Relative Positioning in Europe: Influence of the GPS+Galileo Satellite Geometry

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Introduction

Previous work [1] showed that the use of the additional Galileo constellation improves absolute positioning based on code observables with about 40% in terms of formal errors when simulating urban conditions. For relative positioning based on double difference carrier phase observables, the concept of ***RDOP*** (Relative Dilution of Precision) allowed to demonstrate that using GPS+Galileo, only half the observation time is sufficient to get similar precisions as with GPS only. These results were obtained under error-free ideal conditions. In this poster, we will step down from the error-free conditions and add to our model the different error sources degrading satellite navigation. These error sources being considered individually, we will focus especially on atmospheric errors and following results for relative positioning. Since most of the error sources are eliminated or reduced when using double differences, the concept of the ***RDOP*** values will be adapted to single frequency relative positioning based on single difference carrier phase observables.

For the calculation of the atmospheric errors, estimated values provided by the International GNSS Service were used. The GPS satellite orbits have been created based on the broadcast navigation message, also provided by IGS. Finally for Galileo, we considered a constellation of 27 satellites distributed over three orbits with a right ascension angles of respectively -120°, 0° and 120°, equally spaced on these orbits by a mean anomaly of -160°,-120°, -80°, -40°,0°,40°, 80°,120° or 160°. Other initial values for orbital parameters were: a semi-major axis of 29 994 kilometers [***km***], an inclination angle of 56°, the eccentricity equal to 0, a rate of right ascension of 0° a day, the argument of perigee equal to 0° and finally a period of 14h04min42s.

Impact of the GPS+Galileo Satellite Geometry on the Error Sources

The Error Budget for Absolute Positioning

As well for code as for carrier phase observations, a certain number of systematic errors has to be taken into account when doing absolute positioning. Depending on their properties, those different error sources can be divided in following groups:

- signal propagation errors:
  ionospheric path delay, tropospheric path delay and multipath
- satellite errors:
  clock bias and orbital errors
- receiver errors
  clock bias and ranging error

The square root of the sum of squares of these individual errors, the so-called User Equivalent Range Error (***UERE***), can be seen as a global error and as a measure of the precision for point positioning. Multiplying this value with the Position Dilution of Precision (***PDOP***) consequently provides an approximation of the position error, [3] and [10]. Finally, note that the values of previously mentioned error sources depend on whether we are dealing with code or with carrier phase observations.
From now on, we will only consider carrier phase observations.

Signal Propagation Errors

As mentioned before, estimated values as well for the ionosphere as for the troposphere are available. For the troposphere the IGS provides us with Zenith Path Delay (***ZPD***) files for stations included in the IGS network, containing values for the total ***ZPD***. For the ionosphere, IONosphere map EXchange (IONEX) files give us values of the Vertical Total Electron Content (***VTEC***) for a grid of points representing the earth. Both products are giving values at zenith, while values at the satellite elevation angle on the path between receiver and satellite are needed. We will therefore use mapping functions typically having satellite elevation as input parameter. Since we can make use of estimations to make calculations for the exact atmospheric errors, our investigation about the impact of GPS+Galileo satellite geometry on the atmospheric errors, will only have to focus on the parameter necessary for the computations mentioned above, i.e. the elevation of the GPS and Galileo satellites. **Figure 1**, **(a)** and **(b)** respectively for single GPS and Galileo systems, shows the worldwide daily mean of the mean elevations of visible satellites for a grid of points representing the earth's ellipsoid, using an elevation cut off angle of 5º.

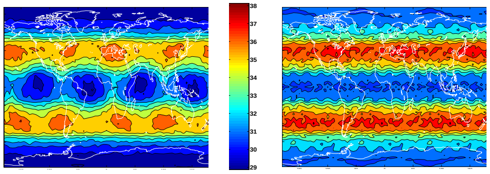

Figure 1a :
Worldwide distribution of the daily mean elevation
of visible GPS satellites using a 5° cut off

Figure 1b :
Worldwide distribution of the daily mean elevation of visible Galileo satellites using a 5° cut off

Worldwide larger mean elevation values for single Galileo system in comparison with GPS only are visible, in particular for approximately 86% of the earth surface. All the differences, positive as well as negative, amount from -2.16° to 2.61°. Adding Galileo to GPS in a new combined system will therefore not yield to very big differences in mean elevation values, but it is still interesting to see how these new values affect the ionospheric and the tropospheric total path delay when using a combined instead of a single GNSS.

Ionospheric Path Delay

The ***VTEC*** values, integrated along the path between receiver and satellite, and expressed in electron per square meters [el/m^2], will be mapped to the corresponding Slant Total Electron Content (***STEC***) by the mapping function ***m(elev)*** of the Klobuchar model, an often used model to calculate zenith ionospheric path delay to the transmitted L1 signal. The ionospheric group delay e_{iono} expressed in meters [m] can be calculated as follows, [4] and [6]:

$$e_{iono} = m(elev) * \frac{40.28}{f^2} * VTEC = \left[1 + 16(0.53 - elev)^3\right] * \frac{40.28}{f^2} * VTEC$$

f is the frequency of the signals on the L1 band 1575.42[MHz], which will be used mutually by GPS and Galileo, while ***elev*** is the mean elevation of the satellites, for this model expressed in number of semicircles of 180°. As could have been expected from the very small amount of evolution in the worldwide mean elevation, yielded by the introduction of a combined system, differences between new and old values for ionospheric path delay seems to be very small. Within the European region these differences had a mean value of -6.5 millimeters [mm] and ranged from -49.8[mm] to 47.4[mm]. Worldwide, those values were a little bigger with a mean difference of -1.35 centimeters [cm] within an interval between -14.31[cm] and 12.83[cm]. **Figure 2** shows the ionospheric path delay at respectively European and worldwide level for the combined system, with respective results of about 2.14 to 4.62 meters [m] and of about 2.06[m] to 7.36[m].

Tropospheric Path Delay

Since values for the total ***ZPD*** are only provided for stations belonging to the IGS network, and this network does not contain enough data to interpolate a complete world-grid, a map for the tropospheric path delay will be made for the European region only. The used mapping function is the one from Black & Eisner, suitable for both hydrostatic as well as for wet delay, [4] and [5]:

$$e_{tropo} = m(elev) * e_{zenith} = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}} * e_{zenith}$$

The mean satellite elevation values are hereby expressed in degrees like usual. Very small differences in tropospheric error between the combined GPS+Galileo and the single GPS system were observed. A mean of -1.92[cm], a minimum of -11.72[cm] and a maximum of 6.32[cm], are the values for the observed differences within the European region. **Figure 3** shows the tropospheric path delay values for the combined GPS+Galileo system varying between respective minimum and maximum values of about 3.41 to 4.63[m] with a mean value of 4.17[m].

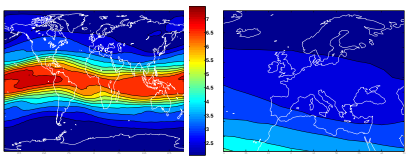

Figure 2 :
Distribution, worldwide and at European level of the ionospheric path delay for the combined GPS+Galileo system

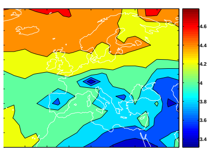

Figure 3 :
Distribution at European level of the tropospheric path delay for the combined GPS+Galileo system

Multipath

Experimental research about the multipath characteristics of permanent GPS stations [9] showed that for the site dependent multipath, it is absolutely necessary to make a correction of 2 to 3[cm] for the slant delay. Within the European region, we will therefore consider a common multipath error of 3[cm] for the remaining part of this poster.

Satellite and Receiver Errors

As for the multipath error, fixed values will be assigned to the satellite and receiver errors. IGS guarantees a maximal value of 5[cm] for the satellite orbit error, while their final clock products, seem to have an accuracy smaller than 0.1 nanoseconds [ns], equivalent with a satellite as well as a receiver clock error of 3[cm], [8]. Finally, receiver ranging errors will not be considered since these errors seems to be negligible (less than one millimeter) for high quality receivers, when measuring carrier phases, [7].

Numerical Overview of the Error Sources

Table 1 gives an overview for the combined system of the all the errors and their values that will be considered further on within the European region. Values for the single GPS system only differ a couple of [cm] for the atmospheric errors and will not be shown. The total ***UERE*** ranges between 4.02[m] and 6.54[m]. An approximated position error is calculated by multiplying the ***UERE*** with the ***PDOP***. The evolution of the values for the individual error sources was considered above, while previous work [1] already showed an improvement ranging from about 30 to 40% for ***DOP*** values in general. **Figure 4** now shows European maps of the approximate horizontal and vertical position error for single GPS system as well as for the combined GPS+Galileo system. The results on **Figure 4a** for the GPS system show horizontal positioning errors ranging between 4.09 and 5.71[m], while for the combined system, **Figure 4b**, equivalent values are all less than 3.87[m]. **Figure 4c** and **Figure 4d** show similar results for the case of the vertical positioning errors, with values ranging from 5.97 to 9.32[m] for the GPS system, while equivalent values for the combined system are all less than 6.22[m]. For both systems, as well in the case of vertical as horizontal positioning errors, we observed a mean improvement for the European region of about 34%, ranging between 30 and 40%.

satellite clock error	0.05[m]
satellite orbit error	0.03[m]
ionospheric path delay	2.14 – 4.62[m]
tropospheric path delay	3.41 – 4.63[m]
multipath	0.03[m]
receiver clock error	0.03[m]
receiver ranging error	/
UERE	**4.02 – 6.54[m]**

Table 1 :
Overview of all the error values and total ***UERE***
at European level

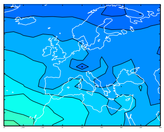

Figure 4a :
Horizontal Positioning Error,
GPS system

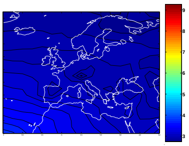

Figure 4b :
Horizontal Positioning Error,
GPS+Galileo system

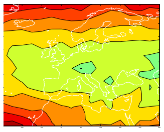

Figure 4c :
Vertical Positioning Error,
GPS system

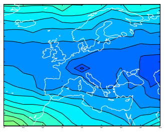

Figure 4d :
Vertical Positioning Error,
GPS+Galileo system

Relative Positioning

Single Difference Carrier Phase Model and Its Error Budget

$$\Phi^j_{pq} = \rho^j_{opq} - \frac{X^j(t) - X_{oq}}{\rho^j_{oq}(t)}\Delta X_q - \frac{Y^j(t) - Y_{oq}}{\rho^j_{oq}(t)}\Delta Y_q - \frac{Z^j(t) - Z_{oq}}{\rho^j_{oq}(t)}\Delta Z_q + c\delta_{pq}(t) + \lambda A^j_{pq} - I^j_{pq} + T^j_{pq} + MP^j_{pq}$$

(ΔX_q , ΔY_q , ΔZ_q) are the unknowns, (X^j , Y^j , Z^j) is the satellite position, (X_{0q} , Y_{0q} , Z_{0q}) is the a priori receiver position, while ρ^j_{0q} is the approximate distance between same satellite j and receiver q. All single differences (***SD***) between receivers p and q are noted as $\bullet^j_{pq} = \bullet^j_q - \bullet^j_p$. The parameters appearing in the model as ***SD*** are: the carrier phase observable (**Φ**), the approximate distance between receiver and satellite (ρ), the ambiguities (A), the receiver clock error (δ), and finally the signal propagation errors (I, T and MP). A property of using ***SD***, is the elimination of the satellite clock error within the model, but because of the use of fixed values for the receiver clock error and for multipath, these errors were also eliminated by using ***SD*** and will not be taken into account any more for the remaining part of this study. In comparison with the errors in **Table 1**, only satellite orbit error and ionospheric and tropospheric errors will be taken into account. For the atmospheric errors, their ***SD*** will now be considered, while the orbital error of a satellite for the case of relative positioning will be equal to its equivalent error for the case of absolute positioning, multiplied by $d / 20000$ with d the distance of the baseline between receiver p and q, expressed in [km]. Using this observation model, we can compute the associated covariance matrix of the unknowns $\Sigma_X = (A^T \Sigma_X^{-1} A)^{-1}$ and convert it to a local topocentric frame, similar to what was done for absolute positioning. The observations are accumulated over sessions varying between ½ and 24 hours, using a 60 seconds measurement interval, using the correlations between the ***SD*** to compute Σ_X. The Relative ***DOP*** (***RDOP***), similar to the ***PDOP*** value for the case of absolute positioning, will be calculated with following formula [2]:

$$RDOP = \sqrt{\frac{trace(\Sigma_X)}{\sigma^2_{DD}}}$$

Results

RDOP values will be calculated for several baselines between EUREF Permanent Network stations, shown in **Figure 5**. Those baselines are subdivided in three groups depending on their orientation: diagonal (= red), north-south (= green) and east-west (= black) baselines. As shown in previous work [1] for the case of double differences, the combined system showed an improvement of the ***RDOP*** value of about 30% in comparison with results obtained from the GPS system using a ***SD*** model. This improvement is shown in **Figure 6** where the magenta and yellow colored lines represent respectively the combined GPS+Galileo and the single GPS system. The position error was considered separately for the north, east and up components, **Figure 7**, showing an improvement of the position error of about 30% for all components. Note that errors for east-west baselines, i.e. baselines between stations with equal latitudes, are systematically less than equivalent errors for the other kinds of baselines. This is mainly due by the fact that for these horizontal baselines, atmospheric errors of both baseline stations don't often differ much, neutralizing each other when considering ***SD*** of these atmospheric errors. Nevertheless, the improvement of 30% was similar for all kinds of baselines.

Figure 5 :
Used baselines within the EPN station network

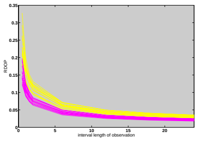

Figure 6 :
RDOP values for single GPS (yellow) and combined GPS+Galileo (magenta) system

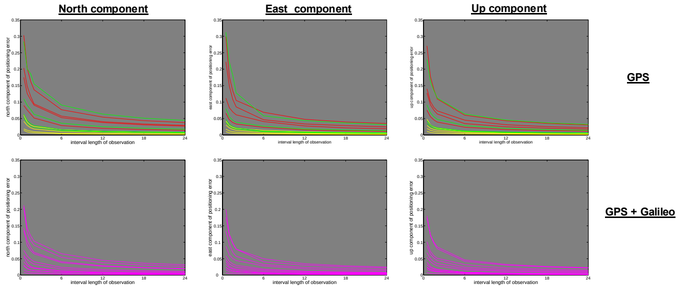

Figure 7 :
Relative position error for diagonal (red), north-south (green) and east-west (yellow) baselines,
for single GPS and combined GPS+Galileo system (all results magenta)

Conclusion

This poster shows the comparison between results for GPS only and the future GPS+Galileo combined system. The worldwide distribution of the mean satellite elevation for the combined system does not seem to differ much from the one for the single GPS system. Using IGS products to compute atmospheric errors with adequate mapping functions, no big improvement will therefore be observed for the values of these errors when considering them individually. Nevertheless, the improvement in ***DOP*** *values, showed in previous work [1], imply a similar improvement for the case of the approximate positioning error. This is true as well for the case of absolute positioning considering* ***PDOP*** *values, as for the case of relative positioning with* ***SD*** *considering* ***RDOP*** *values.*

References

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