Terrestrial reference systems from theory to implementation

• Basic Theory of RS & RF
• Principles for TRF Implementation
• Application to ITRF/ETRF
• Illustrating Examples

Zuheir Altamimi
IGN France
Defining a Reference System & Frames:

• Three conceptual levels [Kovalevsky et al., 1989]:
  - Ideal Terrestrial Reference System (TRS) is a mathematical, theoretical system
  - The Conventional TRS is the sum of all conventions (models, constants,...) that are necessary to realize the TRS
  - A Conventional TRF, which uses above to realize the TRS.

• In effect: Use only TRS and TRF terms
  – The TRS is an ideal, conventional model
  – The TRF is a materialization of the TRS inheriting the mathematical properties of the TRS
  – As the TRS, the TRF has an origin, scale & orientation
  – TRF is constructed using space geodesy observations
Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)
Defined in an Euclidian affine space of dimension 3:

Affine Frame (O,E) where:

O: point in space (Origin)
E: vector base: orthogonal with the same length:
- unit vectors co-linear to the base (Orientation)
- unit of length (Scale)

\[ \lambda = \| \vec{E}_i \|_{i=1,2,3} \]
\[ \vec{E}_i \cdot \vec{E}_j = \lambda^2 \delta_{ij} \]
\[ (\delta_{ij} = 1, \ i = j) \]
"Motions" of the deformable Earth

• Nearly linear motion:
  – Tectonic motion: horizontal
  – Post-Glaciaral Rebound: Vertical & Horizontal

• Non-Linear motion:
  – Seasonal: Annual, Semi & Inter-Annual caused by loading effects
  – Rupture, transient: uneven motion caused by Earthquakes, Volcano Eruptions, etc.
Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch $t$ could be written as:

$$X(t) = X_0 + \dot{X}(t - t_0) + \sum_i \Delta X_i(t)$$

- $X_0$: point position at a reference epoch $t_0$
- $\dot{X}$: point linear velocity
- $\Delta X_i(t)$: high frequency time variations:
  - Solid Earth, Ocean & Pole tides
  - Loading effects: atmosphere, ocean, hydrology, Post-glacial-Rebound
  - ... Earthquakes
TRF Representations

- "Quasi-Instantaneous" Frame: mean station positions at "short" interval:
  - One hour, 6-h, 12-h, one day, one week
  ==> Non-linear motion embedded in time series of instantaneous frames

- Long-Term Secular Frame: mean station positions at a reference epoch \( (t_0) \) and station velocities:
  \[ X(t) = X_0 + V^*(t - t_0) \]

- Non-Linear Frame does not exist, otherwise it is an "Earth model"
TRF Datum Definition

• A TRF should be clearly and unambiguously defined through 7 (14) parameters:
  – 3 (6) origin parameters
  – 1 (2) scale parameters
  – 3 (6) orientation parameters

• The 7 (14) parameters are relative quantities
  – e.g. if we say SLR origin is selected to define ITRF2005 origin, it means zero translations/rates btw SLR and ITRF2005
TRF implementation

- Using Space Geodesy data of a single technique or multiple techniques
- Combination of SG Solutions (ex. ITRFs)

  ==> Construct a singular Normal Equation
  ==> Add constraints to define the TRF in origin, scale or/and orientation
  ==> Produce SSC + Other parameters (e.g. EOPs)

- By transformation formula / 7(14) parameters

  ==> Adjusted (by LS) or Computed parameters
  ==> Ex. ITRF-to-ETRF transformation
Classes of Sets of Station Coordinates (SSC)

- "Primary" SSC: results from LS adjustment:
  - Space Geodesy solutions at the observation level
  - Combined solutions: e.g. ITRFs

- "Secondary" SSC: Expressed/Transformed in a given TRF:
  - IGS00, IGS05, EPN weekly/cumulative solutions
  - National, Regional or Local GPS network solutions
  - ETRFyy lists as results from ITRF-to-ETRF transformation

- CTRF: Set of physical points with coordinate numerical information (values, derivatives) expressed in a selected coordinate system linked to a specific TRS
CATREF Software

\[
\begin{align*}
X_s^i &= X_c^i + (t_s^i - t_0)\dot{X}_c^i \\
&\quad + T_k + D_k X_c^i + R_k X_c^i \\
&\quad + (t_s^i - t_k) \left[ \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \right] \\
\dot{X}_s^i &= \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i
\end{align*}
\]

EOPs

\[
\begin{align*}
x_s^p &= x_c^p + R2_k \\
y_s^p &= y_c^p + R1_k \\
UT_s &= UT_c - \frac{1}{f} R3_k \\
\dot{x}_s^p &= \dot{x}_c^p + \dot{R}2_k \\
\dot{y}_s^p &= \dot{y}_c^p + \dot{R}1_k \\
LOD_s &= LOD_c + \frac{\Lambda_0}{f} \dot{R}3_k \\
LOD &= \int_t^{t+\Lambda_0} dUT
\end{align*}
\]

Derived from relationship btw Celestial & Terrestrial Systems:

\[X_{TRS} = S.N.P.X_{CRS}\]
Time Series Stacking: Frame Definition (1/3)

• (1) Define the frame at a given epoch $t_0$
  $$\Rightarrow 7 \text{ degrees of freedom to be selected/fixed}$$

• (2) Define a linear (secular) time evolution
  $$\Rightarrow 7 \text{ degrees of freedom to be selected/fixed}$$
  $$\Rightarrow \text{Assume linear motion both for stations and frame parameters:}$$
  – Add break-wise approach for discontinuities
  – Investigate the non-linear part in the time series of the residuals
• (1) Minimum Constraints Approach: Select an external frame as a "reference" \((X_R)\)

\[
X_R = X_c + A\theta \quad \theta = 0 \\
(A^T A)^{-1} A^T (X_R - X_c) = 0
\]

• (2) Internal (Intrinsic) Constraints (See next)
Time Series stacking: Frame Definition (3/3)
(Intrinsic Conditions: CATREF Software)

- Consider Transfo. Param. as unknowns in Normal Eq. Sys.
- Estimate time series of Transfo. Param. & long-term solution
- Considering linear transf. parameter $P$:

$$
P(t) = P(t_0) + \dot{P}.(t - t_0)
$$

(1)

- Eq. 1 could be solved by linear regression:

$$
\left(
\begin{array}{c}
K \\
\sum_{k \in K} (t_k - t_0) / \sum_{k \in K} (t_k - t_0) \\
\end{array}
\right)
\left(
\begin{array}{c}
P_k(t_0) \\
\dot{P}_k \\
\end{array}
\right) = 
\left(
\begin{array}{c}
\sum_{k \in K} P_k \\
\sum_{k \in K} (t_k - t_0) P_k \\
\end{array}
\right)
$$

Intrinsic conditions:

$$
P(t_0) = 0 \quad \& \quad \dot{P} = 0 \quad \text{or} \quad \left\{
\begin{array}{c}
\sum_{k \in K} P_k(t_k) = 0 \\
\sum_{k \in K} \frac{P_k(t_k)}{(t_k - t_0)^{-1}} = 0
\end{array}
\right.
$$
Intrinsic Conditions

\[ P(t_0) = 0 \quad \& \quad \dot{P} = 0 \]

\[
\begin{cases}
\sum_{k \in K} P_k(t_k) = 0 \\
\sum_{k \in K} \frac{P_k(t_k)}{(t_k - t_0)^{-1}} = 0
\end{cases}
\]

- Preserve the intrinsic origin of SLR
  - Seen as No-Net-Translation condition
  - Preserve/Realize the long-term CoM as sensed by SLR
- Preserve the intrinsic scale of SLR & VLBI
Some illustrating examples
ITRF2005-IGS05 differences

EUREF Symposium – Session 4 – Brussels, June 2008
ITRF2005-IGS05 differences

Up differences ITRF2005 - IGS05

Zuheir Altamimi

1 cm

EUREF Symposium – Session 4 – Brussels, June 2008
Example of an EPN cumulative solution

Differences when using IGS05 or ITRF2005 RF stations

If relative PCV: Use ITRF2005

If absolute PCV: Use IGS05, but if a user wants to be in ITRF2005, what to do?
TRF Scale

• GM adopted (or estimated) value in case of satellite techniques

• Relativistic corrections (all techniques)

• Troposphere modelling (most of techniques)

• Technique-specific effects
  – VLBI, GPS and DORIS antenna-related effects
  – SLR range and time biases
Scale Annual Average

Scale ANNUAL AVERAGE wrt ITRF2005 (ppb)

VLBI-IVS
(MPT Corrected)
Scale Annual Average

Scale ANNUAL AVERAGE wrt ITRF2005 (ppb)

- VLBI-IVS (MPT Corrected)
- SLR-ILRS (ITRF2005 data +)

EUREF Symposium – Session 4 – Brussels, June 2008
Scale Annual Average

Scale ANNUAL AVERAGE wrt ITRF2005 (ppb)

- VLBI-IVS (MPT Corrected)
- SLR-ILRS (ITRF2005 data +)
- DORIS-IGN (ITRF2005 data +)

EUREF Symposium – Session 4 – Brussels, June 2008
An SLR Solution
Intrinsic Origin & Scale

An SLR Intrinsic Origin and Scale

TX mm

TZ mm

TY mm

Scale mm
An SLR Solution
Intrinsic Origin & Scale

EUREF Symposium – Session 4 – Brussels, June 2008
New DORIS Solution (IGN): WRMS

DORIS - IGN Weekly WRMS (mm)

East
North
Up

Number of satellites used

EUREF Symposium – Session 4 – Brussels, June 2008
DORIS IGN Solution
Intrinsic Origin & Scale

DORIS IGN intrinsic origin and scale

TX (mm)

TZ (mm)

TY (mm)

Scale (mm)

EUREF Symposium – Session 4 – Brussels, June 2008
Network effect on EOPs
Change the RF Core sites by 10 sites

Main difference
Drift:
Xp: 2 µas/yr
Yp: 1 µas/yr

EUREF Symposium – Session 4 – Brussels, June 2008
Concluding Remarks

• Simplify the terminology: use TRS and TRF
  – Work to be done within IAG Commission 1

• Time series analysis for station positions and TRF parameters (origin & scale) are critical for TRF implementation. Two types of TRF are needed:
  – Quasi-Instantaneous TRF
  – Secular TRF

• Objective quality assessment of technique systematic errors

• Continuous observations by space techniques are fundamental

• Equally fundamental is the improvement of the geodetic infrastructure