing strategies 2/2

Outlier rejection

Modelling the time series to estimate discontinuities

Spatial analysis of the discontinuities and clustering in subregions
Examples of time series: MATE
Examples of time series: MATE

Bad data quality (some adjusted station)
Examples of time series: MEDI
Examples of time series: TERA
Examples of time series: TERA

Earthquake
Examples of time series: OCRA
Examples of time series: PAGA
Examples of time series: AQRA
Outliers rejection

Permanent networks are intrinsically redundant

\[ \Downarrow \]

to improve coordinates repeatabilities

a severe automated outliers rejection is useful
Outliers rejection

Permanent networks are intrinsically redundant

\[ \downarrow \]

to improve coordinates repeatabilities

a severe automated outliers rejection is useful

This is a particular case:

few data, manual analysis,

\[ \downarrow \]

conservative approach in outlier rejection

just bad quality sessions before earthquake removed
The results of IGS stations

3 stochastically constrained stations: CAGL, MATE, MEDI

<table>
<thead>
<tr>
<th>Residuals of daily results wrt apriori coordinates</th>
<th>East</th>
<th>North</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>Mean</td>
<td>σ</td>
<td>Min</td>
</tr>
<tr>
<td>East</td>
<td>0.6</td>
<td>2.7</td>
<td>-4.0</td>
</tr>
<tr>
<td>North</td>
<td>0.4</td>
<td>1.0</td>
<td>-2.6</td>
</tr>
<tr>
<td>Height</td>
<td>0.5</td>
<td>4.1</td>
<td>-10.8</td>
</tr>
</tbody>
</table>
Time series interpretation (1/2)

Short time series in the geodetic analysis

↓

constant model to avoid propagation of seasonal effects and localized in time variations into meaningless estimated velocities
Time series interpretation (2/2)

Before earthquake:
not a clear presence of pre seismic signal, just linear trend

linear trend estimation and removal
not to estimate velocities but to better model daily solutions
After earthquake: a postseismic signal is often clear, but few days are available at the present, simple constant model applied, with more data: linear and 2nd order polynomial.
One example

<table>
<thead>
<tr>
<th>(mm)</th>
<th>DE</th>
<th>DN</th>
<th>Dh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.0</td>
<td>2.3</td>
<td>-2.9</td>
</tr>
<tr>
<td>Linear</td>
<td>0.9</td>
<td>0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
Residuals statistics of daily solutions

<table>
<thead>
<tr>
<th>Before (mm)</th>
<th>E</th>
<th>N</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
<td>1.3</td>
<td>3.6</td>
</tr>
<tr>
<td>Min</td>
<td>-6.1</td>
<td>-6.3</td>
<td>-11.1</td>
</tr>
<tr>
<td>Max</td>
<td>7.1</td>
<td>5.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After (mm)</th>
<th>E</th>
<th>N</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
<td>1.4</td>
<td>4.9</td>
</tr>
<tr>
<td>Min</td>
<td>-7.2</td>
<td>-6.5</td>
<td>-23.6</td>
</tr>
<tr>
<td>Max</td>
<td>8.0</td>
<td>8.5</td>
<td>34.7</td>
</tr>
</tbody>
</table>

Worse height results after earthquake: post seismic assessment of 4 stations near L’Aquila
Parameters and covariances estimation

Daily coordinates models in time

↓

Model parameters estimated by LS

Formal daily covariances typically underestimated
and final covariances too much optimistic

↓

Empirical covariances estimation needed
Parameters and covariances estimation

Few observations

\[\downarrow\]

Simplified hypotheses on time series models and covariances
Parameters and covariances estimation

Few observations

\[ \downarrow \]

Simplified hypotheses on time series models and covariances

Joint estimation of parameters and covariances

\[ \downarrow \]

Typically an iterative process up to final results

\[
y_0, \tilde{C}_{yy} \Rightarrow \hat{x}_I, \hat{C}_{yy_I} \Rightarrow y_0 \hat{C}_{yy_I} \Rightarrow \hat{x}_{II}, \hat{C}_{yy_{II}} \Rightarrow \ldots \Rightarrow \hat{x}_F, \hat{C}_{yy_F}
\]
# Hypotheses on network covariances

1. Daily network covariance constant in time
2. No correlations between consecutive days

\[ C(t_k) = C \quad \forall k = 1, \ldots, T, \quad C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1P} \\ C_{21} & C_{22} & \cdots & C_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ C_{P1} & C_{P2} & \cdots & C_{PP} \end{bmatrix}, \]

\[ C_{ij} = \begin{bmatrix} c_{x_{11}x_{1j}} & c_{x_{11}x_{2j}} & c_{x_{11}x_{3j}} \\ c_{x_{21}x_{1j}} & c_{x_{21}x_{2j}} & c_{x_{21}x_{3j}} \\ c_{x_{31}x_{1j}} & c_{x_{31}x_{2j}} & c_{x_{31}x_{3j}} \end{bmatrix} = \left\{ C_{ij \mid m} \right\}_{l,m=1,2,3} \]
Estimation of the model parameters

Constant or linear model

\[ x_{P_i}(t) = \begin{cases} 
  x_{P_i}(t) \\
  x_{P_i}(\bar{t}) + \dot{x}_{P_i} \cdot (t - \bar{t}) 
\end{cases} \]
Estimation of the model parameters

Constant or linear model

\[ x_{P_i}(t) = \begin{cases} x_{P_i}(\bar{t}) \\ x_{P_i}(\bar{t}) + \dot{x}_{P_i} \cdot (t - \bar{t}) \end{cases} \]

1. For each point \( i=1, \ldots, P \), each component \( l=1,2,3 \) an independent regression is estimated by Least Squares

\[ \mathbf{y}_0 = \begin{bmatrix} x_{l_{i0}}(t_1) \\ x_{l_{i0}}(t_2) \\ \vdots \\ x_{l_{i0}}(t_T) \end{bmatrix}, \tilde{C}_{yy} = \sigma_0^2 \mathbf{I} \Rightarrow \text{LS} \Rightarrow \hat{\mathbf{x}}_I = \begin{bmatrix} \hat{x}_{l_{i0}}(\bar{t}) \\ \hat{x}_{l_{i}} \end{bmatrix}_I \]
Empirical estimation of the covariances

Estimated vector of the residuals

\[ \hat{r}_{l,l_I} = \begin{bmatrix} x_{l_0}(t_1) - \left[ \hat{x}_{l_I}(\bar{t}) + \hat{x}_{l_I} \cdot (t_1 - \bar{t}) \right] \\ x_{l_0}(t_2) - \left[ \hat{x}_{l_I}(\bar{t}) + \hat{x}_{l_I} \cdot (t_2 - \bar{t}) \right] \\ \vdots \\ x_{l_0}(t_T) - \left[ \hat{x}_{l_I}(\bar{t}) + \hat{x}_{l_I} \cdot (t_T - \bar{t}) \right] \end{bmatrix} \]
Empirical estimation of the covariances

Estimated vector of the residuals

\[ \hat{\mathbf{r}}_{lI} = \begin{bmatrix} x_{l_0} (t_1) - [\hat{x}_{lI} (\bar{t}) + \hat{x}_{lI} \cdot (t_1 - \bar{t})] \\ x_{l_0} (t_2) - [\hat{x}_{lI} (\bar{t}) + \hat{x}_{lI} \cdot (t_2 - \bar{t})] \\ \vdots \\ x_{l_0} (t_T) - [\hat{x}_{lI} (\bar{t}) + \hat{x}_{lI} \cdot (t_T - \bar{t})] \end{bmatrix} \]

Estimated covariances and correlations

\[ \hat{c}_{ijlmI} = \frac{1}{T - N} \hat{\mathbf{r}}_{lI}^T \hat{\mathbf{r}}_{mI} \]

(N=1/2 for the constant/linear model)
Final results

With the above hypotheses, no need of iterations

\[
\hat{X}_F = \left\{ \begin{array}{l}
\hat{x}_{l_i}(t), \hat{x}_{l_i} \\
_{l=1,2,3; \ i=1,\ldots,P}
\end{array} \right\} = \hat{X}_I
\]

Final parameters
Final results

With the above hypotheses, no need of iterations

Final parameters

\[
\hat{\mathbf{X}}_F = \left\{ \begin{array}{l}
\hat{x}_{l_i}(\bar{t}), \hat{x}_{l_i} \\
\hat{x}_{l_i}(\bar{t})
\end{array} \right\}_{l=1,2,3; \; i=1,...,P} = \hat{\mathbf{X}}_I
\]

Related covariances

\[
\hat{\sigma}_{x_{l_i}x_{l_j}}^2 = \frac{1}{T} \hat{c}_{x_{l_i}x_{l_j}}^2, \quad \hat{\sigma}_{x_{l_i}x_{l_j}} = 0
\]

\[
\hat{\sigma}_{\dot{x}_{l_i}\dot{x}_{l_j}}^2 = \frac{1}{m_t^2 T} \hat{c}_{\dot{x}_{l_i}\dot{x}_{l_j}}^2
\]

\[
m_t^2 = \frac{1}{N} \sum_{i}^{} (t_i - \bar{t})^2
\]
Propagation of coordinates and covariances

Displacement at earthquake epoch

\[ \hat{x}_{iB}(t_E) = \hat{x}_i(t_B) + \hat{x}_i(t_E - t_B) \]

\[ \hat{x}_{iA}(t_E) = \hat{x}_{iA}(t_A) \]

\[ \delta \hat{x}_i(t_E) = \hat{x}_{iA}(t_E) - \hat{x}_{iB}(t_E) \]
Propagation of coordinates and covariances

Displacement at earthquake epoch

\[ \hat{x}_{IB}(t_E) = \hat{x}_i(t_B) + \hat{x}_i(t_E - t_B) \]
\[ \hat{x}_{IA}(t_E) = \hat{x}_{iA}(t_A) \]
\[ \delta \hat{x}_i(t_E) = \hat{x}_{iA}(t_E) - \hat{x}_{iB}(t_E) \]

Covariance of the displacement

\[ C_{IB}(t_E) = C_{\bar{x}x_{IB}} + C_{\bar{x}x_{iB}}(t_E - t_B)^2 \]
\[ C_{IA}(t_E) = C_{\bar{x}x_{iA}} \]
\[ C_{i\delta\delta}(t_E) = C_{iA}(t_E) + C_{iB}(t_E) \]
Covariances of the two propagations

Few data
Horizontal displacements map

10 mm displacement

50 km
Horizontal displacements map

No smooth deformation field but a discontinuity line
Separation of rigid motion from deformation

BAD SPATIAL INTERPOLATION
Separation of rigid motion from deformation

BAD SPATIAL INTERPOLATION

GOOD SPATIAL INTERPOLATION
PIECEWISE INTERPOLATION INVOLVES DISCONTINUITIES = FAULTS!
Spatial covariances and interpolation

A signal could be isolated, but quite arbitrarily ↓

a preliminary clustering of homogeneous areas needed
Spatial clustering

1. L’Aquilla sites: 20-70 mm W displacements
Spatial clustering

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2. Around them: smaller S-W displacements
Spatial clustering

1. L’Aquila sites: 20-70 mm W displacements
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3. East region: 2-30 mm NE displacements
Spatial clustering

1. L’Aquila sites: 20-70 mm W displacements
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3. East region: 2-30 mm NE displacements
4. Around it: no significant displacements, but consistent directions.
Spatial clustering

1. L’Aquila sites: 20-70 mm W displacements
2. Around them: smaller S-W displacements
3. East region: 2-30 mm NE displacements
4. Around it: no significant displacements, but consistent directions.
5. No other significant displacements
Separation of rigid motion from deformation

Horizontal motion of a network on earth surface: rotation of all the points around an axis with angular velocity $\omega$

$$\mathbf{v}_i = [\mathbf{\omega} \times] \mathbf{x}_i$$
Separation of rigid motion from deformation

Horizontal motion of a network on earth surface: rotation of all the points around an axis with angular velocity \( \omega \)

\[ \mathbf{v}_i = [\omega \times] \mathbf{x}_i \]

\( \omega \) can be estimated by minimization of relative kinetic energy of the network

\[ T_{ap} = \sum_{i=1,...,P} \mathbf{v}_i^T \mathbf{v}_i = \min \]

Realization of a Discrete Tisserand reference system
Horizontal analysis in separate regions

Probably no significant rotation of networks but differential displacements

... up to now no Tisserand analysis but statistics on displacements for the two main regions

<table>
<thead>
<tr>
<th></th>
<th>East (14 stations)</th>
<th>L'Aquila (4 stations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
</tr>
<tr>
<td>Mean</td>
<td>E</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>7.0</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>7.6</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>6.6</td>
<td>7.5</td>
</tr>
<tr>
<td>Min</td>
<td>1.0</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>8</td>
</tr>
<tr>
<td>Max</td>
<td>14.4</td>
<td>27.5</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>30.1</td>
</tr>
<tr>
<td></td>
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</tbody>
</table>
Vertical displacements

10 mm displacement

50 km
Vertical displacements

Significant displacements for L’Aquila stations:
-25, -76, -107, -123 mm

No significant displacements in other regions:
mean: 0.5 mm, range -3/+3mm
Conclusions

6th April earthquake in L’Aquila has been accompanied by an extension along an axis oriented NW-SE:

L’Aquila area and an Eastern Adriatic area interested by significant opposite horizontal displacements
Conclusions

6th April earthquake in L’Aquila has been accompanied by an extension along an axis oriented NW-SE:

L’Aquila area and an Eastern Adriatic area interested by significant opposite horizontal displacements

Significant gradients in the horizontal displacements of the Eastern Adriatic area

L’Aquila sites interested by vertical displacements of about 10 cm
Future analyses

Longer time series, to:
- increase the populations after the earthquake,
- analyze the post seismic time series
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increase the populations after the earthquake,
analyze the post seismic time series

More rigorous clustering in separate regions,
rigorous Tisserand analysis,
geometric analysis in Adriatic region
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Longer time series, to:
increase the populations after the earthquake,
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Cross comparison in L’Aquila
with SAR interferograms