The treatment of the permanent tide in EUREF products

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Background

- The time-average of the tide-generating potential of the Sun and the Moon is not zero.
- At the surface of the Earth both the magnitude and the range are a few parts of 10^{-8} of the potential of the Earth.
- To deal with the permanent deformation they cause, there are two concepts that are available for the 3-D shape of the Earth (also called crust or topography) and three concepts for the gravity field.

Non-tidal or tide-free system

- The permanent deformation is eliminated from the shape of the Earth.
- From the potential field quantities (gravity, geoid etc) both the tide-generating potential, and the deformation potential of the Earth (the indirect effect) are eliminated.
- This corresponds to physically removing the Sun and the Moon to infinity.
- Typically the permanent deformation is treated using the same Love numbers h and k (and Shida number ℓ) as for the time-dependent tidal effects, not estimates for secular (fluid) Love numbers.
- I.e., conventional tide-free system

The mean tidal system

- The permanent effect is not removed from the shape of the Earth; the shape therefore corresponds to the long-time average under tidal forcing.
- The potential field retains the potential of this average Earth, and also the time-average of the tide-generating potential (though it is not due to the masses of the Earth).
- Mean-tide potential field describes how water flows and clocks run (general relativity)
- Complications or additional corrections needed in boundary value problems, as the tide-generating masses are outside the Earth

The zero tidal system

- For the potential field quantities
- A “middle alternative”
- Eliminates the tide-generating potential but retains its indirect effect, i.e., the potential of the permanent deformation of the Earth. Thus its “partner” is the mean system for 3-D shape
- because of this, it is often said that for the 3-D we have zero $\equiv$ mean.
- In this alternative, the gravity field is generated only by the masses of the Earth (plus the centrifugal force).

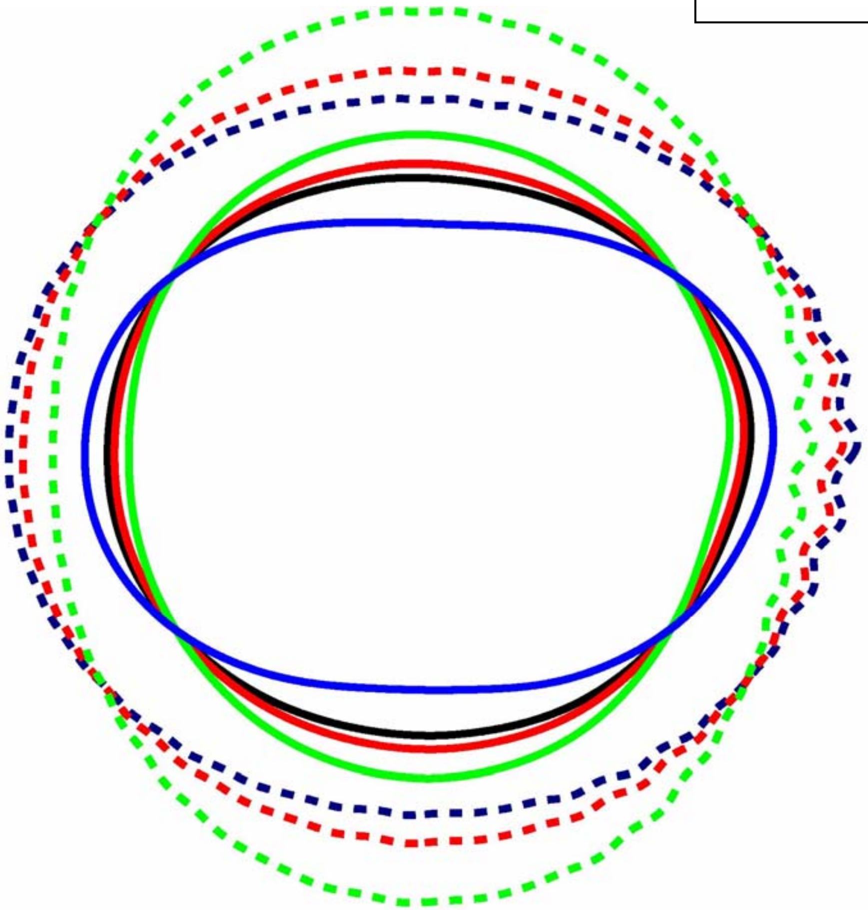
Schematic representation (I)

Dashed line: crust (topography; 3-D)

mean≡zero crust

conventional tide-free crust at $-hW_2/g$, $h \approx 0.6$

fluid tide-free crust at $-hW_2/g$, $h \approx 1.93$



Solid line: geoid

mean at $+W_2/g$

zero

conventional tide-free geoid at $-kW_2/g$, $k \approx 0.3$

fluid tide-free geoid at $-kW_2/g$, $k \approx 0.93$

$$W_2/g \approx -0.296 \sin^2(\phi) + 0.099 \text{ [m]}$$

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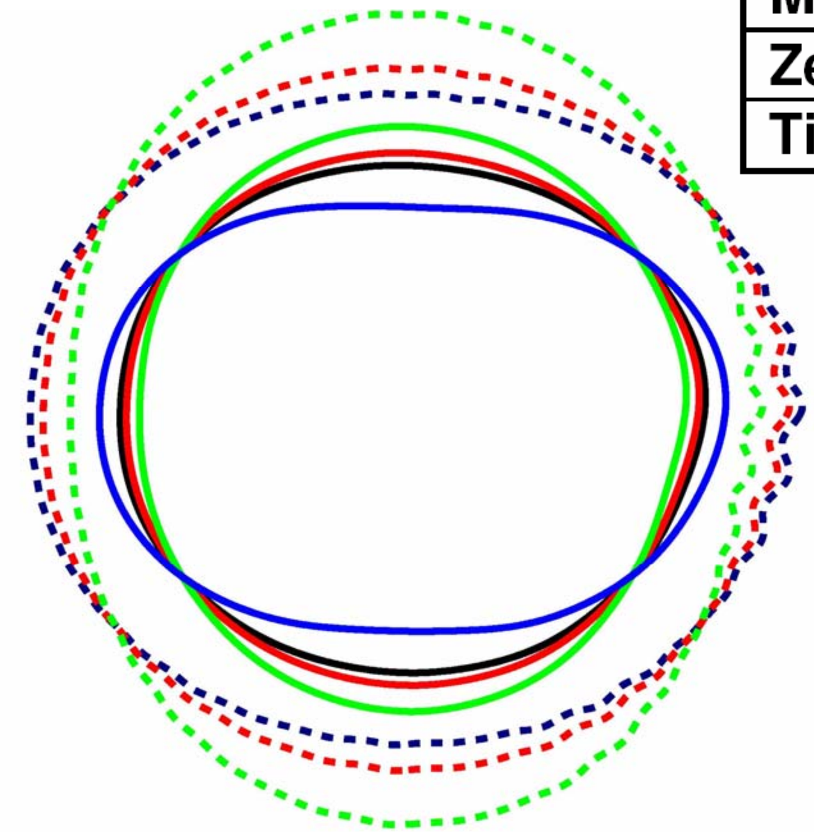
Schematic representation (II)

Various crusts (topography) above various geoids, using
mean ($\equiv$zero) crust above zero geoid as reference:

Geoid \ Crust	Mean	Tide-free
Mean	$-W_2/g$	$(-h+k)W_2/g$
Zero	0	$-hW_2/g$
Tide-free	kW_2/g	$(-h+1)W_2/g$



See e.g., Ekman (1989)

$$W_2/g \approx -0.296 \sin^2(\phi)+0.099 \text{ [m]}$$

Historical note

- there is no ”automatical split” of a tidal correction into a time-variable and permanent part, e.g.
- when simple tidal corrections were calculated on the basis of ephemerides
- the permanent part must be evaluated separately and restored by design
- therefore at the first stage of tidal corrections, the processing programs of various geodetic techniques produced (conventional) tide-free results
- ITRF used them and became in this way tide-free
- When this was discovered in the 1990s (Poutanen et al., 1996) it was “too late” to change the established coordinates
- Thus the ITRFxx and with it all ETRS89 realizations are tide-free

Current situation

- Current IAG resolution (1983) requires the zero system for the gravity field quantities and the zero=mean system for the 3-D Earth
- Current practice is zero for gravity, tide-free for 3-D (e.g., ITRFxx), and mixed (overwhelmingly mean) for potential differences determined with precise levelling.
- Earth geopotential models (EGMs) are usually provided both in conventional tide-free and in zero-tide versions as far as the potential coefficients are concerned. The difference is only in the C_{20} term.
- The EGMs are evaluated using ITRF coordinates
- Regional geoids inherit the tidal system from the EGM used

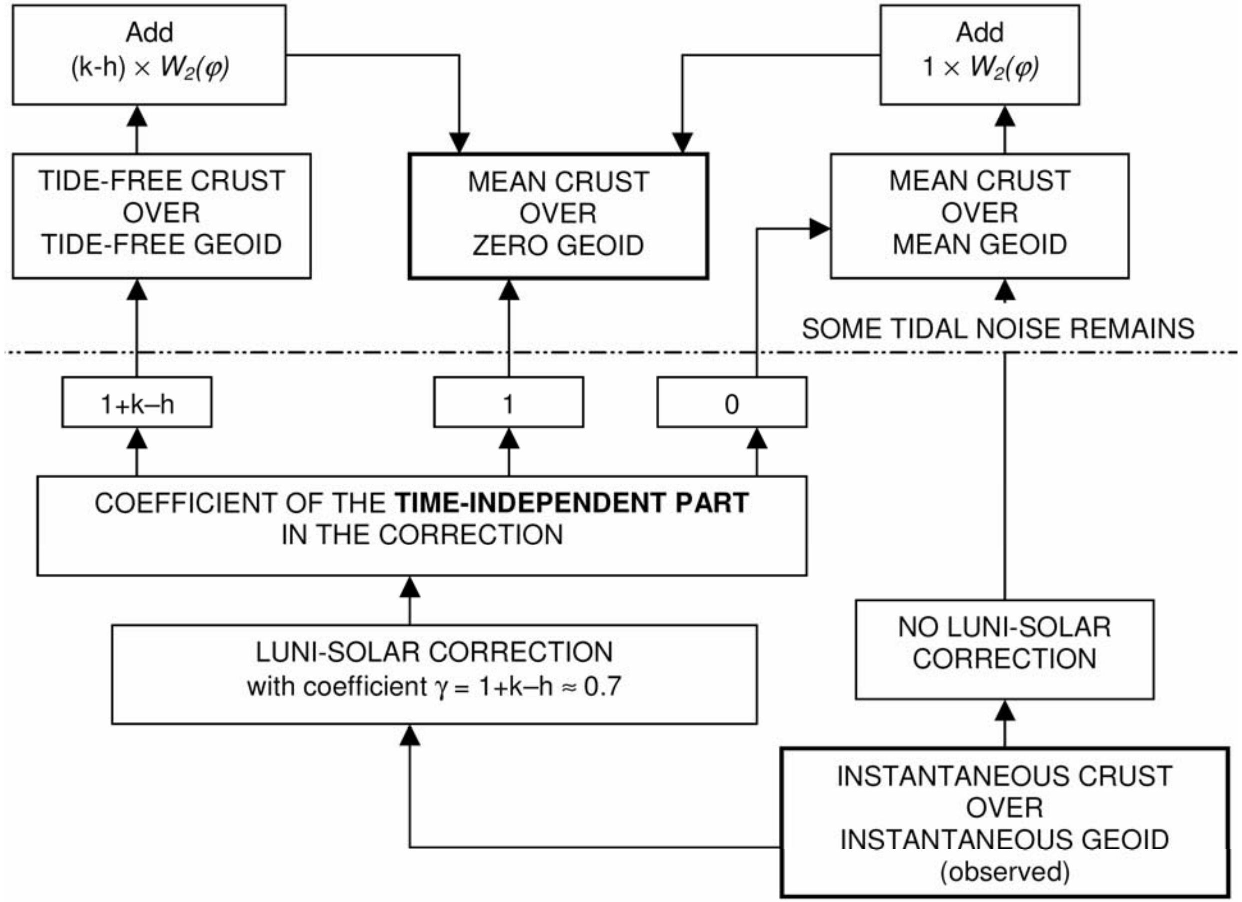
Current situation (II): national height systems

- Legacy systems are overwhelmingly mean-tide systems
- Usually not by design: if no ”luni-solar correction” is applied to precise levelling, then the mean system emerges automatically
- The application of the ”luni-solar correction” has almost invariably (and unwittingly) resulted in a conventional tide-free system
- Zero-tide systems are very recent: Sweden (2005), Finland (2007)

Current situation (III): European height systems

- The European Vertical Reference System (EVRS) is zero-tide by definition
- but due to the mixed systems (mostly mean, some non-tidal) of the national observations going into the United European Levelling Network UELN-95/98, the realization EVRF2000 is mixed
- The EVRF2007 is zero-tide

How to put the present UELN data into the zero system



Formulas for the permanent tidal potential and derived quantities

- Are available e.g. in Ekman (1989), in geodetic coordinates (geodetic latitude on ellipsoid) with 1 mm precision
- Are available in e.g. in IERS standards in Cartesian 3-D with sufficient precision (0.01 mm or 0.01 mgpu), with the exception of the restoration of the permanent tidal deformation of the crust (0.1 mm)
- I have compared IERS standards with computations of the tide-generating potential: Cartwright-Taylor-Edden (1973) reduced to 2000, Tamura (1987), Roosbeek (1996), Hartmann-Wenzel (1995), Kudryavtsev (2004).
- Differences 0.01 mgpu at ellipsoid

Formulas for the permanent tidal potential and derived quantities (II)

$$W_2 = B\left(\frac{r}{R}\right)^2 P_2(\sin\psi) = B\left(\frac{r}{R}\right)^2\left(\frac{3}{2}\sin^2\psi - \frac{1}{2}\right) = A\left(\frac{r}{a}\right)^2\left(\sin^2\psi - \frac{1}{3}\right)$$

(ψ, r) geocentric latitude and radius vector

R scale factor, a semi-major axis of GRS80 ellipsoid

$$W_2(r,\psi) = A\left(\frac{r}{a}\right)^2\left(\sin^2\psi - \frac{1}{3}\right), \quad A = -291.66\,\text{mgpu}$$

(IERS2003), in the epoch 2000.0

The rate in A is –0.09 mgpu/century

Coefficient *A* for selected calculations of tidal spectra. Epoch 2000.

Source	*A*, mgpu	Remarks
Cartwright and Tayler (1971) and Edden (1974)	–291.65(2)	HW95 normalization by Wenzel (1995)
Hartmann and Wenzel (1995)	–291.65(5)	HW95
Roosbeek (1996)	–291.66(5)	HW95 normalization by Roosbeek (1995)
Kudryavtsev (2004)	–291.65(7)	HW95 normalization by Kurdryatsev (2004)
IERS2003	–291.66	HW95 normalization by me

Formulas for the permanent tidal potential and derived quantities (III)

$$W_2(\varphi) = -288.41\sin^2\varphi - 1.95\sin^4\varphi + 97.22 \quad [\text{mgpu}]$$

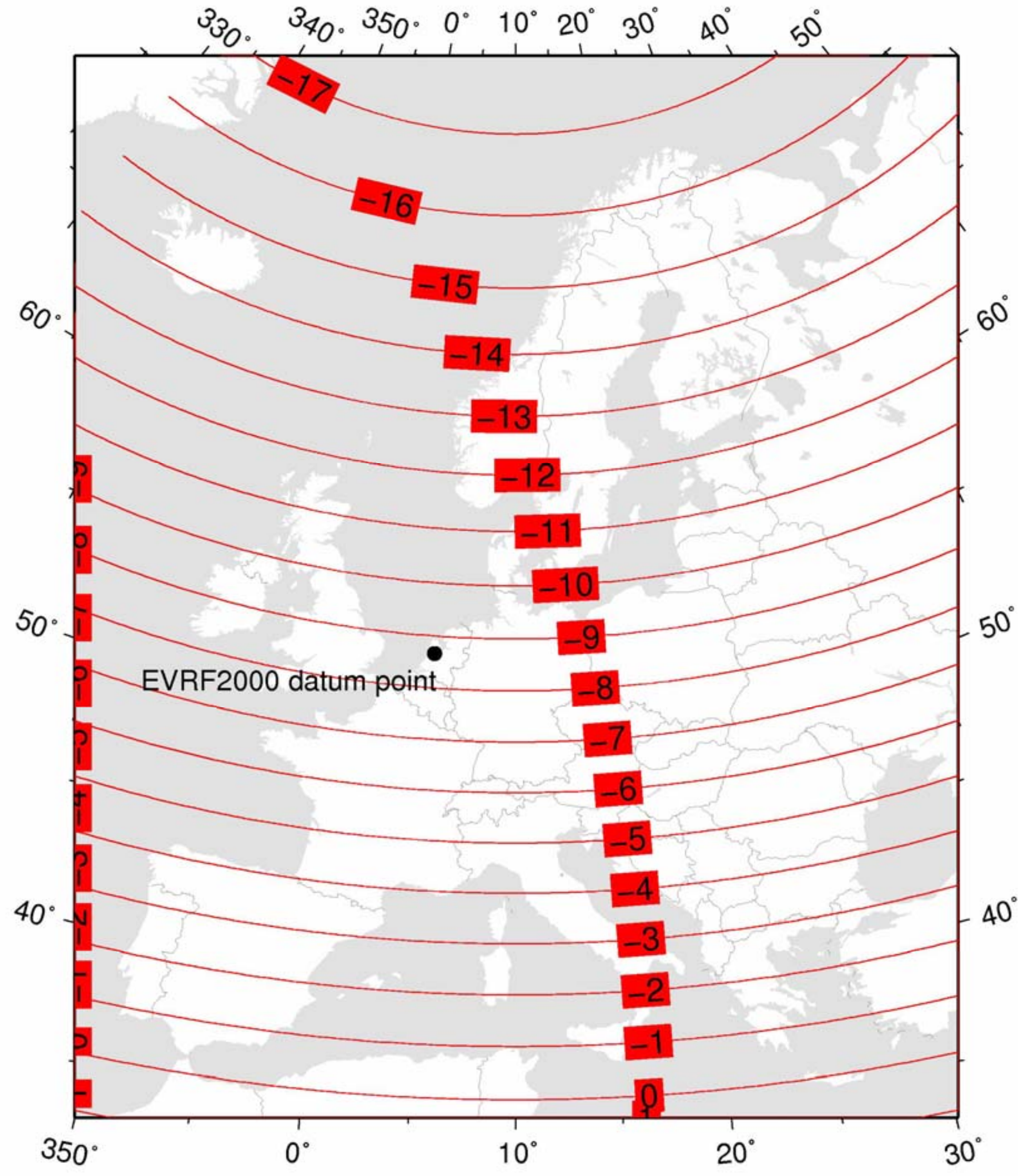
On the surface of the GRS80 ellipsoid

φ geodetic latitude

Add above figure to geopotential numbers in the mean tidal system to get geopotential numbers in the zero tidal system

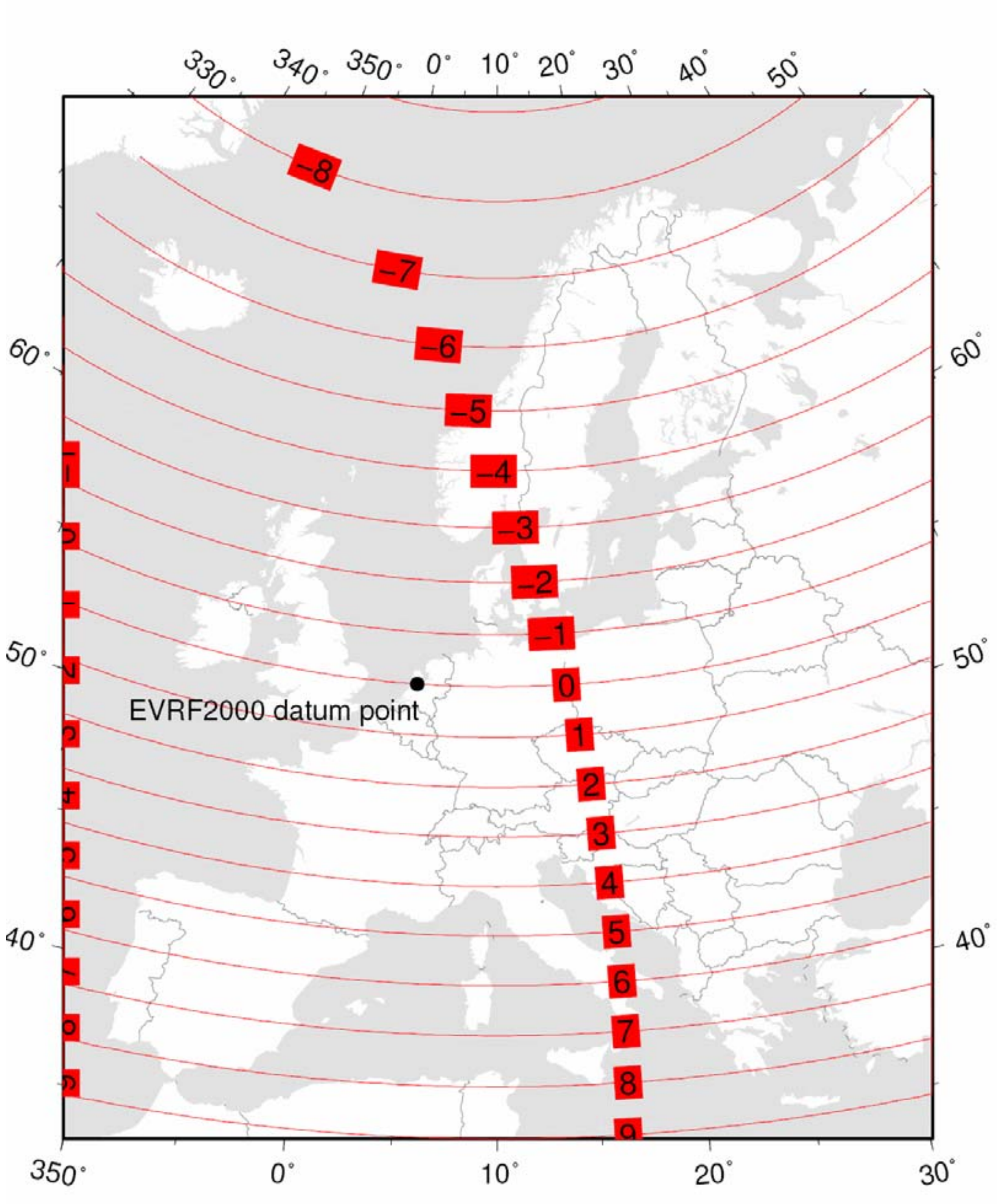
$$H_2(\varphi) = W_2(\varphi)/\gamma_0(\varphi) = -294.94\sin^2\varphi - 1.83\sin^4\varphi + 99.40 \quad [\text{mm}]$$

Add above figure to normal heights in the mean tidal system to get normal heights in the zero tidal system



Correction from heights in the mean tide system to heights in the zero tide system

in cm



Correction from heights in the mean tide system to heights in the zero tide system

Relative to Amsterdam latitude

In cm

Tidal considerations for realization of EVRF2007 through Earth Geopotential Models (EGM) and European Gravimetric Quasigeoid (EGG)

ITRFxx and with it ETRS89 realizations are tide-free.

The EGMs and the EGG are specified in the (tide-free) ITRFxx coordinates, whatever the tidal system of the represented potential

What does that imply? Apparently nothing special. The EGM or EGG at the tide-free ITRFxx coordinates gives us the geopotential at the tide-free position (x,y,z) of the BM. If we want to have the geopotential (which could be mean, zero, or tide-free) at the mean crust position of the BM, we must restore the (conventional) permanent tidal deformation before evaluating the EGM.

How to realize the EVRF2007 using an EGG or an EGM, at a point with 3-D coordinates (ITRFxx, or an ETRS89 realization)

- The EGM or EGG potential must have the same tidal system (zero) as the EVRF2007
- The EGG is presented in the ITRFxx/ETRS89 coordinates
- But before evaluating the potential the (conventional) permanent tidal deformation must be restored.
- Thus instead of P(x,y,z) in ETRS89 we evalulate the EGG at P'(x+Δx, y+Δy,z+Δz) where (Δx,Δy,Δz) represents the restoration according to IERS2003 standards

Tidal considerations for realization of EVRF2007 through Earth Geopotential Models (EGM) and European Gravimetric Quasigeoid (EGG), continued

In the EVRS draft conventions circulated with the EVRF2007 results the corresponding section written by me is wrong!

In every point P with accurate 3-D coordinates the EVRS can be realized by evaluating the total geopotential $W_{EGG} = U_{EGG} + T_{EGG}$ from a (possibly detailed European) quasigeoid solution EGG which gives the geopotential in the ITRF or ETRS89, in the zero tide system.:

$$c_{PE} = W_{0E,\,EGG} - W_{P,\,EGG}$$

Here $W_{EGG} = U_{EGG} + T_{EGG}$ is the total geopotential in the EGG solution, U_{EGG} its normal potential (not necessarily the GRS80 or even a level ellipsoidal field), T_{EGG} the disturbing potential. $W_{0E,\,EGG}$ is the potential of the European height datum in the EGG, calculated e.g. through

$$W_{0E,\,EGG} = (1/N)\,\Sigma(W_{P,\,EGG} + c_{PE})$$

where the summation goes over N points P with both accurate 3-D coordinates, and geopotential numbers from the UELN adjustment.

Note that no corrections for different tidal systems of ITRF and EGG are needed.

Tidal considerations for realization of EVRF2007 through Earth Geopotential Models (EGM) and European Gravimetric Quasigeoid (EGG)

The IERS2003 conventions provide for the restoration of the permanent tidal deformation in 3-D

The formulas are given with the precision of 0.1 mm.

I have computed the component in the direction of the GRS80 ellipsoidal normal

$$h_{PT}(\varphi) = 60.3 - 179.0 \sin^2 \varphi - 1.9 \sin^4 \varphi \quad [\mathrm{mm}]$$

How to realize the EVRF2007 using an EGM or EGG, at a point with 3-D coordinates (ITRFxx, or an ETRS89 realization)

We calculate the potential $W_{0E,EGG}$ of the European height datum ("NAP") in the EGG

$$W_{0E,EGG} = \frac{1}{N}\sum_{i=1}^{N}\left(W_{P'_i,EGG} + c_{P'_i,E}\right)$$

Here

$$W_{EGG} = U_{EGG} + T_{EGG}$$

is the total geopotential of the EGG (or EGM).

(The normal potential U_{EGG} is typically NOT identical with say the GRS80)

Then EVRF2007 geopotential number at P from EGG is obtained evaluating W_{EGG} at P'

$$c_{PE} = W_{0,EGG} - W_{P',EGG}$$

The correction for the fact that ITRFxx coordinates are tide-free appears in the P'

Or approximately using the original ITRFxx coordinates and Bruns' formula

$$W_{0E,EGG} = \frac{1}{N}\sum_{i=1}^{N}\left(W_{P_i,EGG} - hW_{2,P_i} + c_{P_i,E}\right)$$

$$c_{PE} = W_{0,EGG} - W_{P,EGG} + hW_{2,P}$$

where *h* is the conventional Love number of the ITRF tide-free crust

Ellipsoidal heights as an intermediate step

Let us use the GRS80 ellipsoid. Denote the GRS80 normal potential field U_{GRS80} and split W_{EGG} anew

$$W_{EGG} = T_{EGG} + U_{EGG} = T_{EGG} + U_{EGG} - U_{GRS80} + U_{GRS80}$$
$$= T + U_{GRS80}$$

where

$$T = T_{EGG} + U_{EGG} - U_{GRS80}$$

is the disturbing potential of the EGG relative to the GRS80 normal field

Ellipsoidal arithmetics

$$-c_P = W_{P',EGG} - W_{0E,EGG}$$
$$= (W_{P',EGG} - U_{P',GRS80}) + (U_{P',GRS80} - U_{0,GRS80}) + U_{0,GRS80} - W_{0E,EGG}$$
$$= T_{P'} - \bar{\gamma} h_{P'} + U_{0,GRS80} - W_{0E,EGG}$$

Since

$$c_P = \bar{\gamma} H \qquad T_{P'} = \gamma \zeta$$

we have

$$-\bar{\gamma} H = -\bar{\gamma} h_{P'} + \gamma \zeta + U_{0,GRS80} - W_{0E,EGG}$$

Ellipsoidal arithmetics (continued)

$$-\bar{\gamma}H = -\bar{\gamma}h_{P'} + \gamma\zeta + U_{0,GRS80} - W_{0E,EGG}$$

Putting $h_{P'} = h_P + h_{PT}$ $\quad U_{0,GRS80} - W_{0E,EGG} = \gamma h_{offset}$

we have

$$-\bar{\gamma}H = -\bar{\gamma}(h_P + h_{PT}) + \gamma\zeta + \gamma h_{offset}$$

and identifying the γ

$$H = h_P + h_{PT} - \zeta - h_{offset}$$

Ellipsoidal arithmetics (end)

$$H = h_P + h_{PT} - \zeta - h_{offset}$$

Here

H is the normal height in EVRF2007

h_P is the ellipsoidal height of P (in ITRFxx/ETRS89) over the GRS80 ellipsoid

h_{PT} is the correction from tide-free ellipsoidal heights to mean-tide ellipsoidal heights

ζ is the quasigeoid height from the EGG or EGM, but re-calculated relative to the GRS80 normal gravity

h_{offset} is implied by the difference of the GRS80 normal potential at ellipsoid and the potential of the datum of the EVRF2007

$$U_{0,GRS80} - W_{0E,EGG} = \gamma h_{offset}$$

Thank you!!