

# Towards a Dense European Velocity Field

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## Abstract

The ETRS89 is intended to be a European-wide reference system for all geodetic applications. The EUREF Technical Working Group has initiated a project to establish a Dense European Velocity Field (DEVF) whose main objective is to ensure the long term maintenance of the European Terrestrial Reference Frame 1989 (ETRS89). The DEVF will allow a kinematic instead of somewhat static ETRS89 realization, permitting an accurate positioning in the ETRS89.

## 1. Introduction

The ETRS89 definition stipulates that its realizations should be co-moving with the rigid part of the Eurasian tectonic plate (Boucher and Altamimi, 1992, Altamimi and Boucher 2001). Taking into account that the ETRS89 should cover all the European continent including the deforming part, all kind of crustal deformation and seasonal variation should be taken into account in the establishment of the DEVF. It is anticipated that a grid or and a formula should be properly provided allowing precise kinematic realization of the ETRS89. The DEVF is an ongoing project with continuous improvement and network extension and densification. The input data sought to be included in the DEVF consists in primarily time series of station positions of the diverse European permanent networks, anchored on the European Permanent Network (EPN). National and local GPS permanent networks will be added as well as possibly sufficiently repeated GPS campaigns. In addition to GPS, European VLBI, SLR and DORIS stations will be added, as appropriate. One of the main reasons of using station positions times series in the DEVF development is to be easily able to monitor station non linear motions.

## 2. Analysis Strategy

The analysis strategy will be based on the following steps:

- individual solutions to be considered should be free from any constraints, or with easily removable constraints

- minimum constraints will be used solely for frame definition
- well maintained Reference Stations will be selected for an accurate datum definition
- a raw combination will first be computed in order to identify outliers and jumps for efficient handling of discontinuities
- the final refined combination will be expressed in the ETRS89 from which the DEVF will be extracted.

CATREF software developed for ITRF combination will be used in DEVF time series combination. The combination model of CATREF software is based on a generalized 7-parameter similarity. Assuming that for each individual solution  $s$ , and each point  $i$ , we have position  $X_s^i$  at epoch  $t_s^i$  and velocity  $\dot{X}_s^i$ , expressed in a given TRF  $k$ , the combination consists in estimating:

- Positions  $X_c^i$  at a given epoch  $t_0$  and velocities  $\dot{X}_c^i$ , expressed in the combined frame  $C$ ;
- Transformation parameters  $T_k$ ,  $D_k$  and  $R_k$  at an epoch  $t_k$  and their rates  $\dot{T}_k$ ,  $\dot{D}_k$  and  $\dot{R}_k$  from the combined frame  $C$  to each individual frame  $k$ .

The general combination model is given by the following equation:

$$\begin{cases} X_s^i = X_c^i + (t_s^i - t_0)\dot{X}_c^i \\ \quad + T_k + D_k X_c^i + R_k X_c^i \\ \quad + (t_s^i - t_k) [\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i] \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{cases} \quad (1)$$

where for each individual frame  $k$ ,  $D_k$ ,  $T_k$  and  $R_k$  are respectively the scale factor, the translation vector and the rotation matrix. However, the normal equation constructed using the above set of equations is singular, having a rank deficiency of 14, corresponding to the datum definition parameters. There are several ways implemented in CATREF software to

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define the combined frame. An efficient way is to use an equation of minimum constraints, given by:

$$(A^T A)^{-1} A^T (X_R - X_c) = 0 \quad (2)$$

where  $X_c$  is the vector of estimated station positions and velocities,  $X_R$  is the reference solution containing a selected set of stations and  $A$  is the design matrix of partial derivatives. Unlike the classical constraints applied over station coordinates, minimum constraints are applied over the frame parameters, thus allowing to express the combined solution in any external frame (e.g. ITRF2000), without altering the quality (or internal consistency) of the estimated solution. For more practical details see for instance (Altamimi, 2002).

### 3. An illustrating Test Example

As a test example to illustrate the DEVF project, we used the EUREF combined weekly solutions of the EPN. Figure 1 shows the distribution of the used EPN stations. We used the raw combination outputs of the time series to identify outliers and discontinuities. After outlier rejections, discontinuities were handled using break-wise modelling, e.g. considering different station positions, before and after each jump. Velocity before and after the jump is

constrained to be the same. Figure 2 illustrates the jump in the height component of EUSK EPN station taken as example.

Seasonal variations may occur on some stations which, if not taken into account, may impact the estimated velocities. Amplitude and phase are generally estimated for stations exhibiting such seasonal variations using

$$dx(t) = A \cdot \cos(2\pi f(t - t_0) + \phi) \quad (3)$$

where  $dx$  designates one of the three residual cartesian components,  $A$  and  $\phi$  are annual amplitude and phase, respectively, and ( $f = 1$ ) is the frequency in cycles per year. To illustrate this phenomena, Figure 3 shows clear annual variations of EBRE station.

After properly handling the discontinuities and seasonal variations, a refined combination was performed and expressed in the ITRF2000. Figure 4 shows the extracted horizontal velocities expressed in the ITRF2000 and Figure 5 illustrates the horizontal ETRS89 velocities expressed in the ETRF2000, following Boucher and Altamimi (2001). In addition Figure 6 plots the vertical velocities which are obviously expressed in both ITRF2000 and ETRF2000. Note that in plot 6 one could distinguish upward vertical velocities in the nordic countries which may reflect the Post Glacial Rebond effect.

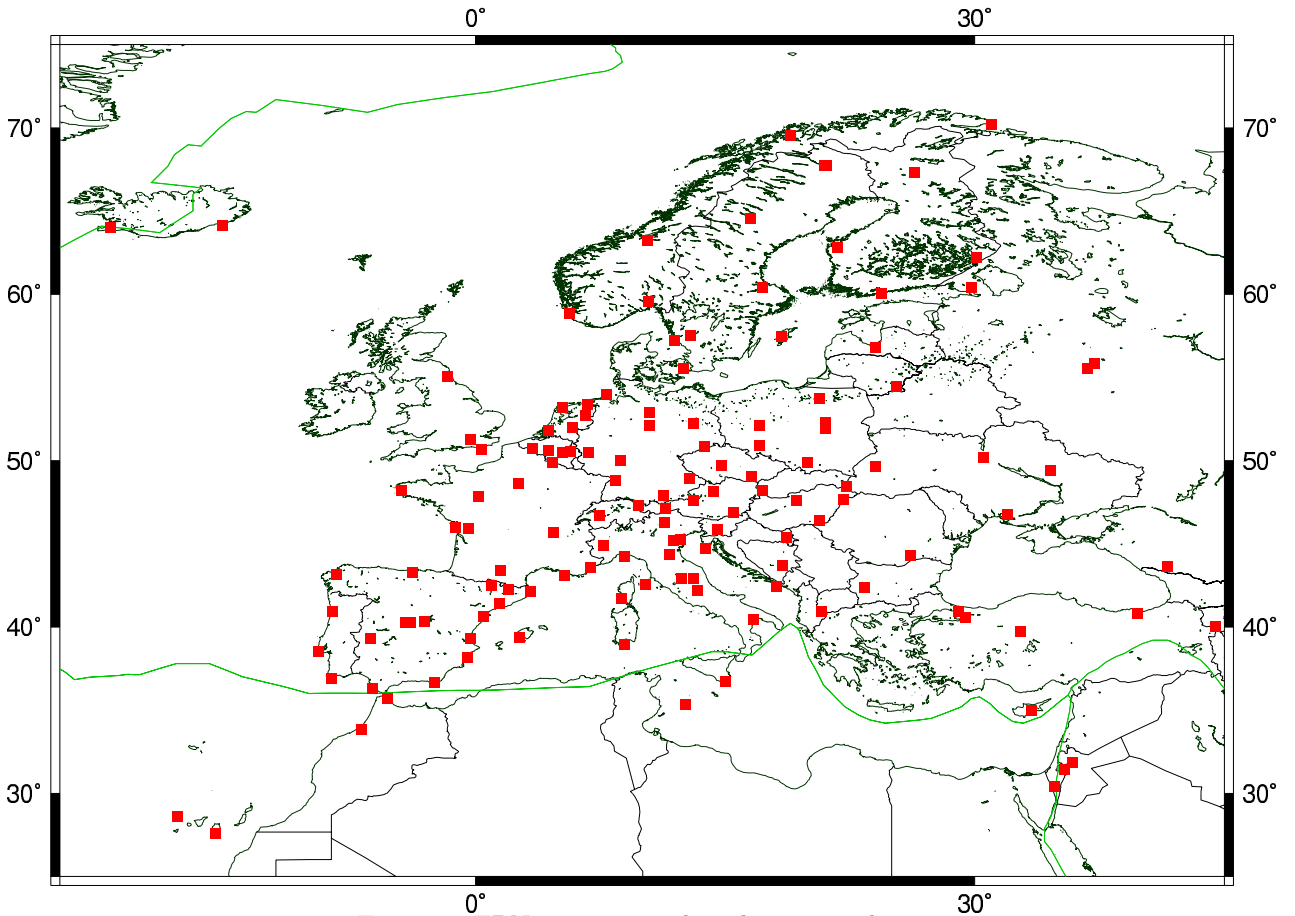


Figure 1. EPN stations used in the test analysis.

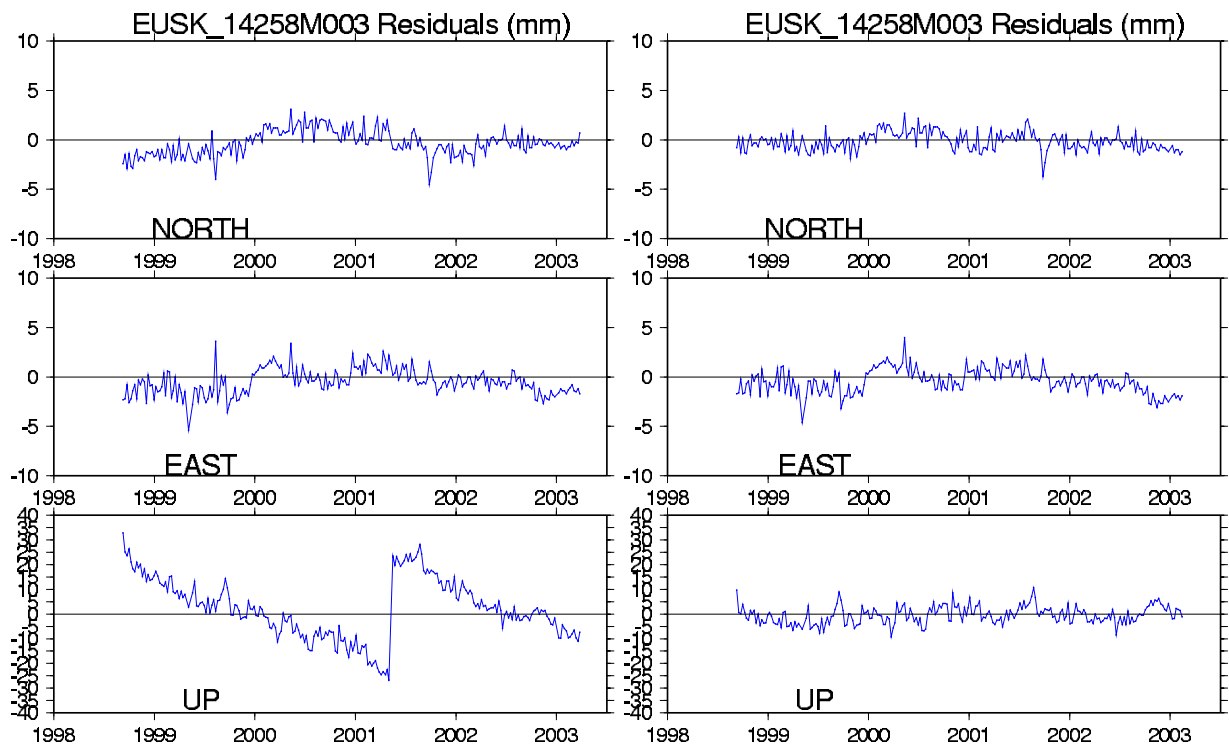


Figure 2. Example of time series discontinuity before and after break-wise modelling.

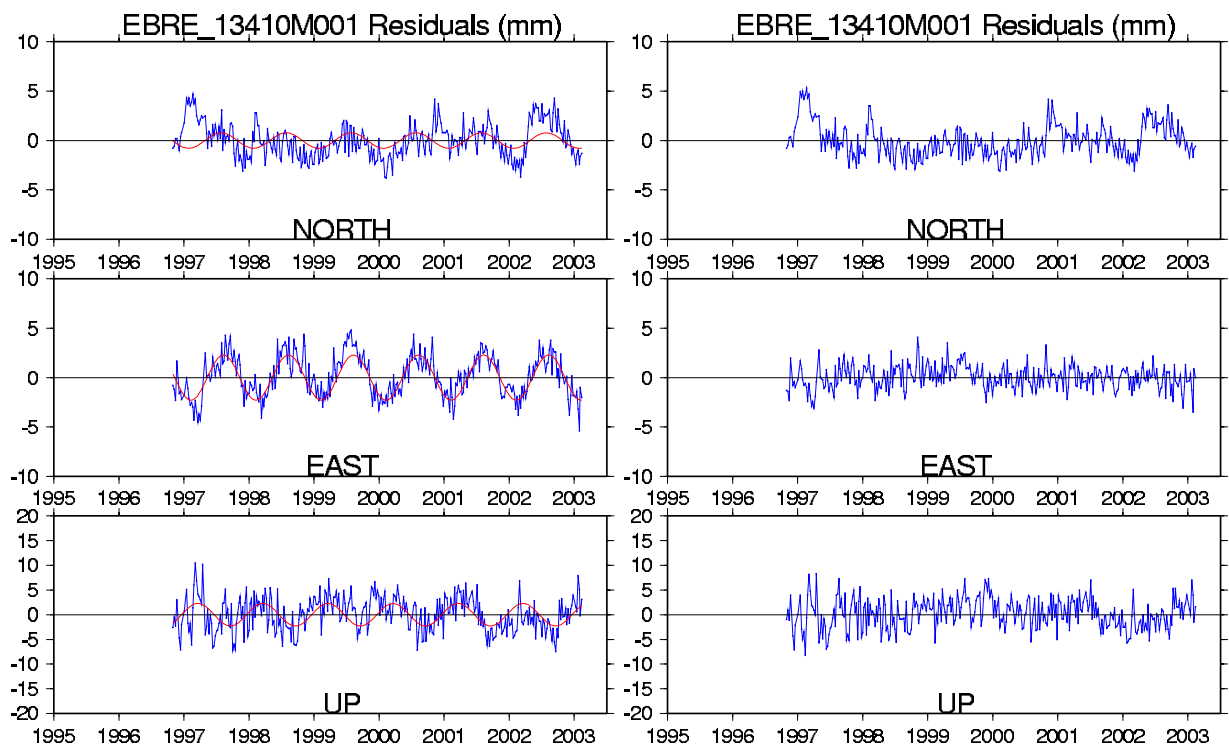


Figure 3. Example of annual variations before and after amplitude and phase modelling.

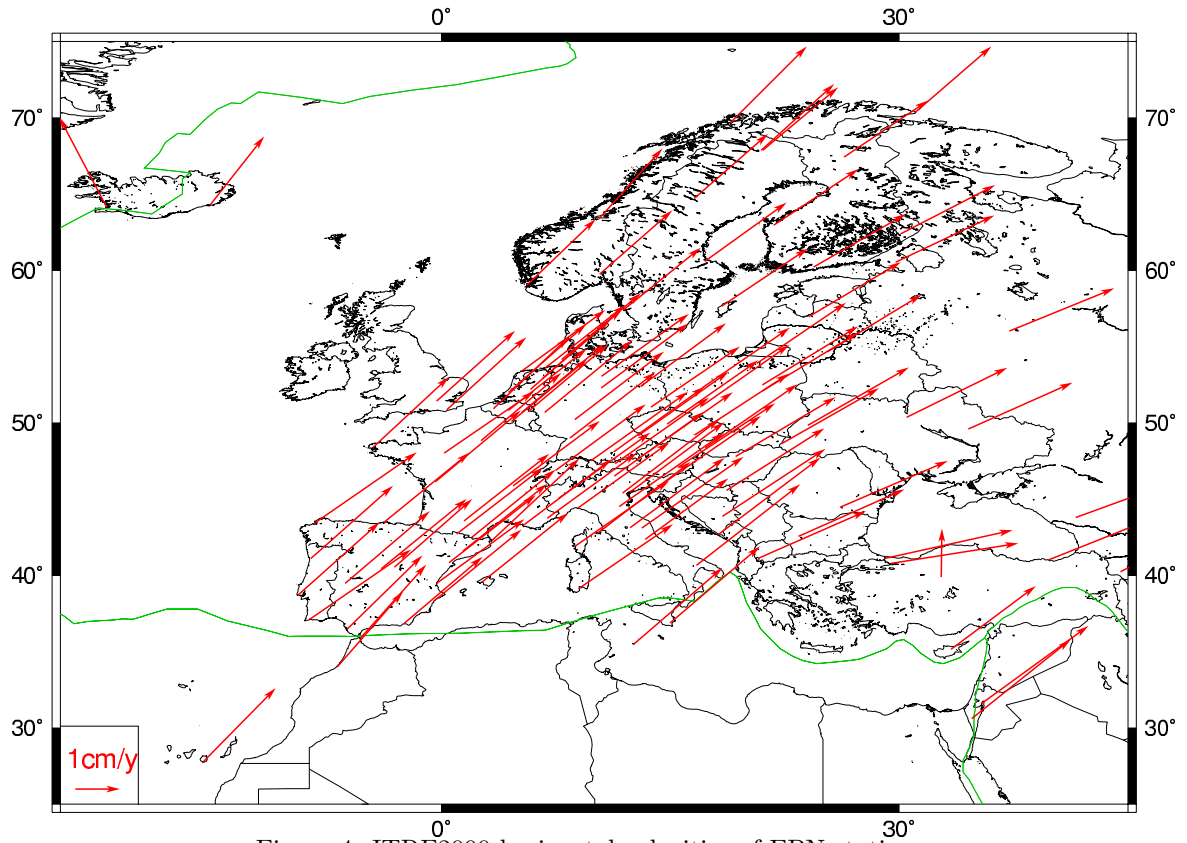


Figure 4. ITRF2000 horizontal velocities of EPN stations.

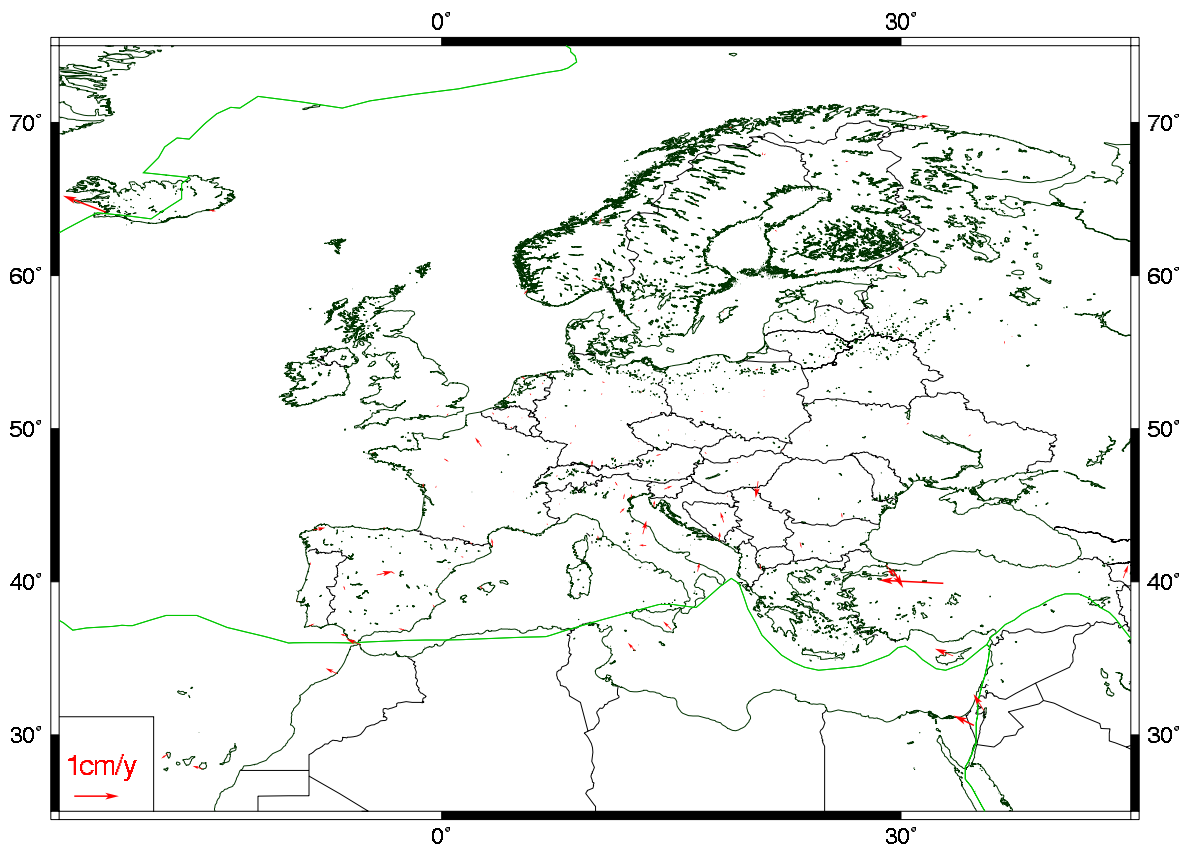


Figure 5. ETRF2000 horizontal velocities of EPN stations.

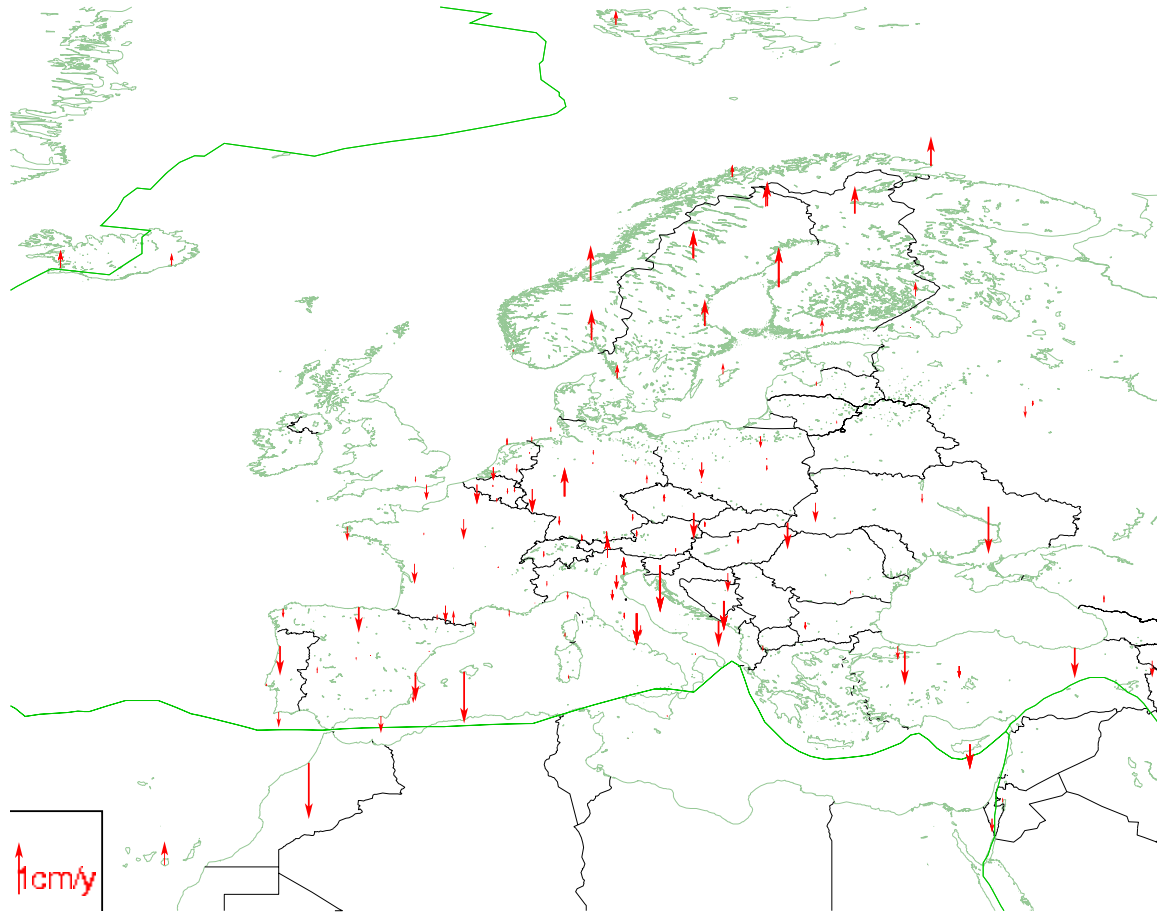


Figure 6. ITRF2000/ETRF2000 vertical velocities of EPN stations.

#### 4. Conclusions

Data span of most European GPS Permanent Networks, covering the whole continent, is now larger than three years. If used, these data will allow the establishment of an accurate and dense velocity field. This ongoing project is open for all contribution from all national and local networks. Contributions are solicited either as weekly and campaign-epoch solutions or as position& velocity solutions. Future work will be concentrated in particular on Post-Glacial Rebound modelling and velocity grid or formula allowing precise and kinematic positioning in the ETRS89.

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