ED50-ETRS89 TRANSITION MODELS FOR THE SPANISH GEODETIC NETWORK

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Abstract

In this paper we describe the methods tested to perform ED50-ETRS89 datum transition based on the Spanish REGENTE (National Network using Spatial Techniques) campaign. These methods vary from the classical ones (5P, 7P, complex and real polynomials) to distortion modelling where tree different methods: Least Squares Collocation, Minimum Curvature Surfaces and Rubber Sheeting have been analysed Finally, we show the present status of the tools developed to make this change including distributing the grid generated in one of the most widely used format (NTV2) to make more easy integrating the transformation from the user point of view.

Introduction

Once the REGENTE campaign (Figure 1) was finished in November 2001, strongly constrained to IBERIA network (ETRS89 B-class observed in cooperation with cooperation of the Instituto Portugues de Cartografía e Cadastro [25]) this gave rise a complete set of dual coordinates to perform a test of which method is most appropriate.

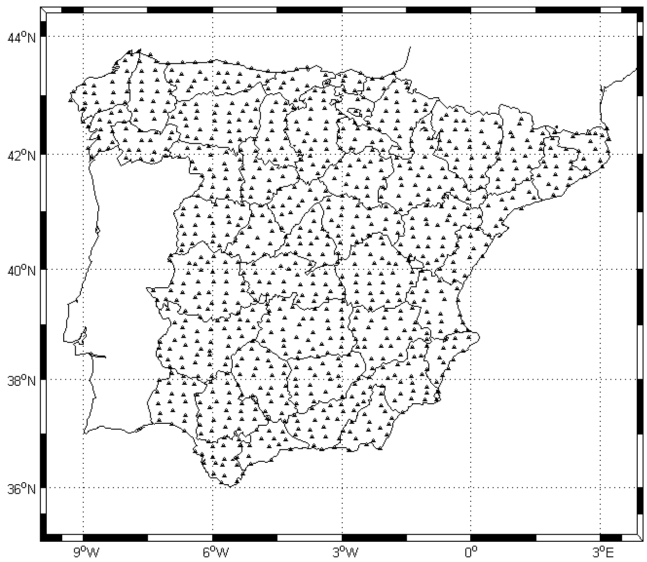

Figure 1REGENTE campaign

If one thinks of the final users of this transformation, the method must ([4]):

- Involve a single transformation.
- Be simple to apply.
- Be available in a user-friendly format for space information users.

- Be capable of transforming large amounts of information efficiently.
- Be integrable in Geographic Information Systems.
- Handle the theoretical conversion from one system to another.
- Imitate the results of a network readjustment, modelling small systematisms and changes of shape.

In this paper we describe some of the methods tested with REGENTE network to achieve such requirements.

Five parameters model

Simply consists on assuming that both systems are parallel and their relationship is just a translation plus Δa y Δf. The result for the whole network is ΔX=87.835m, ΔY=105.771m y ΔZ=122.500m (ED50 to ETRS89 sense)

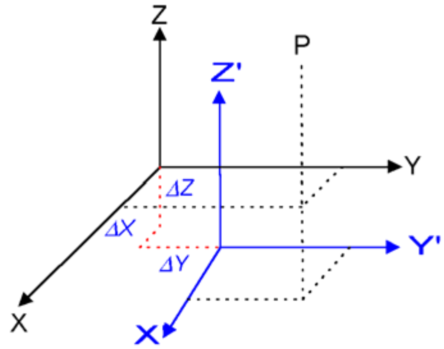

Figure 2 Five parameters transformation model

Statistics	E	N
# points	992	992
Average	-0.05	-0.09
Std Dev	3.44	2.47
Max	8.60	6.84
Min	-7.80	-4.75
Range	16.40	11.59
95%	6.88	4.95

Table 1 5P Residuals for the Spanish mainland

The residuals are considerably high, but if one thinks that this kind of transformation is widely used for most handheld GPS users, with positioning precision between 3 and 10 meters and, that the local behaviour is not high than 4 meters, a 5-parameter solution can be used by several groups of users.

Seven parameters model

The relationship of two revolution ellipsoids in space, with different shape and position, must be mathematically expressed in terms of translation, rotation and scale change. This must be true if the geodetic networks were perfect but the lack of homogeneity of

ED50 due to observation techniques and computational methods of adjustment causes that the 7P model does not fit the full territory with residuals below 2m.

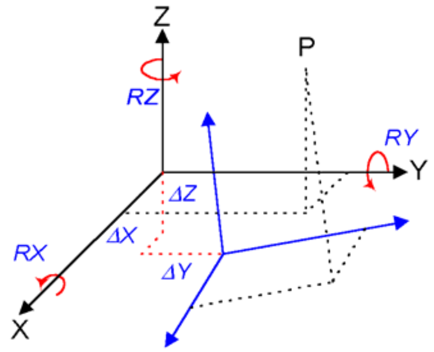

Figure 3 Seven Parameters transformation model

	NW_PENINS.	PENINSULA	BALEARIC ISLANDS
ΔX_0 (m)	178.383	131.032	181.4609
ΔY_0 (m)	83.172	100.251	90.2931
ΔZ_0 (m)	221.293	163.354	187.1902
μ (ppm)	-21.2	-9.39	-17.57
Ω_X (“)	0.5401	-1.2438	0.1435
Ω_Y (“)	-0.5319	-0.0195	0.4922
Ω_Z (“)	-0.1263	-1.1436	-0.3935

Table 2 Seven Parameters for Spain (except Canary islands)

Statistics	E_P	N_P	E_{NWP}	N_{NWP}
# points	829	829	162	162
Average	0.03	0.18	0.02	-0.07
Std Dev	0.62	0.56	0.40	0.25
Max	1.80	2.24	1.28	0.57
Min	-2.28	-2.37	-0.80	-0.76
Range	4.08	4.61	2.08	1.33
95%	1.23	1.11	0.79	0.50

Table 3 Statistics for the Seven Parameters Model

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ED50} = \begin{bmatrix} \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \end{bmatrix} + (1+\mu) \begin{bmatrix} 1 & \Omega_z & -\Omega_y \\ -\Omega_z & 1 & \Omega_x \\ \Omega_y & -\Omega_x & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ETRS89} \qquad \textbf{(Ec. 1)}$$

Achieving 2m residuals is only possible splitting peninsular Spain in two parts: 41º30’N<φ<43º50’N y 9º25’W<λ<4º30’W) and rest. The parameters (ETRS89->ED50 sense and sign convention as Ec.1) are shown in Table 2 and the residuals in Table 3

Complex and real Polynomial model

If one observes the Figure 4 that shows the difference between both datums it is logical to think that a polynomial must fit the datum differences better than the 7P model. In accordance with this assumption it is possible to make two models: real and complex. Complex model were not used in the past for datum transformations but this was true for cartography, for instance between different conformal projections.

$$\varphi' = C_1 + C_2\lambda + C_3\varphi + C_4\lambda^2 + C_5\lambda\varphi + C_6\varphi^2 + C_7\lambda^3 \ldots$$

(Ec. 2)

$$\lambda' = C'_1 + C'_2\,\lambda + C'_3\,\varphi + C'_4\,\lambda^2 + C'_5\,\lambda\varphi + C'_6\,\varphi^2 + C'_7\,\lambda^3 \ldots$$

$$\lambda' + i\,\psi' = \sum_{j=0}^{n} (K_j + iK_j^{'})(\lambda + i\,\psi)^j$$

(Ec. 3)

The method used to accept/reject significant terms was “progressive elimination” (see [10] for more details) including most of the regression controls commonly used in literature. The complex model provides more numerical stability for high polynomial degree, this explain why the residuals in the Table 4 are lower in the complex solution than in the real one.

Statistics	E $_{Real}$	N $_{Real}$	E	Ni
# points	991	991	991	991
Average	0.00	0.00	0.00	0.00
Std Dev	0.37	0.36	0.24	0.22
Max	0.98	1.48	0.94	1.03
Min	-1.66	-1.56	-1.09	-0.77
Range	2.64	3.04	2.03	1.80
95%	0.75	0.75	0.48	0.44
99%	0.93	0.89	0.60	0.55

Table 4. Residuals of real and complex polynomial transformation model

The main problem using polynomial rises when transformations are done outside of the data area. One must be careful when computing datum differences outside the data area, Appelbaum [1] used in the past “dummy points” outside the data area to ensure a linear behaviour outside the border, the same concept as the virtual points in the rubber sheeting transformation mentioned in the next section.

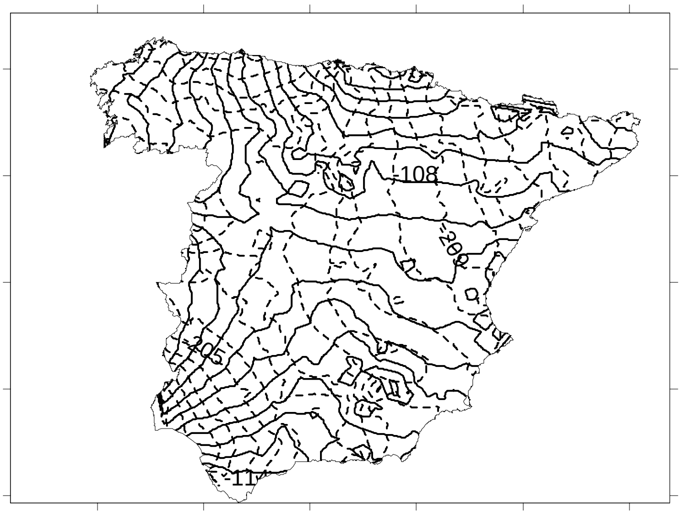

Figure 4. ED50-ETRS89 datum differences. North component with solid line.

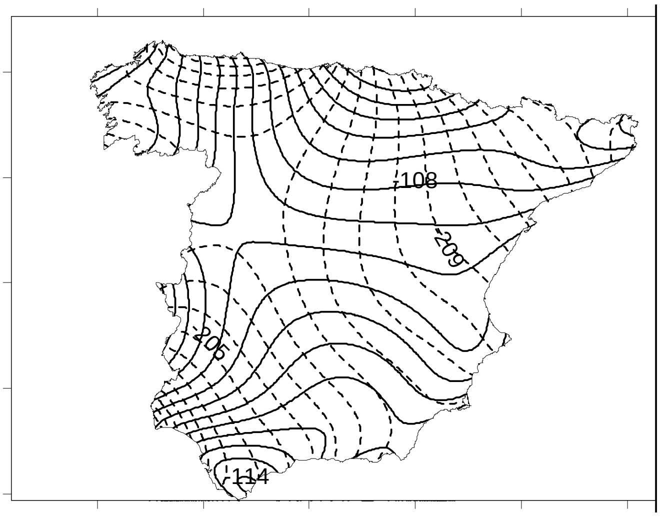

Figure 5. Real polynomial model

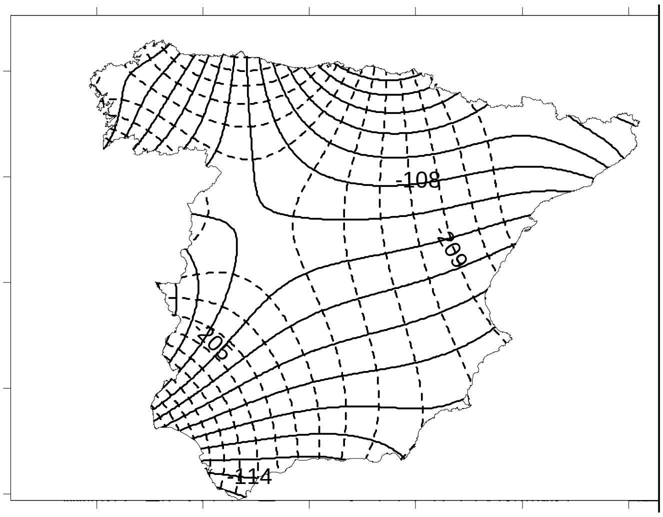

Figure 6 Complex polynomial model

Distortion modelling

Let's assume there is no mathematical model that fits exactly the two datums, then ED50 must have any kind of distortion, if such distortion were known could be removed and generate an "exact" relationship between the two datums.

The goal is to model the distortion and at the same time trust on the theoretical 7-parameter datum model. If we think of two ellipsoids of different shape and sizes in two different positions of the space but near parallel each other a conformal 7-parameter transformation should be the ideal mathematical model. There are several causes of this distortion: many points considered as fixed ones with no good quality, different measurement techniques, human observation factors, computation and adjustment strategy etc. What we must do is to compute the best 7-parameter model and then remove the distortion with some model in grid form.

Once the final transformation model (7P+distortion grid) is defined, an important issue is to test the model with the low order network dual coordinates if available.

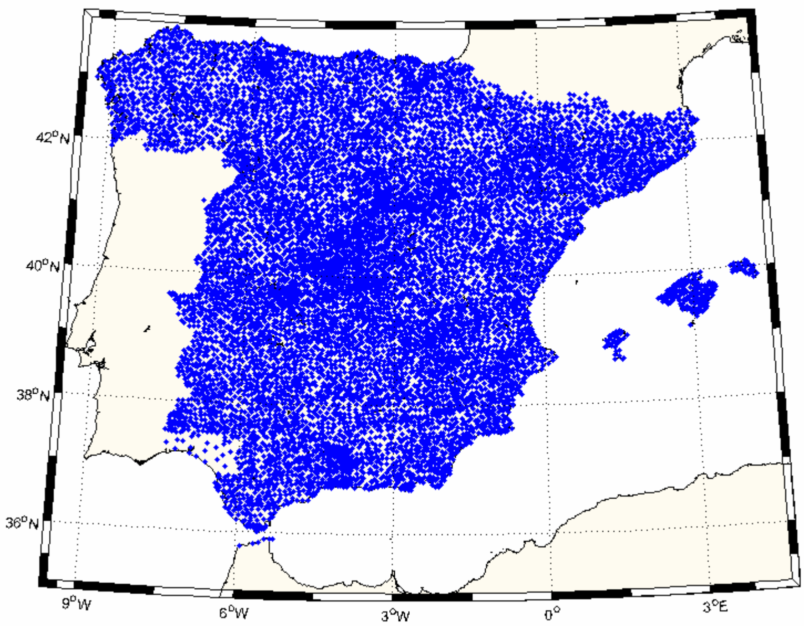

Figure 7 Low order network

Least Squares Collocation

The common least squares model is expanded to $\boldsymbol{Ax+s+n=K}$ where: $\boldsymbol{A}$ is the design matrix, $\boldsymbol{x}$ parameters, $\boldsymbol{K}$ independent term, $\boldsymbol{s}$ signal and $\boldsymbol{n}$ the noise; $\boldsymbol{s}$ and $\boldsymbol{n}$ are considered random variables. Least Squares Collocation seeks to solve the parameters, filter out the noise and compute the signal at points other than the measuring points. This is exactly the case of datum transformation. $\boldsymbol{Ax}$ represents the conformal transformation, $\boldsymbol{s}$ the distortion of the network and $\boldsymbol{n}$ the measuring errors. Using a set of m points with known signal ($\boldsymbol{\Delta\lambda_i}$ $\boldsymbol{\Delta\varphi_i}$, i=1,…, m), it is possible to predict in a point P the value of the signal $\Delta\lambda_P$ $\Delta\varphi_P$ by applying the prediction algorithm given in [18] shows:

$$\Delta\lambda_P = C_I C_D{}^{-1}\Delta\lambda \qquad \textbf{(Ec. 4)}$$

with

$$C_I = \left(C(d_{P1}) \quad \ldots \quad C(d_{Pn})\right) \qquad \textbf{(Ec. 5)}$$

and

$$C_D = \begin{pmatrix} C(d_{11}) & C(d_{12}) & \ldots & C(d_{1n}) \\ . & & & . \\ . & & & . \\ C(d_{n1}) & C(d_{n2}) & \ldots & C(d_{nn}) \end{pmatrix} \qquad \textbf{(Ec. 6)}$$

where C_D is the signal+noise covariance matrix and C_I is a pure signal covariance between the point P and the ones used for the signal estimation. Finally, $\Delta\lambda$ is the distortion at the known points. The same procedure is applied for the latitude.

$$\Delta\lambda = \left(\Delta\lambda_1 \quad \Delta\lambda_2 \quad \ldots \quad \Delta\lambda_m\right)^T \qquad \textbf{(Ec. 7)}$$

Covariance function is empirically obtained, as can be seen in the red dots of Figure 2, and approximately follows a Gaussian function. However, there is a negative covariance from 400Km, so it is better to use Reilly's covariance function (Ec. 8), even though this part of the function is seldom used.

$$C(s) = C_0(1 - \frac{1}{2}(s/d)^2)e^{-\frac{1}{2}(s/d)^2} \qquad \textbf{(Ec. 8)}$$

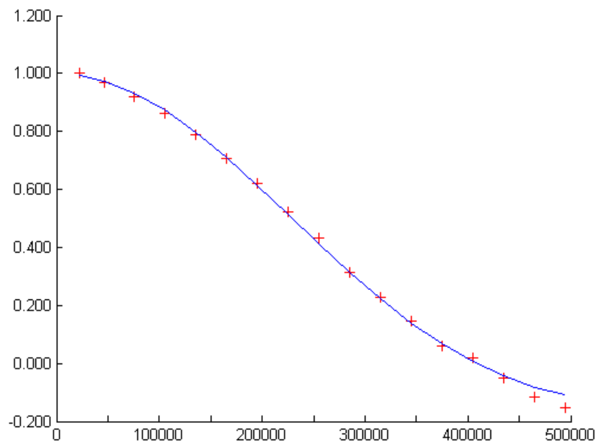

Figure 8. Longitude covariance

Minimum Curvature Surfaces

This method was initially developed by Briggs [3], fortran programmed by Swain [23] and used for the NAD27 to NAD93 datum transition by Dewhurst [9]. The original idea comes from mechanical engineering where, if an ideal metal plate is bent by perpendicular forces, there are no shear forces or tensions in the plate. The forces are the datum distortions at dual coordinate points. The theory developed for creating MCS mainly stems from elasticity theory and splines in tension, and both methods achieve the same results inside the data area, but the second one gives poorer extrapolation results.

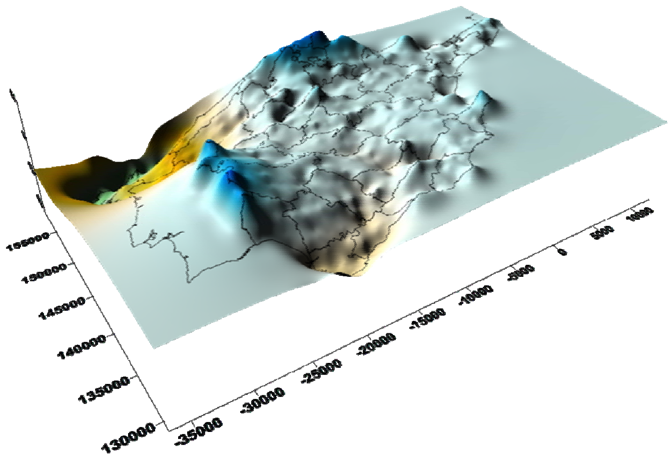

Figure 9. MCS distortion surface

As can be seen in Figure 9 the behaviour of the MCS surface beyond the data area trends to zero, it means that only the former 7P are used when a Ed50<>Etrs89 computation is done.

Rubber Sheeting

This method is usually employed for matching different maps (digital or analogical) in a common coordinate frame deforming as it was a rubber sheet material of one of them. It is more rigorous to say that a topologically equivalent figure is built from the first one to the second, this can be accomplished by different methods, the most used is based on a Delaunay triangulation, then each vertex has its own longitude and latitude shift and a linear transformation can be done inside the triangles.

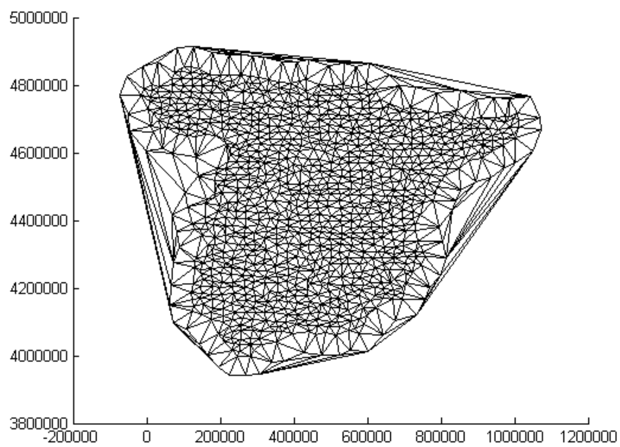

Figure 10.REGENTE Delaunay triangulation

Some virtual points have been created to ensure a linear behaviour in the datum shifts outside the convex hull.

Method comparison

The three methods tested show similar residuals, and the overall performance of each method has been evaluated using simply the sum of the squares of the residuals (Figure 20 and conclusions) as well as in Castilla La Mancha area. MCS was chosen as the method for the Spanish datum transition on account of the extrapolation advantages. The properties of an ideal metallic plate not bent by forces beyond the data area set the value of the model to zero, so the transformation only consists of 7P. The results with independent points are approximately accurate to 15cm (95%), while another test conducted in the north of Spain obtained an accuracy of 17cm (95%), and if one considers that the global relative accuracy of the ED50 network is 10-20cm, the results of the transformation using distortion modelling are below the quality threshold of the worst network.

Statistics	E_{TEST}	N_{TEST}
# points	1400	1400
Mean	0.01	0.02
Std. Dev.	0.05	0.05
Max.	0.24	0.22
Min.	-0.24	-0.24
Range	0.48	0.46
95%	0.10	0.10
99%	0.13	0.12

Table 4. Points below 25cm, MCS solution

Statistics	E_{TEST}	N_{TEST}
# points	1400	1400
Mean	0.00	-0.01
Std. Dev.	0.07	0.05
Max.	0.24	0.19
Min.	-0.25	-0.25
Range	0.49	0.43
95%	0.13	0.11
99%	0.16	0.13

Table 5. Points below 25cm, LSC solution

Statistics	E_{TEST}	N_{TEST}
# points	1395	1395
Mean	0.00	-0.00
Std. Dev.	0.05	0.05
Max.	0.23	0.22
Min.	-0.24	-0.24
Range	0.48	0.46
95%	0.10	0.09
99%	0.13	0.12

Table 6. Points below 25cm, Rubber-Sheeting solution

Range (m)	Longitude	Range (m)	Latitude
Minimun Curvature surfaces			
[-28.10 -28.00]	1	[-1.30 -1.20]	1
[-0.40 -0.30]	1	[-0.30 -0.20]	2

[-0.30 -0.20]	3	[-0.20 -0.10]	1
[-0.20 -0.10]	1	[-0.10 +0.00]	3
[-0.10 +0.00]	1	[+0.00 +0.10]	3
[+0.00 +0.10]	2	[+0.10 +0.20]	3
[+0.20 +0.30]	4	[+0.20 +0.30]	1
[+0.30 +0.40]	2	[+0.30 +0.40]	1
[+0.40 +0.50]	1	[+0.40 +0.50]	1
[+0.50 +0.60]	1	[+0.50 +0.60]	1
[+0.90 +1.00]	1	[+12.30 +12.40]	1
Least Squares Collocation			
[-28.10 -28.00]	1	[-1.40 -1.30]	1
[-0.40 -0.30]	2	[-0.40 -0.30]	1
[-0.30 -0.20]	2	[-0.30 -0.20]	1
[-0.10 +0.00]	1	[-0.10 +0.00]	4
[+0.00 +0.10]	1	[+0.00 +0.10]	4
[+0.10 +0.20]	1	[+0.10 +0.20]	3
[+0.20 +0.30]	6	[+0.20 +0.30]	2
[+0.30 +0.40]	1	[+0.50 +0.60]	1
[+0.40 +0.50]	1	[+12.30 +12.40]	1
[+0.50 +0.60]	1		
[+0.90 +1.00]	1		
Rubber Sheeting			
[-28.10 -28.00]	1	[-1.30 -1.20]	1
[-0.40 -0.30]	2	[-0.30 -0.20]	4
[-0.30 -0.20]	6	[-0.20 -0.10]	1
[-0.20 -0.10]	1	[-0.10 +0.00]	4
[+0.00 +0.10]	3	[+0.00 +0.10]	6
[+0.20 +0.30]	3	[+0.10 +0.20]	2
[+0.30 +0.40]	4	[+0.20 +0.30]	1
[+0.40 +0.50]	1	[+0.30 +0.40]	2
[+0.50 +0.60]	1	[+0.50 +0.60]	1
[+0.90 +1.00]	1	[+12.30 +12.40]	1

Table 7. Anomalous residuals

MCS Test areas

Choosing one method to see what kind of results can we expect from different areas let's make a complete test over three areas. We can not forget that the grid has been computed using 1000 points belonging to REGENTE and separated 25-30Km. If the main purpose is use this grid to perform datum changes at any scale level the prediction values of the grid inside REGENTE points must ensure a certain level of quality. For testing the grid we have employed 5 different areas (aprox 4000 points). Three of them can be seen in Figure 11, Figure 12 and Figure 13, red points have a residual higher than 30cm.

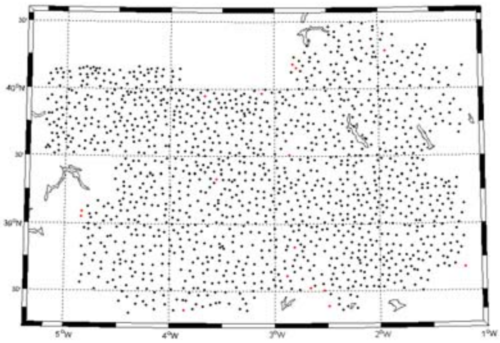

Figure 11. Castilla La Mancha low order network

Statistics	E TEST	N TEST
# points	1400	1400
Mean	0.01	0.02
Std. Dev.	0.05	0.05
Max.	0.24	0.22
Min.	-0.24	-0.24
Range	0.48	0.46
95%	0.10	0.10
99%	0.13	0.12

Table 5. Castilla la Mancha statistics

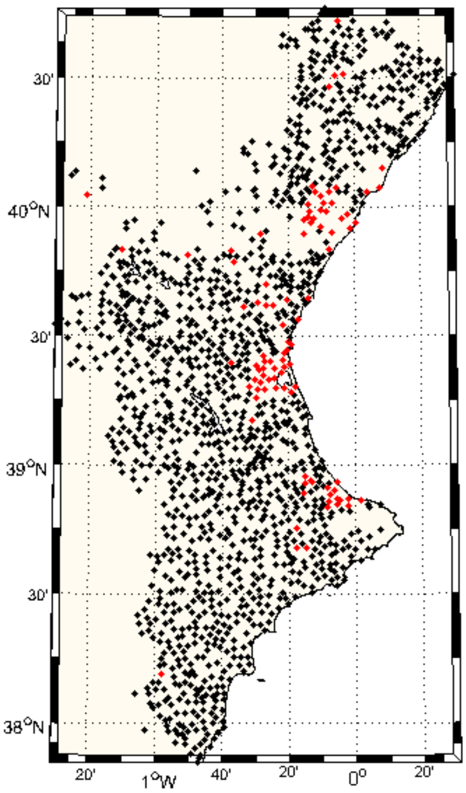

Figure 12. Valencia low and 4th order network (data from ICV)

```
Longitude <0.30                      Latitude <0.30
[   -0.30     -0.20]   23    [   -0.30     -0.20]   10
[   -0.20     -0.10]  144    [   -0.20     -0.10]   49
[   -0.10     +0.00]  560    [   -0.10     +0.00]  482
[   +0.00     +0.10]  477    [   +0.00     +0.10]  661
[   +0.10     +0.20]  124    [   +0.10     +0.20]  145
[   +0.20     +0.30]   38    [   +0.20     +0.30]   19
Longitude >0.30                      Latitude >0.30
[   -0.60     -0.50]    3    [   -0.60     -0.50]   1
[   -0.50     -0.40]    6    [   -0.40     -0.30]   2
[   -0.40     -0.30]   11    [   -0.30     -0.20]   8
[   -0.30     -0.20]   22    [   -0.20     -0.10]  16
```

[-0.20 -0.10]	9	[-0.10 +0.00]	16
[-0.10 +0.00]	7	[+0.00 +0.10]	11
[+0.00 +0.10]	3	[+0.10 +0.20]	2
[+0.20 +0.30]	7	[+0.20 +0.30]	11
[+0.30 +0.40]	18	[+0.30 +0.40]	15
[+0.40 +0.50]	8	[+0.40 +0.50]	8
[+0.50 +0.60]	1	[+0.50 +0.60]	4
		[+1.10 +1.20]	1

1367 <0.300m
96 >0.30m (7.02%)

Table 6. Valencia statistics. (data from Instituto Cartografico Valenciano)

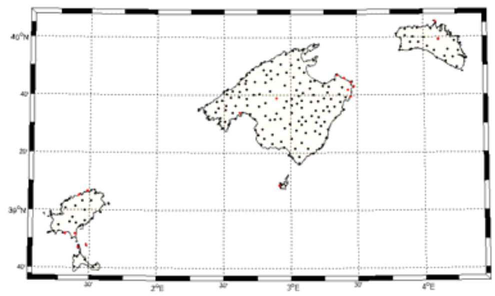

Figure 13. Balearic Islands low order network

Longitud <0.30		Latitud <0.30	
[-0.30 -0.20]	4	[-0.30 -0.20]	1
[-0.20 -0.10]	11	[-0.20 -0.10]	5
[-0.10 +0.00]	54	[-0.10 +0.00]	100
[+0.00 +0.10]	99	[+0.00 +0.10]	76
[+0.10 +0.20]	25	[+0.10 +0.20]	11
[+0.20 +0.30]	1	[+0.20 +0.30]	1

Longitud >0.30		Latitud >0.30	
[-0.90 -0.80]	1	[-0.40 -0.30]	1
[-0.70 -0.60]	1	[-0.30 -0.20]	2
[-0.60 -0.50]	1	[-0.20 -0.10]	4
[-0.50 -0.40]	2	[-0.10 +0.00]	2
[-0.40 -0.30]	2	[+0.30 +0.40]	4
[-0.30 -0.20]	2	[+0.40 +0.50]	1
[-0.10 +0.00]	1	[+0.60 +0.70]	1
[+0.00 +0.10]	3	[+1.50 +1.60]	1
[+0.20 +0.30]	1	195 <0.300m	
[+0.50 +0.60]	1	17 >0.30m (8.72%)	
[+0.70 +0.80]	1		

Table 7. Balearic Islands statistics

Error detection

It is not easy to distinguish between distortion and error, in a general sense we can assume distortion if all the surrounding points follow a similar pattern and error if not. Then, when a "spike" appears it is supposed to have an error, obviously with points separated 25-30km is a risk to assume this. A full dual coordinate set, including low order networks, is needed to be completely sure. There are two methods tested to detect the “spikes”: median difference and “outlier index”. Both are based on a significant change of behaviour of the point investigated when is compared with the surrounding points. In the Figure 14 is shown in red how the median difference is able to detect the spikes.

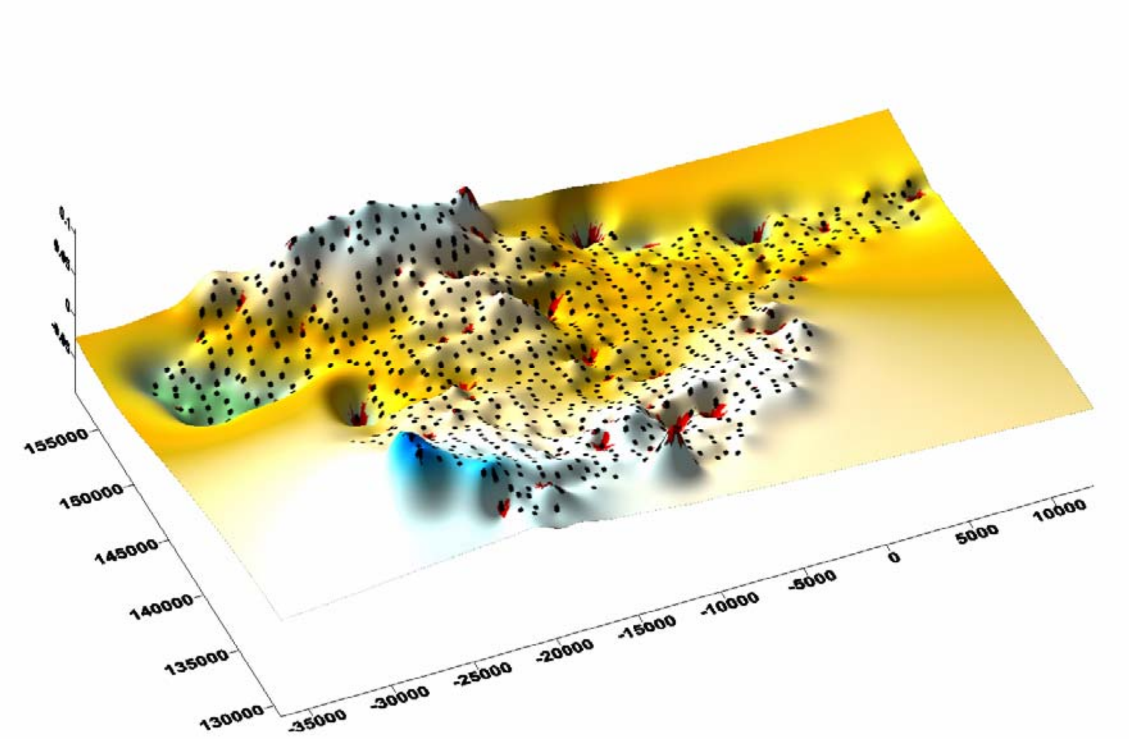

Figure 14. Spikes detection

The median difference consists on comparing the median value of the surrounding points minus the value of the point investigated. The outlier index is based on the similar concept but a inverse weighted mean is used instead of a median value.

Tools created

NTV2 grid

An inportan issue is to determine the way the distortion model+7P is distributed. There are two options: build our own format or try to adapt the model to one existing. The second option is the recommended one from our point of view. There are many advantages using a format already developed by others who had the same problem in the past. Again, there are two alternatives: NADCON (NAD27 to NAD83 format) and NTV2 (National Transformation Version 2). Both formats are easy to implement but the second one is perhaps more universal; this NTV2 grids are the usual way to distribute the Canadian NAD27>NAD83 and AGD to GDA84 Australian transformation models.

The advantages are of course that the grid is already implemented in many commercial packages, for instance:

- Autodesk
- Blue Marble Geographics
- Trimble Total Control
- ESRI Canada Limited (ARC-INFO)
- PCI Geomatics
- Bentley (MGE)

- Guild International Inc.
- xwave
- GeoAnalytic Inc.
- Mapinfo
- Genawarehouse (GenaMap WorldServer Release 8:0)
- Mentor Software, Inc.
- Safe Software Inc. (Feature Manipulation Engine, FME. SpatialDirect)
- Geocomp. (Terramodel for Windows 9.30)
- Schreiber Instruments, Inc. (GeoWindows 1.0)
- MicroSurvey Software Inc. (MicroSurvey 98,MicroSurvey CadPro 3.1)

(*from Australian GDA website*)

But the disadvantage is that the longitudes are negative east . One have to change the sign of the longitudes when filling the grid, it is no big inconvenience but can create some kind of problems in the future (Figure 16)

The biggest advantage is that one binary file can hold as many grids ordered hieratically as needed with virtually any grid interval (Figure 15 and Figure 17).

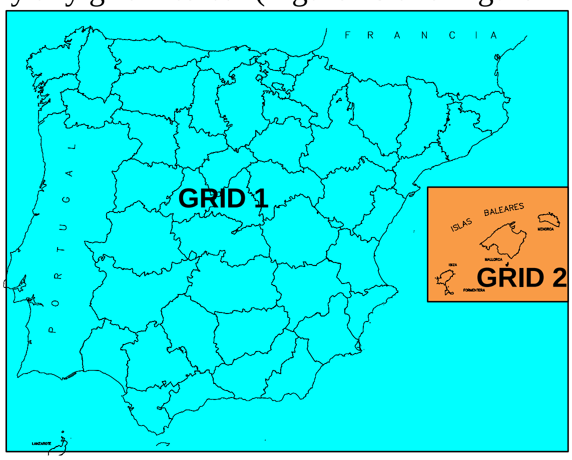

Figure 15. Grids for mainland and Balearic islands

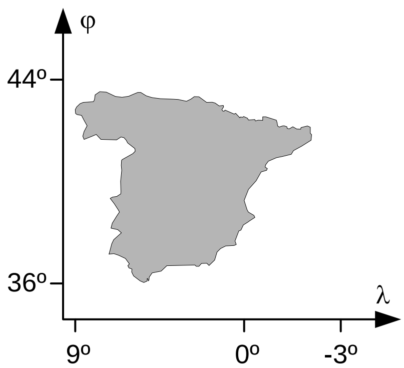

Figure 16. Longitude sign as must be written in the grid

Main header

grid 1: header
grid 1: node values
...
grid N: header
grid N: node values

Figure 17. Hieratical structure of grids

JSP utilities

Is also important to give non-expert users all the necessary tools to perform datum transformations. The most suitable method is to setup a website containing the most useful operations. The technology chosen for this is JSP (Java Server Pages) which gives us a powerful tool to develop any kind of calculus. For instance the JAMA (Java Matrix Package) developed at NIST (National Institute of Standards and Technology) agency belonging to the U.S. Commerce Department's Technology Administration, has been implemented. This package allow us to perform all the matrix operations needed, for instance: SVD decompositions, inverse, transpose…..

The reality and the level of development of Internet let us make tools based on Client-Server architecture, oriented to all kind of applications.

There are different languages and Client-Server architectures, but the Java Server Page (JSP) technology, developed in Java Object-Oriented Programming is the most versatile, attending to the following reasons:

- JSP is independent from the platform (Solaris, Windows, Mac OS, Linux, UNIX,...) and supports all Web Server (Apache, Netscape, ISS,...).

- JSP vs Applet: JSP is a programming Server-Side.

- JSP vs Servlet: JSP separates the esthetical and static web components (HTML code) from the logical and dynamic web components (java classes).

- JSP makes the use of files, which contain necessary data for the mathematical process: DataBase, Grid in binary format, easier

- The maintenance and the update of the server-side application is easier too.

This programming technology, used in all kind of geodetic calculation, let us centralize all mathematical processes in one server point, update the calculation with advantage models and get better results to the client submit.

The result will be a group of web pages (JSP´s) which do geodetics calculations and transformations through Internet. These JSP´s are programmed in Object-Oriented programming and in the server-side.

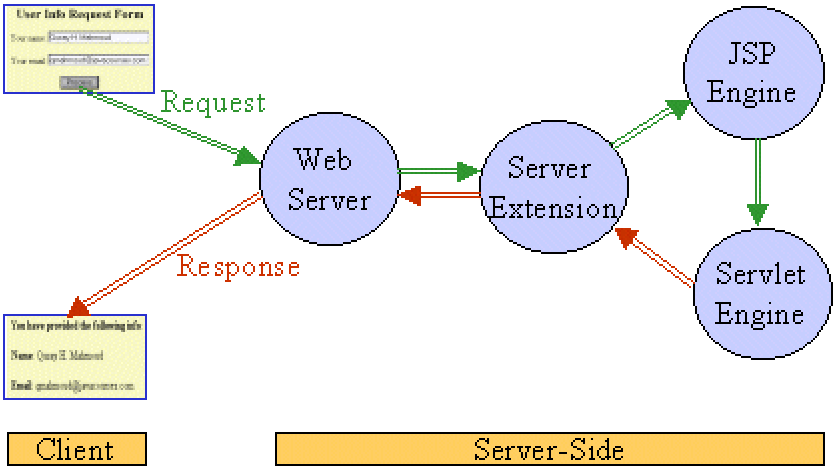

Figure 18. Flow Request/**Response** in a Java Server Page (JSP)

Now we will explain the application to the transformation between the ED50 system and the ETRS89 system, using the Minimum Curvature Surface (NTV2 grid), which has been calculated for Spain mainland.

First, we design the esthetical components of the JSP. This page has a form, in which the client introduces the necessary data in (latitude and length in ED50 system). The java language (JavaBeans) is embedded in the HTML code of the web page.

Next, the JavaBeans class is programmed. This class sends the client´s request, through the JSP form ,to all java components, which calculate the result. It also returns this result, so that it could be incorporated in the final JSP.

And finally, the rest of the java components are codified. The final classes, which can build object, access to file (NTV2 grid), process matrix and calculate result, are programmed and these respond to the final result for the client.

The group of these files (JSP+JavaBeans+Java Classes) are stored in the server. The client sends the request, the server receives it, calculates and gives the result to the client.

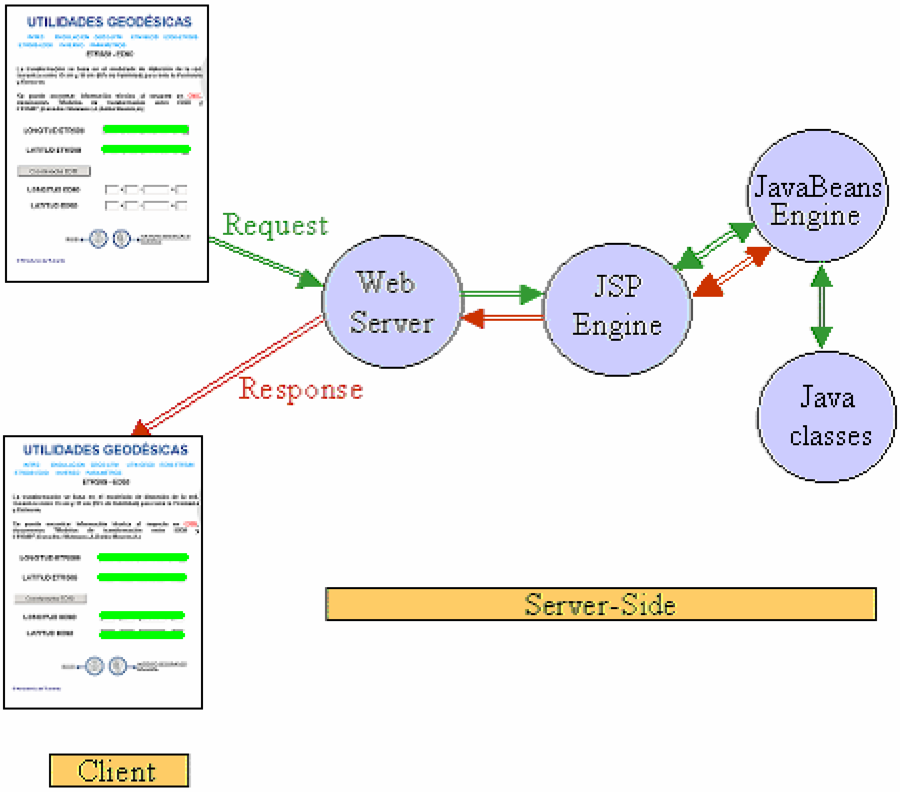

Figure 19. Flow **Request**/Response in ED50ETRS89.JSP

Conclusions

The three methods give results accurate enough for performing a datum transformation. In terms of goodness of fit, the RS method affords the best results, although virtual points have to be built in order to ensure linearity and avoid sharp triangles at the border. LSC affords the advantage of estimating the prediction error. Finally, MCS is a balanced option of goodness of fit and use beyond the limits of the data area. The best conclusion is obviously the results show in the method comparison.

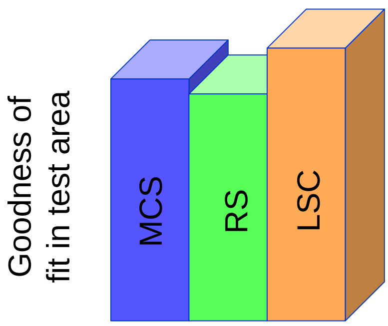

Figure 20. **Global perfomance**

All the tools developed have been published at www.cnig.es ("calculadora geodesica") as well as it will be done at www.ign.es in a near future.

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