Reference frame activity: Combination of National (RGP) and Regional (REGAL) Permanent Networks Solutions with EUREF-EPN and the ITRF2000

J.-M. NOCQUET[1], E. CALAIS[2], P. NICOLON[1]

Introduction

The determination of an accurate, dense, continental scale consistent reference frame system constitutes one of the goals of the current development of permanent GPS arrays in Europe. The EUREF-EPN defines a frame where weekly solutions provided by the analysis centres are combined and expressed in the current ITRF. However, most of the analysis centres submit to EUREF combination centre a solution including only a subset of the network they are routinely processing. For instance, the IGN Local Analysis Centre (RGP) submission to EUREF-EPN includes only 26 of the ~40 stations actually processed. Moreover, some regional networks set up for geophysical investigations such as the REGAL network around the western Alps are routinely processed but are not included in EUREF weekly combination. In order to densify the realisation of the EUREF-EPN reference frame and cross-check the results coming from different permanent GPS networks and analysis centres, we performed a combination of different permanent network in western Europe.

This work was realised in the frame of geodynamics investigations in collaboration with the CNRS (Nice) and Purdue University. The authors were primary interested in the determination of horizontal velocities for geodynamics investigations, but the combination provided also some correlated results that are described in this contribution.

We present hereafter a combination of position-velocities solutions coming from (1) a selection of 36 ITRF2000 sites, (2) a solution from a subset of sites of the European Permanent GPS Network (EUREF-EPN), (3) a solution of the French national geodetic permanent GPS network (RGP), and (4) a solution of a permanent GPS network in the western Alps (REGAL). The resulting velocity field describes horizontal crustal motion at 64 sites in western Europe with an accuracy better than 1 mm/yr. It is then used it to assess the level of rigidity of the Eurasian plate interior in Europe.

Input data and realisation of individual solutions

ITRF2000

Since our goal is to determine a highly accurate position-velocity solution enabling to test crustal motions at the 1 mm/yr level, we selected ITRF2000 sites that satisfy strict quality criteria (NOCQUET et al., 2001): (1) standard deviation of horizontal velocity <1 mm/yr; (2) weighted rms of horizontal velocity residuals <2 mm/yr in the combination; (3) velocity obtained from at least three different individual solutions; (4) agreement between at least three individual solutions and the ITRF2000 final value better than 1.5 mm/yr; and (5) minimum of 4 years of continuous GPS data in individual solutions for sites not collocated with other techniques. 36 sites in Europe were selected. MEDI (Medicina), with a wrms of 3.3 mm/yr, does not fulfill criterion (3) but was nevertheless included in this study because of its geodetic and geophysical interest (multi-technique geodetic site and active deformation in the Apennines).

EUREF-EPN

45 EUREF-EPN sites located in central and western Europe were selected with at least two years of continuous data. 24 of them are also included in our ITRF2000 site selection (Table 1). Our input data consists of weekly SINEX files from the EUREF-EPN for these 45 sites, spanning the period July 1996 to July 8, 2001.

RGP

In the solution we present, 18 of the stations included in the IGN/LAREG solution are EPN stations and are therefore processed by at least two other EUREF analysis centres. The IGN/LAREG weekly position solution is produced using the Bernese 4.2 software (BEUTLER et al, 2001), following the standard strategy defined in the EUREF recommendations (*ftp://ftp.epncb.oma.be/pub/centre/analysis/IGN.LAC*). Weekly repeatabilities are 2.2 mm and 4.1 mm in the horizontal and vertical components, respectively. A previous RGP solution, with less sites and a shorter data time span included in the ITRF2000 showed a wrms of 0.5 mm/yr for horizontal velocities. Our input data consists of weekly SINEX files for these 40 sites, spanning the period January 1, 1998 to August 28, 2001.

REGAL

The REGAL network is a permanent GPS array covering the western Alps and their surroundings, dedicated to crustal deformation monitoring (CALAIS et al., 2000). The REGAL network started operating in 1997 and currently consists of 19 stations, 4 of them contributing to the RGP. We processed the REGAL network using the GAMIT software v.10.05

[1] J.-M. Nocquet, P. Nicolon, Institut Géographique National, Laboratoire de Recherche en Géodésie, Marne-la-Vallée, France

[2] E. Calais, Purdue University, Department of Earth and Atmospheric Sciences, West Lafayette, Indiana, USA

(KING and BOCK, 2001), including 4 additional RGP stations and 25 EUREF-EPN stations. We solve for station coordinates, satellite state vectors, 7 tropospheric delay parameters per site and day, and phase ambiguities using double-differenced GPS phase measurements, with IGS final orbits and IERS earth orientation parameters relaxed. We obtain long term repeatabilities on the order of 2-3 mm for the horizontal components, and 8 mm for the vertical component. For this study, we selected REGAL sites that have been operating continuously for at least 2 years. Our input data therefore consists of daily SINEX files for 32 sites, spanning the period January 1, 1996, to July 20, 2001.

Realization of individual solutions

Rather than combining the results of individual solutions at the weekly position level, we chose to first derive position-velocity solution for each individual network and to combine the results at the position-velocity level in a second step. We use the general concepts developed for reference frame definition and coordinates/velocities solutions combination (e.g. BROCKMANN, 1997, DAVIES and BLEWITT, 2000, ALTAMIMI et al., in press). SILLARD and BOUCHER (2001) recently pointed out the influence of reference frame constraints in geodetic results and combination. They proposed a strategy using ``minimum constraints'' in order to handle properly reference frame definition in geodetic solutions. We followed their approach. We first started by removing the constraints that were added in individual solutions for reference frame definition. For instance, EUREF weekly combined solutions are provided with a priori constraints of 10^{-4} mm on the position components of a subset of 14 ITRF97 well-determined stations (BOR1, GRAZ, KOSG, MATE, ONSA, POTS, REYK, WTZR, ZWEN, VILL, GRAS, NYA1, TRO1, and THU1, see *http://www.epncb.oma.be/products.html*). Such tight constraints can significantly modify original relative position and the derived velocities. We therefore remove these constraints using the a priori variance-covariance matrix, following the relation:

$$\Sigma_{uncons}^{-1} = \Sigma_{cons}^{-1} - \Sigma_{a\ priori}^{-1}$$

where $\sum_{uncons}^{-1}$, $\sum_{cons}^{-1}$, $\sum_{apriori}^{-1}$ are, the inverses of the variance-covariance matrices of, respectively, the resulting unconstrained solution, the constrained solution, and the ``a priori'' constraints that were applied to the original solution. At this step, the reference frame is only ``loosely'' defined through the final IGS orbits, that were kept fixed during the GPS analysis. The unconstrained variance matrix $\sum_{uncons}^{-1}$ contains both contributions from the natural measurement noise and from the reference system effect. This latter contribution can be reduced by adding the so called ``minimal constraints''. Minimal constraints are the algebraic expression on the variance-covariance matrix that the reference frame implementation is performed through a geometric (usually 7 parameters) transformation. Minimal constraints are added to the coordinate variance-covariance matrix. Using the unconstrained weekly solutions with their associated minimally constrained variance matrices, we then simultaneously compute a position-velocity solution and a time series for each site using the following equation:

$$X_{t_s}^i = X_{sol}^i + (t_s - t_0)\dot{X}_{sol}^i + T_s + D_s X_{sol}^i + R_s X_{sol}^i$$

where X_{ts}^i is the position of site *i* of the weekly solution *s* at the epoch *ts*, X_{sol}^i is the estimated position at the chosen epoch of combination t^0, $\dot{X}_{sol}^i$ is the estimated velocity, and Ts, Ds, Rs are the 7 transformation parameters between the resulting and the weekly solutions at epoch ts. Since velocities are estimated, the temporal evolution of the reference frame must also be defined. This is done by applying a condition that (Ts, Ds, Rs)= 0 at two epochs of the time series (for instance at its beginning and end). The time series $X_i(t)$ is derived using:

$$X_i(t) = X_{sol}^i + \dot{X}_{sol}^i (t - t_0) + v_i(t)$$

where $v_i(t)$ is the residual of weekly solution for site *i* at the time *t* in the estimation of the velocity.

For both EUREF-EPN and RGP data, we noticed that unexplained jumps in the time series can impact the velocity estimate at a 1 mm/yr level (e.g. GOPE station). In order to minimise this problem, we solved for two different positions (before and after the jump) for a point but constrained the velocity to be identical for the entire time series. By doing so, we took benefit of the whole time span available for the velocity estimation. We also excluded EUREF data before GPS week 860 (July 1996) because of a jump in the time series at most sites, probably caused by the change from ITRF93 to ITRF94 in the estimation of precise orbits by the IGS. We name hereafter ``EUREF-IG'' our solution derived from the EUREF-EPN network.

The daily solutions derived from REGAL network are handled differently. We first pass the loosely-constrained daily estimates and their associated variance-covariance matrices to a Kalman filter (GLOBK, HERRING et al., 1990) in order to estimate velocities and positions. At this stage we apply tight constraints on orbits and Earth Orientation Parameters (EOP), but loose constraints on site positions (100 m) and velocities (10 m/yr) at all stations. We obtain a loosely-constrained position-velocity solution, to which we apply minimal constraints on positions and velocities as defined above. A previous REGAL solution including less sites and a shorter data time span produced using this same strategy, was submitted and included in the ITRF2000 definition. It showed a wrms of 0.6 mm/yr on horizontal velocities.

Combination

The combination uses the sites shared by several solutions to tie these solutions into a single and consistent solution. These common sites also serve to cross-check individual solutions and detect outliers. Reference frame constraints applied in individual geodetic solutions can modify significantly the information included in the individual original solutions (SILLARD and BOUCHER, 2001). The combination methodology presented above handles reference frame constraints simultaneously and homogeneously for all individual solutions. We apply a weighting scheme that rescales the variance-covariance matrices of each individual solution and provides realistic formal errors on final estimates.

Table 1: Number of sites of individual solutions used in the combination and number of sites shared by solutions

	EUREF-IG	ITRF2000	RGP	REGAL
EUREF-IG	45			
ITRF2000	22	34		
RGP	18	7	23	
REGAL	24	15	17	32

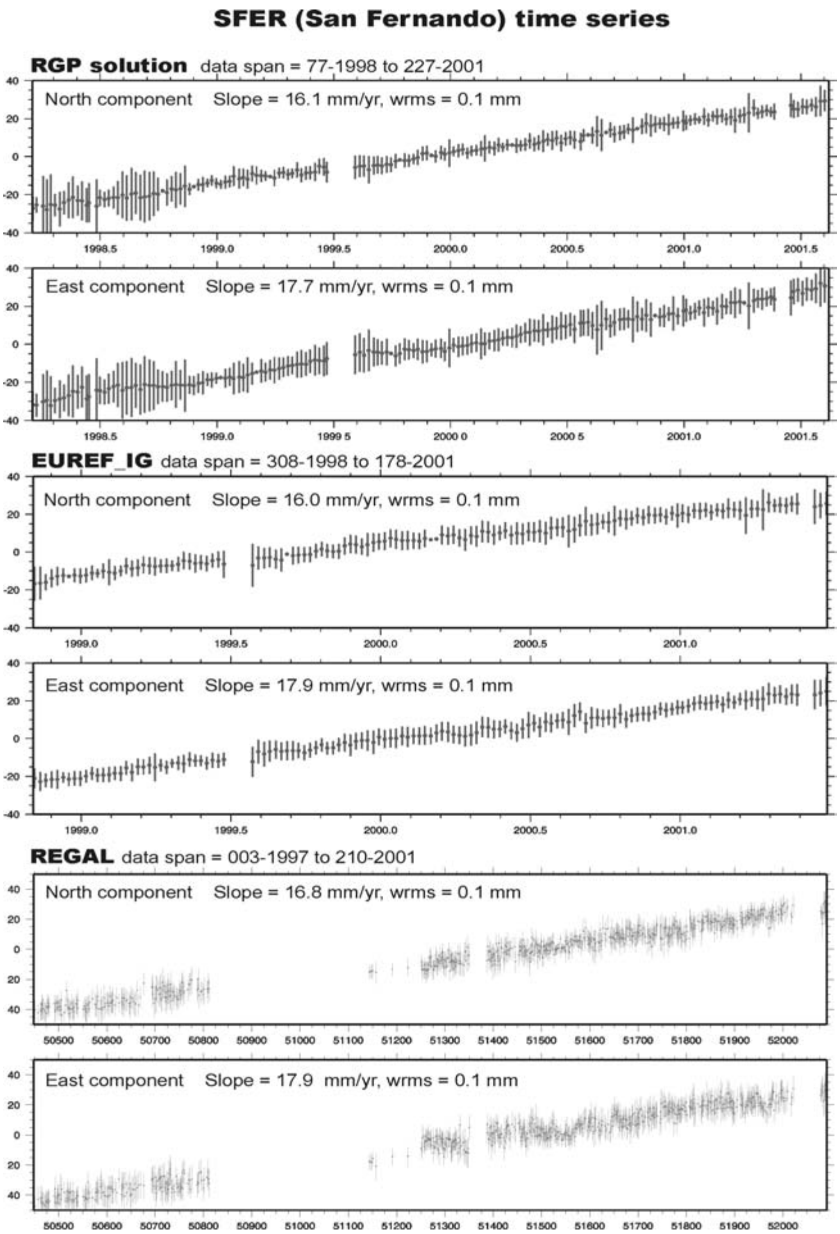

Figure 1: Time series obtained for permanent GPS site San Fernando (Southern Spain) for the results of RGP, EUREF IG and REGAL networks. Velocities are expressed in the ITRF2000. The maximum discrepancy on horizontal velocity component between the 3 analysis is 0.8 mm/yr. The standard deviation on horizontal velocity for the combined solution is 0.3 mm/yr for both East and North velocity component.

Methodology

The input data to the combination consist of individual solutions with minimal constraints applied (see above). We use a combination methodology similar to the one used for the definition of the ITRF (ALTAMIMI et al., in press). For each site i in solution s (s = RGP, REGAL, EUREF-IG, ITRF2000), we simultaneously estimate the position X_{comb}^{i} at epoch t_0 (epoch of the combination), the velocity $\dot{X}_{comb}^{i}$, and a 14-parameters transformation between the individual and the combined solution using (after ALTAMIMI et al., in press):

$$\begin{aligned} X_s^i &= X_{comb}^i + (t_s - t_0)\dot{X}_{comb}^i \\ &+ T_k + D_k X_{comb}^i + R_k X_{comb}^i \\ &+ (t_s - t_k)[\dot{T}_k + \dot{D}_k X_{comb}^i + \dot{R}_k X_{comb}^i] \end{aligned}$$

$$\dot{X}_s^i = \dot{X}_{comb}^i + \dot{T}_k + \dot{D}_k X_{comb}^i + \dot{R}_k X_{comb}^i$$

where X_s^i is the position of site i in solution s at epoch t_s, X_{comb}^i the estimated position of site i at epoch t_0, and $\dot{X}_{comb}^i$ its final velocity in the combination. Tk, Dk, Rk and $\dot{T}$, $\dot{D}$, $\dot{R}$ are the transformation parameters between individual solutions s and the combined solution and their time derivatives. ts is the epoch of minimal position variance for the solution s, which is generally the middle point of the observation time span included in the solution. t_k is the epoch of expression of the transformation parameters. The reference frame definition in the combination is implemented by imposing the 14-parameters transformation between ITRF2000 and the combined solution to be zero (no translation, scale factor, or rotation and no rate of change of these parameters). Our velocity field is therefore expressed in the ITRF2000 reference frame. From this preliminary combination, an a posteriori variance factor s^2_s for each individual solution s is estimated in the inversion, which is then applied to the variance-covariance matrix of the corresponding individual solution in an iterative way until both individual s^2_s and the global a posteriori variance factor equals 1. Normal residuals in the combination are used for outliers detection.

Quality assessment of the results

The wrms of each individual solution for horizontal and vertical position and velocity components provides a first assessment of the solution accuracy (Table 2). For positions, the level of agreement between individual solutions is better than 1 mm for horizontal components (rms 0.6) and 1-3 mm for the vertical component (rms 2.4). We find that all the solutions used here have a wrms on horizontal velocities less than 0.4 mm/yr (rms 0.3 mm/yr). The solution accuracy can also be assessed using the level of agreement between solutions, given by the wrms in the combination for each site. For most sites, we find an agreement between solutions on the order of 0.5 mm/yr. CASC (Cascais), however, shows a disagreement between EUREF-IG, RGP, and ITRF2000 of about 2.5 mm/yr. Also, we find that the EUREF-IG and REGAL solutions significantly disagree on the east component at LAMP (Lampedusa, difference 1.5 mm/yr).

Table 2: Wrms of individual solution in the combination – values are mm (position) and mm/yr (velocity)

solution	position		velocity	
	horizontal	vertical	horizontal	vertical
EUREF-IG	3	26	2	27
ITRF2000	4	31	2	11
RGP	2	20	2	31
REGAL	11	13	4	6

The formal errors of the combined solution depend on the variance of the individual solutions before combination but also on the level of agreement between solutions in the combination and is usually greater than the standard deviation coming from each individual solution. We find formal errors on horizontal velocities lower than 1 mm/yr at all sites except RIGA and GLSV. The best determined sites have a formal error of about 0.2 mm/yr on horizontal velocities.

Stability analysis

As a result of this combination, we obtained a solution for 64 sites in western Europe with standard deviation on horizontal velocities on the order of 1 mm/yr or better. In order to test the long-term stability sites located in western Europe, we used an automatic algorithm using a ``blind'' statistical approach to search for the subset of sites defining a rigid rotation (consistent with their velocity uncertainty) (NOCQUET et al., 2001). We found that the subset [POTS, BOGO, JOZE, GOPE, OBER, WTZR] provides the best fit, with residual velocities less than 0.3~mm/yr. All these sites belong to the supposedly tectonically stable part of the Eurasian plate and an area where post-glacial rebound effect on horizontal velocity does not exceed 0.2 mm/yr (PELTIER, 1995). We then progressively augment this initial subset of sites by adding one site at a time and testing the consistency of the new site subset with a rigid rotation using c^2 and F ratio tests. We find that a 29 sites subset satisfies these statistical tests, given their velocity uncertainties. This domain extends from Central Europe to the westernmost part of Europe, including Spain and Sardinia. Velocity residuals at these 29 sites are less than 0.8 mm/yr. The overall wrms of the residual velocities is 0.4~mm/yr. The best fit Euler vector defined by this site subset is given in Table 3. Figure 2 shows the residual velocities after subtracting the rigid rotation defined above from the velocities. It shows that velocities in the reference frame defined by the 29 sites subset significantly differs from zero at the sites located south of the Iberian Peninsula, in Italy, and in the Alps.

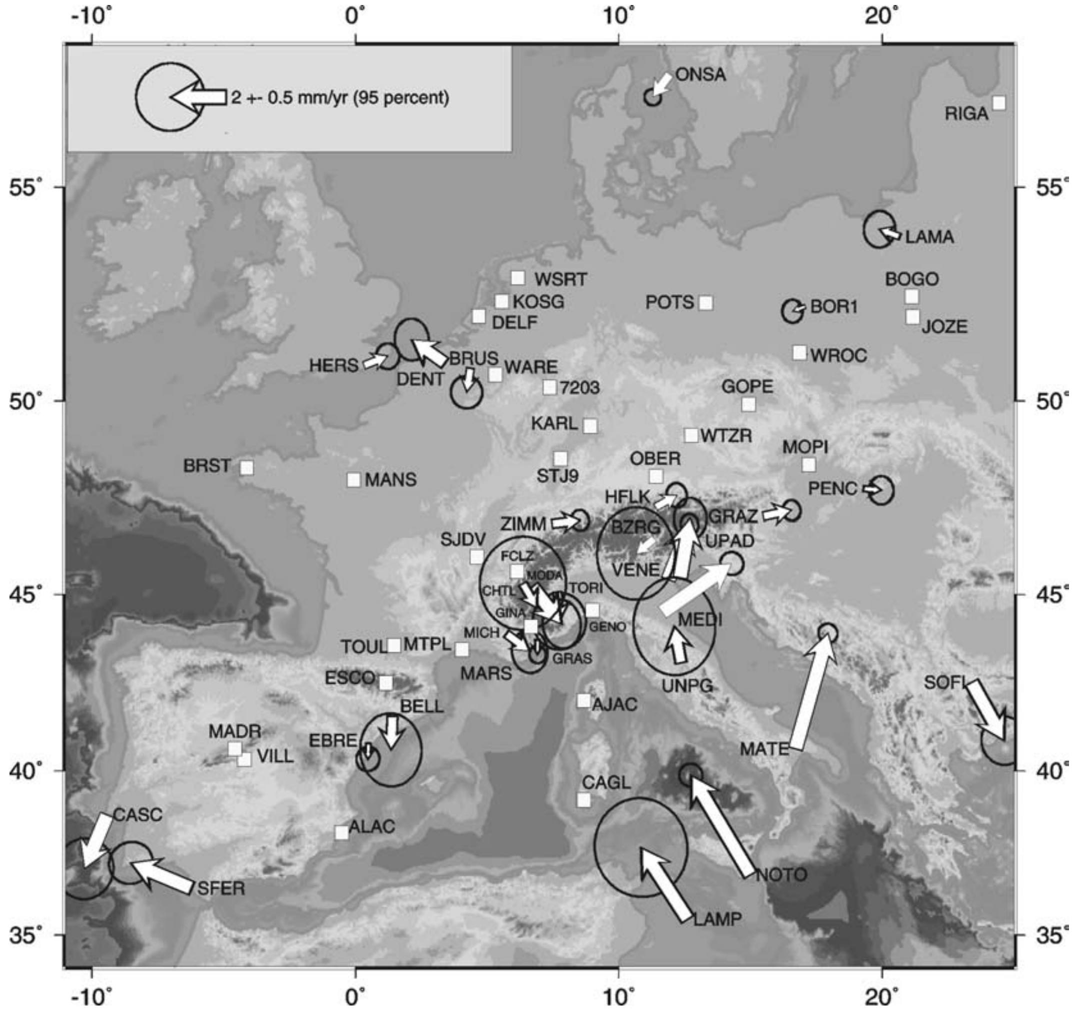

Figure 2: Result of an automatic algorithm that searches for the subset of sites defining a rigid rotation with residual velocities within their uncertainty. White squares indicate the sites with insignificant velocities (except for the Alpine region).

Table 3: Euler parameters derived from an automatic algorithm that searches the subset of sites

Euler vector values for Europe: results of the automatic search algorithm.

Latitude deg.dec	Latitude deg.dec	Angular velocity deg/Myr
560	-1015	25

Euler pole error ellipse

major semi axis deg.dec	minor semi axis deg.dec	Azimuth deg.dec	s (angular velocity) deg/myr
69	15	-153	1

In summary, the results of this ``blind'' automated approach indicate that most of western Europe behaves rigidly at the 0.4 mm/yr level (wrms) providing a new quantitative assessment of plate interior rigidity in western Europe. In particular, it shows that post-glacial rebound influence on horizontal velocities should be less than 0.4 mm/yr for the part of Europe located south of Fennoscandia.

Combination with the RRF network

The Réseau de Référence Français (RRF) is the french national reference GPS network (non-permanent). It is established since 1989 and the 1993 campaign solution was used to define the RGF93 reference frame which is the french national realisation of the European Reference Frame System ETRS89. This network consists of 23 sites and was observed completely in 1993 and 1996 and partially in 2000 for geophysics purposes. 4 permanent GPS stations of the RGP are installed on RRF sites with an high accurate local tie between the RRF marker and GPS station available (TOUL, GRAS, AJAC, MARS). In order to assess the consistency of the solutions derived from the RRF campaigns and the solutions from new permanent GPS networks in France, we performed a combination of both solutions. Different combination strategies were tested, and the final solution includes the local ties, enabling to determine a common velocity from both campaign and permanent network results. Table 4 shows the wrms on position residuals in the combination for RRF 1993, 1996 and 2000 campaigns solution. It indicates an agreement at a sub-centimetre level for horizontal components and a 1-1.5 cm level agreement for the vertical component. The residuals for RRF sites that now benefit from a permanent station indicate an agreement on about 1 cm for horizontal component (maximum residual 1.1 cm at GRAS).

Table 4: wrms residuals of RRF campaign solution in the combination with permanent networks – values are mm

Solution	Position	
	Horizontal	Vertical
RRF93	54	134
RRF96	37	99
RRF00*	20	53

* only 4 sites were observed.

Conclusion

The combination of solutions derived from weekly or daily analysis for EUREF, RGP and REGAL networks with a selection of ITRF2000 sites provides a way to assess the level of agreement between the different network solutions coming from different analysis. The level of agreement is on the order of 1 mm for the horizontal components and 1.5-3 mm for the vertical component for positions and ~0.5 mm/yr and 3 mm/yr for respectively the horizontal and vertical velocity component. This study illustrates a possible strategy to express solutions derived from regional permanent GPS networks in a consistent way with EUREF-EPN solutions.

Bibliographie

ALTAMIMI Z., C. BOUCHER, *The ITRS and ETRS89 Relationship: New Results from ITRF2000*, Proceedings of the EUREF Symposium, Dubrovnic, 2001.

ALTAMIMI Z., P. SILLARD, C. BOUCHER, *ITRF2000: A New Release of the International Terrestrial Reference Frame for Earth Science Applications*, J. Geophys. Res., in press.

BEUTLER G. et al., *Bernese GPS software version 4.2*, Astronomical Institute and University of Berne, 2001

BROCKMANN E., *Combination of solutions for geodetic and geodynamics applications of the Global Positionning System*, PhD Thesis, University of Bern, 55, 1997. 211, 1997.

CALAIS E., *Continuous GPS measurements across the western Alps*, 1996-1998, Geophys. J. Int., 38, 221-230, 1999.

CALAIS E. et al., *REGAL: A permanent GPS network in the western Alps*: Configuration and first results, C.R. Acad. Sci. Ser. II, 331, 435-442, 2000

CALAIS E., J.-M. NOCQUET, F. JOUANNE, M. TARDY, *Current strain regime in the western Alps from continuous GPS measurements*, 1996-2001, Geology, in press.

DAVIES, P., G. BLEWITT, *Methodology for global geodetic time series estimation: A new tool for geodynamics*, journal, J. Geophys. Res., 105, 11083-111000, 2000

HERRING, T. A., J. L. DAVIS, I. I. SHAPIRO, *Geodesy by Radio Interferometry: The application of Kalman filtering to the analysis of Very Long Baseline Interferometry Data*, J. Geophys. Res., 95, 12561-12581, 1990.

KING, R. W., Y. BOCK, *Documentation for the GAMIT GPS software analysis*, release 10.05, Mass. Inst. of Technol. Cambridge, 2001.

MAO, A., C.G.A. HARRISON, T.H. DIXON, *Noise in GPS coordinate time series*, J. Geophys. Res., 104, 2797-2816, 1999.

MILNE, G.A., J.L. DAVIS, J.X. MITROVICA, H.-G. SCHERNECK, J.M. JOHANSSON, M. VERMEER, H. KOIVULA, *Space-geodetic constraints on glacial isostatic adjustment in Fennoscandia*, Science, 291, 2381-2385, 2001

NOCQUET, J.M., E. CALAIS, Z. ALTAMIMI, P. SILLARD, C. BOUCHER, *Intraplate deformation in western Europe deduced from an analysis of the ITRF-97 velocity field,* J. Geophys. Res., 106, 11239-11258, 2001.

NOCQUET, J.M., E. CALAIS, *Crustal velocity field of western Europe from permanent GPS array solutions*, 1996-2001, submitted.

PELTIER, W.R., *VLBI baseline variations from the ICE-4G model of post-glacial rebound*, Geophys. Res. Lett., 22, 465-468, 1995.

RAO, C.R., J. KLEFFE, *Estimation of Variance Components and Applications*, North-Holland, New York, 1988.

SILLARD, P., C. BOUCHER: A *review of algebraic constraints in terrestrial reference frame datum definition* Journal of Geodesy, 75, 63-73, 2001

VANICEK, P., E. KRAKIWSKY, *Geodesy the Concepts*, Elsevier Sci., New York, 1986

ZHANG, J., Y. BOCK, H. JOHNSON, P. FANG, S. WILLIAMS, J. GENRICH, S. WDOWINSKI, J. BEHR, *Southern California Permanent GPS geodetic array: Error analysis of daily position estimates and sites velocities*, J. Geophys. Res., 102, 18,035-18,055, 1997.