EUREF 2004 Symposium Bratislava - Slovakia 2 - 5 June 2004

Using Residuals for Improving the Accuracy of Co-ordinate Transformation

Manuela Vasconcelos (mvasconcelos@igeo.pt)
Instituto Geográfico Português

João Casaca (jcasaca@lnec.pt)
Laboratório Nacional de Engenharia Civil

Abstract

The positions of the vertices of the Portuguese geodetic network (first-order and a large part of the second-order), relative to the European Terrestrial Reference System (ETRS89) have been measured by the Portuguese Geographical Institute (IGP) and are widely used to support differential GPS. The observation of the second-order network will be completed by the end of 2004. However, there are still thousands of trig points (third-order network) that will not be measured with GPS, and for which we need to provide co-ordinates in the ETRS89. Therefore, it is essential to achieve a transformation method as accurate as possible.

A statistical analysis of the residuals of the Bursa-Wolf (BW) transformation between the national Portuguese mainland datum (Datum73) and the ETRS89 was carried out over the first-order vertices, in order to evaluate the global and regional compatibility of the two sets of co-ordinates.

In this poster, we present the first results of a method to obtain the co-ordinates of the third order network, making use of the residuals of the BW transformation between Datum 73 and ETRS89, as a technique to improve the accuracy of the transformation. This procedure is being tested for the second-order points measured until now. Consequently we intend to develop a proceeding that in the future will allow us to produce ETRS89 co-ordinates for the whole third-order network.

1. The Geodetic Network of Portugal Mainland

IGP measured with GPS the vertices of the first-order geodetic network of Portugal mainland (111 points) from 1997 to 1999. This network was computed in **ETRS89** constrained by the points established during European campaigns.

The observation of the second-order network began in 1999 and will be completed by the end of 2004. At the present approximately 50% of the second-order vertices have ETRS89 co-ordinates.

The most commonly used geodetic datum of Portugal mainland is **Datum73**, which was established with classical observations. Datum73 has its origin in the centre of Portugal and is associated with the Hayford ellipsoid.

Since Datum73 is the official reference system for Portugal mainland it is necessary to provide transformation methods from this system to the ETRS89.

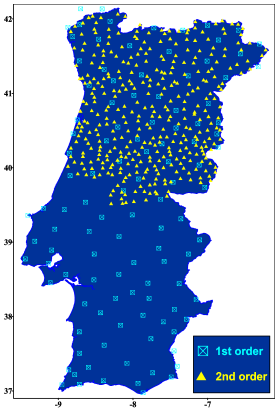


Figure 1 – Geodetic Points with ETRS89 co-ordinates.

2. Datum73 to ETRS89 Transformation for the First-Order Network

The Bursa-Wolf (**BW**) seven parameters (three translations, three rotations and a scale factor) were estimated with the two sets of co-ordinates (in Datum73 and ETRS89) for the 111 vertices of first-order network (Table 1a).

The system of equations for the determination of the parameters is very redundant; the mean redundancy of the system is 0.979. The posterior standard deviation of the unit of weight is 0.39m.

The residuals in X, Y and Z for this transformation are shown in Figure 2 and in Table 2b is presented a statistical analysis of the residuals.

The analysis of the residuals correlation matrix showed that there is a large correlation between the residuals corresponding to the same co-ordinate, and a small correlation between the residuals of different co-ordinates.

From the residuals empirical variance matrix the following empirical standard deviations were averaged:

$s_X = 0.31\text{m}$ $s_Y = 0.45\text{m}$ $s_Z = 0.39\text{m}$

The residuals in latitude, longitude and height were computed and linearized, and the correspondent empirical variance matrix was also computed. The resulting standard deviations of the residuals were:

$s_\varphi = 0.43\text{m}$ $s_\lambda = 0.43\text{m}$ $s_h = 0.29\text{m}$

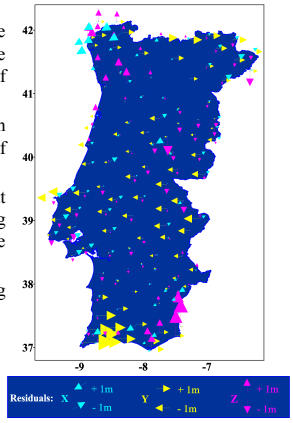


Figure 2 – Residuals of the Bursa-Wolf Transformation from Datum 73 to ETRS89 for the 1st order network.

Tables 1 – Results of the the Bursa-Wolf Datum73 to ETRS89 Transformation for the 1st order network. (a) Parameters. (b) Statistical analysis of the residuals.

Parameters		Standard Deviation
ΔX (m)	- 231.89	± 1.54
ΔY (m)	+ 101.34	± 3.84
ΔZ (m)	+ 27.23	± 1.39
εX (")	- 0.61	± 0.09
εY (")	+ 0.17	± 0.05
εZ (")	- 0.93	± 0.10
δ (ppm)	+ 1.84	± 0.21

Residuals (m)	X	Y	Z
Maximum	+ 0.53	+ 1.44	+ 1.28
3rd Quartile	+ 0.25	+ 0.34	+ 0.22
Median	+ 0.03	- 0.10	- 0.12
1st Quartile	- 0.16	- 0.35	- 0.28
Minimum	- 0.81	- 0.84	- 0.84
Standard Deviation	± 0.31	± 0.45	± 0.39

3. Testing the Bursa-Wolf Parameters on the Second-order Network

A BW transformation, with parameters derived from the first-order network, was applied to the second-order vertices. Since these 362 points already have been observed with GPS, the differences between the measured and the transformed ETRS89 co-ordinates were computed. The standard deviation of the residuals in X, Y and Z was 0.32m and the maximum residual was 0.84m.

The results achieved with the BW parameters were not satisfactory for the application of this method to the entire third-order triangulation points.

4. Improving the Accuracy of the Transformation with the Weighted Average Interpolation

A simple method for spatial interpolation is the Weighted Average (**WA**), which is resumed in Table 2. The residuals from the BW transformation of the first-order network were used to interpolate corrections to be applied to the ETRS89 co-ordinates of the second-order vertices resulting from the BW transformation of the Datum73 co-ordinates.

The differences between corrected transformed ETRS89 co-ordinates and measured ETRS89 co-ordinates produced a sample of 362 residuals. The standard deviation of the residuals in X, Y and Z was 0.12m and the maximum residual was 0.46m.

Table 2 – Formulas used to compute the Weighted Average.

$$\begin{bmatrix} C_{Xk} \\ C_{Yk} \\ C_{Zk} \end{bmatrix} = \sum_{i=1}^{n} \frac{\frac{1}{S_k^3}}{\sum_{j=1}^{n} \frac{1}{S_j^3}} \times \begin{bmatrix} R_{Xi} \\ R_{Yi} \\ R_{Zi} \end{bmatrix}$$

C = corrections
R = residuals
S = distances

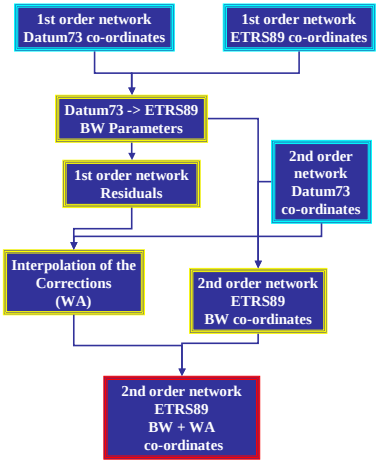


Figure 3 – Scheme of the Transformation Method applied to the second-order network.

The following empirical standard deviations were averaged:

$s_X = 0.14\text{m}$ $s_Y = 0.08\text{m}$ $s_Z = 0.12\text{m}$

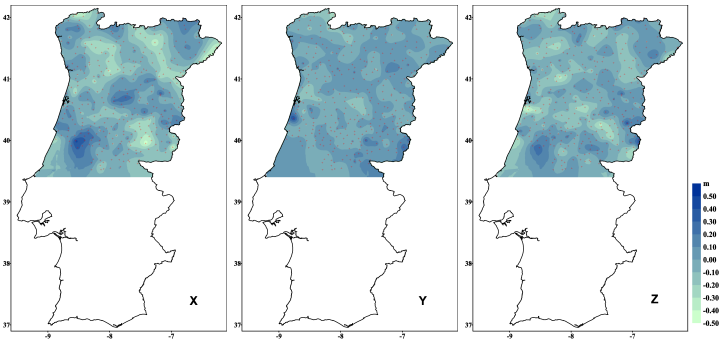
The residuals in latitude, longitude and height were also calculated and linearized. The resulting standard deviations of the residuals were: $s_\varphi = 0.09\text{m}$ $s_\lambda = 0.07\text{m}$ $s_h = 0.16\text{m}$

The WA interpolation reduced the mean standard deviation of the residuals from 0.32m to 0.12m.

This method will be applied to improve the BW transformation of the third-order network.

Table 3 – Comparison of the residuals (X, Y, Z) obtained with BW and with BW plus Weighted Average Interpolation.

Residuals (m)	Bursa-Wolf Transformation	Bursa-Wolf + Weighted Average
Minimum	- 0.82	- 0.46
1st Quartile	- 0.23	- 0.10
Median	+ 0.01	- 0.02
3rd Quartile	+ 0.23	+ 0.04
Maximum	+ 0.84	+ 0.44
Interquartil Range	+ 0.46	+ 0.14
Mean	- 0.02	- 0.03
Standard Deviation	± 0.32	± 0.12
5th Percentil	- 0.60	- 0.23
95th Percentil	+ 0.45	+ 0.15



Figures 4 – Residuals in meters obtained for the Datum 73 to ETRS89 transformation of second-order vertices with Bursa-Wolf parameter plus the Weighted Average Interpolation.