# Regional gravity field from TOPEX/POSEIDON, ERS-1, ERS-2 altimetry and BGI gravimetry data in the Mediterranean and Black Sea area

A. MARCHENKO, Z. TARTACHYNSKA, P. ZAZULYAK

National University "Lviv Polytechnic", S. Bandera St., 12, Lviv, Ukraine, email: march@pancha.lviv.ua

# Introduction

The application of different approaches to data processing of satellite altimetry in such closed regions as the Mediterranean Sea, Marmora Sea, Black Sea, and Azov Sea can be found in numerous studies (see, for instance, MAZZEGA and HOURY, 1989; KNUDSEN and ANDERSEN, 1998; KILIÇOĞLU 2001; MARCHENKO and TARTACHYNSKA 2003; etc.) where final solutions for gravity anomalies and geoid undulations have been constructed on adopted regular grids with different resolution and accuracy.

This paper represents further continuation of the recent study (MARCHENKO and TARTACHYNSKA, 2003) on the inversion of Sea Surface Heights (SSH) into the gravity anomalies in the Black Sea area (based on restricted by short time-period data sets) and will extend here to the Mediterranean and Black Sea areas with better resolution. The recovery of the gravity anomalies  $\Delta g$ and geoid heights N from the combination of ERS1, ERS2, and TOPEX/POSEIDON altimetry and BGI gravimetry data is made for the Mediterranean Sea (from 7°W to 36°E in longitude and from 30°N to 46°N in latitude) and Black Sea area (from 26°E to 42°E in longitude and from 40°N to 48°N in latitude). Nearly 3000000 satellite altimetry data have been applied in this analysis for the approximately 10 year period from 1992 to 2001.

Two solutions for gravity anomalies and geoid heights at  $(2' \times 2')$  grid points over the marine part of the study areas are evaluated by means of the Tikhonov regularization method using the remove-restore procedure and reproducing kernels described by singular point harmonic functions. In the first step the empirical and analytical covariance functions are derived through SSH data only. Then the inverted gravity anomalies are used in combination with the BGI gravimetry data to compute, by the moving average technique, the field of the variance of residual gravity anomalies for the improvement of local empirical and analytical covariance functions within the tested areas. Hence the final combined solution (2×2)E1E2TP for geoid heights and gravity anomalies was constructed in the second step using all mentioned altimetry and gravimetry data and different covariance functions of gravity anomalies within western, central, and eastern sub-regions of the Mediterranean Sea and separately for the Black Sea area. The comparison with the KMS1999 gravity anomalies results in better agreement in the western basin but rather large differences in the eastern basin of the Mediterranean Sea and Black Sea. The comparison of the  $(2\times2)E1E2TP$  geoid heights with EUVN GPS/leveling data (IHDE et al., 2000) leads in general, despite the used extrapolation, to a better accordance in the Black sea area in contrast to the comparison with the EGG97 quasigeoid.

## Data

The following data sets corrected by CSL AVISO for different geophysical phenomena and instrumental effects are used:

- Subset 1 represents 453990 ERS-1 corrected Sea Surface Heights (SSH) taken for the period from 1992 to 1996 year;
- Subset 2 represents 1035626 ERS-2 corrected SSH taken for the period from 1995 to 2001 year;
- Subset 3 represents 1458659 TOPEX-POSEIDON corrected SSH also extracted from the AVISO database and taken for the period from 1992 to 2001 year.

Because of the absence of altimetry data around some islands and near the coastline of the Mediterranean Sea, Marmora Sea, Black Sea, and Azov Sea, the two additional data sets of point gravimetry data are used to support the SSH-only solution:

- Subset 4 represents 247080 values of the BGI point marine gravimetry data ∆g;
- Subset 5 represents 219791 values of the BGI point land gravimetry data  $\Delta g$  over the study region (islands and countries around the Mediterranean and Black Sea).

## Method

All data sets were converted beforehand to the WGS84 reference ellipsoid. Taking into consideration a large total number of the corrected SSH (2627220) in the case of the Mediterranean Sea (region I), in the preprocessing step the regular distributed SSH values were predicted at 94171 ( $3' \times 3'$ ) grid points by the least-

squares prediction method using the Gaussian covariance function. In the case of the Marmora Sea, Black Sea, and Azov Sea (region II) the regular distributed SSH values were predicted in the same manner from 321056 irregular SSH data points to the 36034 ( $2' \times 2'$ ) regular grid points. BGI gravity anomalies were averaged to the total numbers 37422

and 4693 of  $(5' \times 5')$  mean values for the Mediterranean and Black Sea areas, accordingly. Note that additional crossover verification of BGI gravimetry was made to detect and delete gross discrepancies from these data. Fig. 1 and Fig. 2 illustrate such regular SSH and  $\Delta g$  data distribution over the marine part of the Mediterranean and Black Sea regions, respectively.



**Fig. 1**. Distribution of the corrected SSH data (•) at the  $(3'\times3')$  grid points and gravity anomalies (•) at the  $(5'\times5')$  grid points adopted as initial data in the Mediterranean Sea area (region I)



**Fig. 2**. Distribution of the corrected SSH data (•) at the  $(2'\times2')$  grid points and gravity anomalies (•) at the  $(5'\times5')$  grid points adopted as initial data in the Black Sea area (region II)

After these preliminary computations the traditional "remove-restore" procedure for the geoid and gravity anomalies estimation is realized in the following three steps for each region (I and II) separately.

- 1. The contribution of the geopotential model EGM96 up to degree/order 360 is removed from the gravity anomalies  $\Delta g$  ( $\delta \Delta g = \Delta g \Delta g_{EGM96}$ ) and SSH altimetry data ( $\delta N = SSH N_{EGM96}$ ), which in this case are adopted as the geoid heights *N*.
- 2. Residual gravity anomalies  $\delta \Delta g$  and the residual geoid heights  $\delta N$  are predicted at the (2'×2') grid points inside the studying marine regions using the regularization method (MORITZ, 1980; MARCHENKO et al, 2001; MARCHENKO and TARTACHYNSKA, 2003).
- 3. The gravity anomalies  $\Delta g = \delta \Delta g + \Delta g_{EGM96}$  and the geoid heights  $N = \delta N + N_{EGM96}$  are restored by means of the EGM96 gravity field model at the  $(2' \times 2')$  grid points for each marine area.

#### **Regional gravity field solution**

The anomaly heights N and the gravity anomalies  $\Delta g$ , called here the (2×2)E1E2TP solution, are constructed in the form of a (2'×2') grid using the mentioned remove-restore procedure with the additional improvement of covariance functions by two iterations. After removing the contribution of the geopotential model EGM96 (360,360) from altimetry data (SSH) the residual geoid heights  $\delta N$  and residual gravity anomalies  $\delta \Delta g$  were adopted as initial information. The empirical covariance functions (ECF) are estimated on the first stage from the *residual geoid heights*  $\delta N = \delta SSH = SSH - N_{EGM96}$ . Only two ECF for the I and II regions, respectively, were constructed and approximated by the analytical covariance functions (ACF) based on the radial multipoles potentials or ACF of the so-called point singularities (MARCHENKO and LELGEMANN, 1998). The optimal degree n=1 of the ACF was chosen in both cases from the ECF approximation that corresponds to the dipole kernel function or the modified Poisson kernel without zero degree harmonic.



Fig. 3. Geoid heights of the  $(2\times2)E1E2TP$  combined solution in the Mediterranean Sea area. Contour interval: 2 m.

Having such ACF for  $\delta N$  (and through the covariance propagation also for  $\delta \Delta g$ ) the regularization method is used directly to predict  $\delta N$  and  $\delta \Delta g$  at the grid points  $(2' \times 2')$  for both regions. Then obtained  $\delta \Delta g$  were applied for the computation of new ECF and ACF, based on the *residual*  $\delta \Delta g$  gravity anomalies.

As is well known (see, for instance, MAZZEGA and HOURY, 1989) the Mediterranean Sea has a complex structure of gravity field and an evident difference between its Western and Eastern parts. For that reason, the inverted gravity anomalies were used in the combination with BGI gravimetry data to compute, by the moving average technique, the field of the variance of residual gravity anomalies for the detection of the following three basins.

• The Western basin is the area from 7°W to 12°E in longitude.

- The Central basin is the area from 12°E to 21°E in longitude.
- The Eastern basin is the area from 21°E to 36.2°E in longitude.

The second and final prediction of  $\delta N$  and  $\delta \Delta g$  is made using the corresponding three different local empirical and analytical covariance functions for the Mediterranean Sea and only one covariance function for the Black Sea area (based on the residual  $\delta \Delta g$  gravity anomalies). The same initial data and new ACFs are adopted in the form of the modified Poisson kernel. As a result, the (2×2)E1E2TP solution was restored after the second prediction at the chosen 232356 grid points (2'×2') for the region I and 47164 grid points for the region II. Fig. 3 and Fig. 4 illustrate the obtained geoid heights and gravity anomalies in the Mediterranean Sea. Fig. 5 and Fig. 6 demonstrate the same geodetic functionals in the Black Sea area.



Fig. 4. Gravity anomalies of the (2×2)E1E2TP combined solution in the Mediterranean Sea area. Contour interval: 50 mGal

**Table 1**. Statistics of the gravity anomalies  $\Delta g$  restored at grid points  $(2' \times 2')$  and their accuracy estimation  $\sigma_{\Delta g}$  in different basins of the Mediterranean Sea

Statistics	Western basin		Central basin		Eastern basin	
	∆g, mGal	$\sigma_{\Delta g}$ , mGal	Δg, mGal	$\sigma_{\Delta g}$ , mGal	∆g, mGal	$\sigma_{\Delta g}$ , mGal
Minimum	-142.6	3.9	-196.9	4.4	-309.5	4.4
Maximum	126.2	12.2	210.8	13.5	258.9	17.5
Mean	-6.9	5.4	-12.2	5.9	-42.4	5.8
Standard deviation	29.2		41.1		80.8	



Fig. 5. (2×2)E1E2TP geoid heights in the Black Sea area. Contour interval: 0.5 m.



Fig. 6. (2×2)E1E2TP gravity anomalies in the Black Sea area. Contour interval: 20 mGal.



**Fig.** 7. Accuracy estimation of the (2×2)E1E2TP gravity anomalies in the Black Sea area. Contour interval: 1 mGal.

Additionally, Table 1 shows statistics of the inverted (2×2)E1E2TP gravity anomalies  $\Delta g$  and their accuracy estimation  $\sigma_{\Delta g}$  (standard deviation) in the three considered basins of the Mediterranean Sea. Note also that mean values of the predicted accuracy for the whole Mediterranean region are  $\sigma_N$ =4.7 cm and  $\sigma_{\Delta g}$ =5.7 mGal. Standard deviation and mean deviation of the restored gravity anomalies in this 1st region are 57.1 mGal and -20.5 mGal, respectively.

Fig.7 illustrates the distribution of accuracy  $\sigma_{Ag}$  of the (2×2)E1E2TP gravity anomalies in the Black Sea area. Mean values of predicted accuracy for the Black Sea region are  $\sigma_N$ =3.6 cm and  $\sigma_{Ag}$ =4.5 mGal. Statistics of the restored gravity anomalies in the 2nd region are: mean deviation=-24.1 mGal, standard deviation=37.6 mGal, maximal and minimal deviations are equal to 96.5 mGal and -120.3 mGal, respectively.

## Conclusions

In conclusion, Table 2 illustrates the comparison, given here within the Western, Eastern, and Central basins of Mediterranean Sea, of the above-constructed  $(2\times2)E1E2TP$  gravity anomalies with the  $(2'\times2')$ KMS1999  $\Delta g$  derived also from multi-mission satellite altimetry data (see, ANDERSEN and KNUDSEN, 1998; KNUDSEN and ANDERSEN, 1998).

**Table 2.** Differences between the predicted $(2' \times 2')$ E1E2TP  $\Delta g$  and KMS1999 gravity anomalies

Statistic	Western basin	Central basin	Eastern basin
Minimum	-64.9	-105.7	-89.6
Maximum	100.0	277.7	178.1
Mean	7.7	7.0	15.9
Standard deviation	10.9	12.4	20.7

Note here a better accordance (in terms of standard deviations) with the KMS1999 solution of the predicted gravity anomalies within Western and Central basins. Nevertheless, we get rather large differences between both results demonstrated by Table 2, especially in the Eastern basin, where initial altimetry and gravimetry data may be absent near the islands and coastline. Thus the inverted gravity anomalies may reflect, in this case, only the result of the extrapolation.

The comparison with the KMS1999 gravity anomalies in the region II leads to a similar conclusion about smaller deviations in the western part and greater differences in the eastern part of the Black Sea area. Statistics of comparison for the area II are: mean deviation = -11.8 mGal; standard deviation = 10.8 mGal.

Fig. 8 illustrates the comparison of the  $(2\times 2)E1E2TP$  with EGG97 and EUVN geoids. Differences between

the  $(2\times2)E1E2TP$  geoid heights and EUVN GPS/leveling data (IHDE et al., 2000) leads generally, despite extrapolating from the Black Sea to GPS/leveling stations, to a better accordance of the  $(2\times2)E1E2TP$  solution than EGG97 quasigeoid in the considered EUVN GPS/leveling sites on the whole, and in the case of TR05 and UK04 stations especially.



**Fig. 8**. Comparison of the  $(2\times2)E1E2TP$  and EGG97 geoids is given after datum shift transformation. (Contour interval: 0.25 m). Differences between the EUVN GPS/leveling solution (IHDE et al., 2000), EGG97 quasigeoid, and extrapolated  $(2\times2)E1E2TP$  geoid heights are given separately in parenthesis and square brackets, respectively

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#### References

- ANDERSEN O.B., KNUDSEN P. (1998) Global marine gravity field from the ERS-1 and GEOSAT geodetic mission altimetry. Journal of Geophysical Research, Vol.103, No C4, pp. 8129-8137
- IHDE J., ADAM J., GURTNER W., HARSSON B.G., SACHER M., SCHLÜTER W., WÖPPELMANN G. (2000) The Height Solution of the European Vertical Reference Network (EUVN). Veröffentlichungen der Bayerischen Kommission für die Internationale Erdmessung der Bayerischen Akademie der Wissenschaften, Astronomisch-Geodatische Arbeiten, München, Heft Nr. 61 (IAG/EUREF Publication No. 9, Ed. by J.A. Torres and H. Hornik). pp. 132-145.
- KILIÇOĞLU A. (2001) Determination of the Gravity Field from Satellite Altimetry Data in the Black Sea. Paper presented at the IAG Scientific Assembly. Budapest, Hungary, September, 2001

- KNUDSEN, P., ANDERSEN O. B. (1998) Global marine gravity and mean sea surface from multimission satellite altimetry, In: "Geodesy on the Move, Gravity, geoid, geodynamics and Antarctica", IAG scientific assembly, Rio de Janeiro, Brazil, sept 3-9 1997, Eds, Forsberg, Feissel and Dietrich, IAG symposia, 119, pp. 132-138, Springer Verlag
- MARCHENKO A.N., LELGEMANN D. (1998) A classification of reproducing kernels according to their functional and physical significance. IGeS Bulletin, No 8, Milan, pp. 49-52
- MARCHENKO A., BARTHELMES F., MEYER U., SCHWINTZER P. (2001) Regional geoid determination: An application to airborne gravity data in the Skagerrak, Scientific Technical Report, STR01/07, GeoForschungsZentrum Potsdam.
- MARCHENKO A.N., TARTACHYNSKA Z.R. (2003) Gravity anomalies in the Black sea area derived from the inversion of GEOSAT, TOPEX/POSEIDON and ERS-2 altimetry. Bolletino di Geodesia e Scienze Affini, ANNO LXII, n.1, pp. 50-62
- MAZZEGA P., HOURY S. (1989) An experiment to invert Seasat altimetry for the Mediterranean and Black Sea mean surfaces. Geophysical Journal, Vol. 96, pp. 259-272
- MORITZ H. (1980) Advanced Physical Geodesy. Wichmann, Karsruhe