# Co-ordinate Transformation in Portugal Mainland – a brief study

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## Abstract

The Instituto Geográfico Português is the national authority responsible for the establishment and maintenance of the reference systems in Portugal. As such, it has the task of supplying to the citizens, co-ordinates in the several reference systems employed in the Country, and also parameters for performing the required transformations between them.

In Portugal Mainland are used two local reference systems: Hayford-Gauss Datum Lisboa (HGDLx) and Hayford-Gauss Datum 1973 (HGD73). Following EUREF recommendations, in the near future the ETRS89 (European Terrestrial Reference System 1989) should be adopted as the reference system for Europe.

This poster concerns to the first results of a study where some of the possible co-ordinate transformation methods were compared. The analysed transformations were: HGDLx to HGD73 and ETRS89 to HGD73. The goal was, not only trying to obtain a method of transformation with the required accuracy, but also easy to implement procedures. The methods of transformation studied by us were Bursa-Wolf's 7 parameters applied to geocentric co-ordinates; Polynomial Functions of the cartographic co-ordinates; and a Local Interpolation performed over the cartographic plane.

The calculations were based on the first order geodetic network of Portugal Mainland, completed with a group of fictitious points. This fictitious network was created with the same density as the 1st order network, and it encircles the whole territory of the Country.

## 1. Introduction

The Instituto Geográfico Português (IGP), being the national authority responsible for the creation and maintenance of the geodetic systems in Portugal, has the task of providing co-ordinates in the different reference systems used in the country, and also parameters for performing the transformations between them.

In Portugal Mainland two local reference systems are often used: Hayford-Gauss Datum Lisboa (HGDLx) and Hayford-Gauss Datum 1973 (HGD73). HGDLx is an earlier system, with the first order network established by chains of triangulation. HGD73 was conceived later and the co-ordinates for the first order trig points were obtained by a global adjustment of the network.

Following EUREF (European Reference Frame) recommendations, in the near future the ETRS89 (European Terrestrial Reference System 1989) should be adopted as the reference system for Europe. Accordingly, we have also employed this system in our experiments.

We expose here our first studies on the following transformations:

- HGDLx to HGD73
- ETRS89 to HGD73

The tested transformation techniques were:

- Bursa-Wolf's 7 parameters applied to the geocentric co-ordinates;
- Polynomial Functions of the cartographic co-ordinates;
- A local interpolation, performed over the cartographic plane.

The choice for these methods of transformation had in mind, not only the accuracy needed for the different applications (cartographic, topographic, etc…), but also the easiness of their implementation by someone less expert in these matters.

## 2. Transformation between HGDLx and HGD73

The transformation from HGDLx to HGD73 was performed using the whole geodetic network of Portugal Mainland, since there are co-ordinates in both reference systems for every trig point.

For each technique, the parameters were calculated with the co-ordinates of the first order network (Base Points). The obtained parameters were tested for the second and third order geodetic networks (Control Points). Figures 1 and 2 show the distribution of these points.

The transformation with **Bursa-Wolf's 7 parameters** was tested because there are several software that use this technique, and also because its precision is sufficient for some kinds of applications.

In the transformation between HGDLx and HGD73, the residuals obtained for the Control Points produced the following results:

- root mean square (r.m.s.) = 1,6m;
- highest residual = 4,7m.

Figure 3 shows the distribution of the residual obtained with this method, for the transformation between the systems HGDLx and HGD73.

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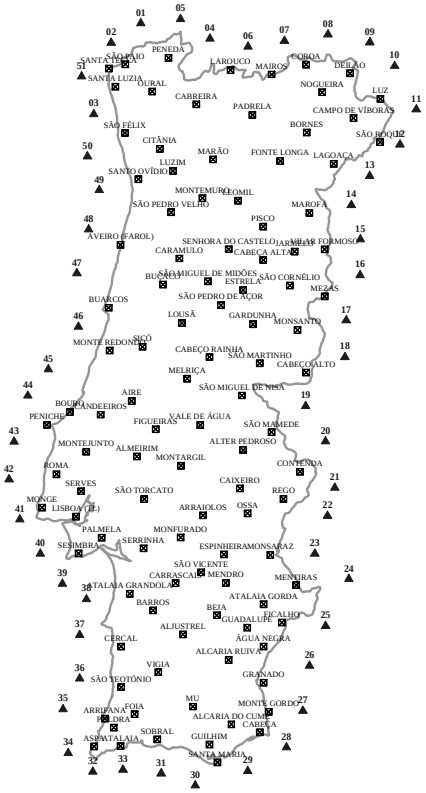


*Figure 1: The First Order Network in black, and the Fictitious Points, in blue.*


*Figure 2: Control Points: 2nd and 3rd Order Networks.*

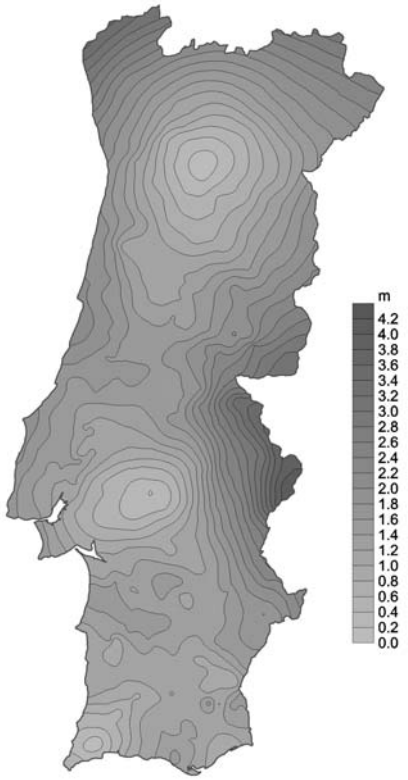


*Figure3: HGDLx to HGD73 transformation – residuals obtained for the 2nd order network with Bursa-Wolf's parameters.*

The *Polynomial Functions* establish a mathematical relationship between the two reference systems. Each co-ordinate of the final system is modelled by a polynomial function of the co-ordinates in the starting reference system. This method was executed with the cartographic co-ordinates.

Polynomial transformations from the 2nd to the 13th degree were tested. The best results were produced by the 7th degree functions:

- r.m.s. = 0,3m;
- highest residual = 1,6m.

For all the examined polynomials, the worst results appeared near the edges of the country. In order to try to overcome this problem, it was decided to create a group of Fictitious Points that would encircle the whole territory of the country (see Figure 1). This fictitious network, consisting of 51 points, was build with a similar density as the first order network, by a simple process of Intersection. The points were co-ordinated in both systems, by simulated directions from two trig points near them.

After acquiring these points, the polynomial functions were recalculated using as base points both the 1st order and the fictitious networks. The best results were then achieved with polynomial transformations of the 9th degree:

- r.m.s. = 0,2m;
- highest residual = 1,2m.

In figures 4 and 5 are plotted the residuals obtained with this technique without and with the fictitious points.

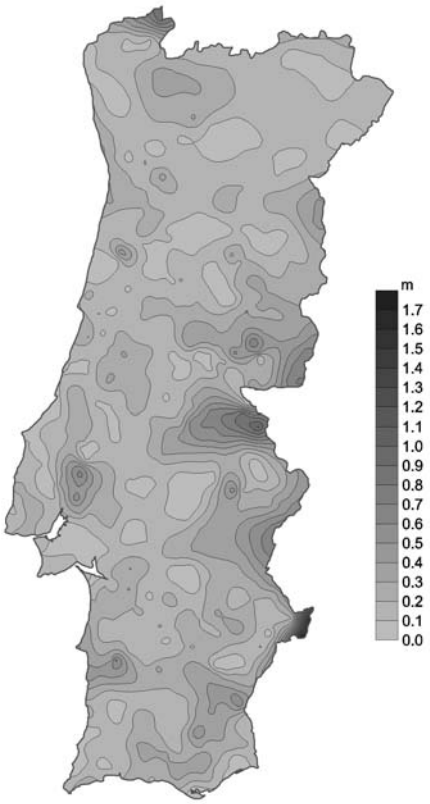


*Figure 4: HGDLx to HGD73 transformation - residuals obtained .for the 2nd order network using Polynomial Functions of the 7th degree, built with the 1st ordernetwork*

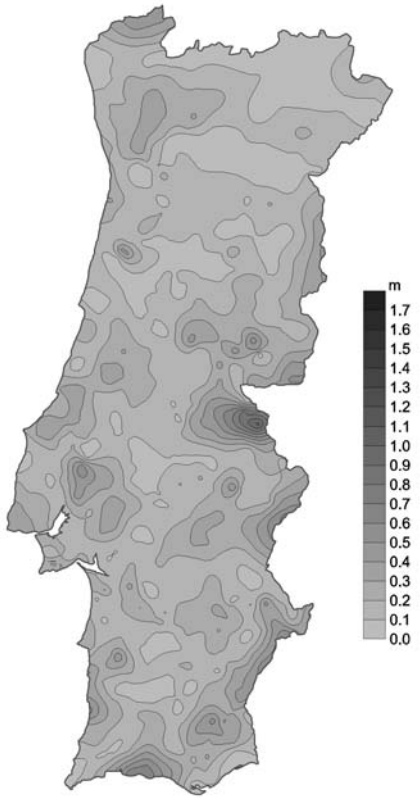


*Figure 5: HGDLx to HGD73 transformation - residuals obtained .for the 2nd order network using Polynomial Functions of the 9th degree, built with the 1st orderand the fictitious networks.*

The analysis of the pictures above reveals that the introduction of the fictitious points improved slightly the results obtained with the polynomial functions.

The method which is here called as **Local Interpolation** was implemented using Delauney's triangulation. The Triangulated Irregular Network (TIN) was built from the cartographic co-ordinates of the 1st order network and the fictitious points, independently for each of the co-ordinates (N, E). The transformed co-ordinate of a given point is obtained by a planar interpolation over the triangle in which the point fits in. For the application of this technique the fictitious points are indispensable, in order to ensure that every region of the country should be covered by triangles.

The evaluation of the quality of this method was made, again, using the 2nd and 3rd order networks. The residuals of the control points produced the following results:

r.m.s. = 0,15m;

highest residual = 1,1m.

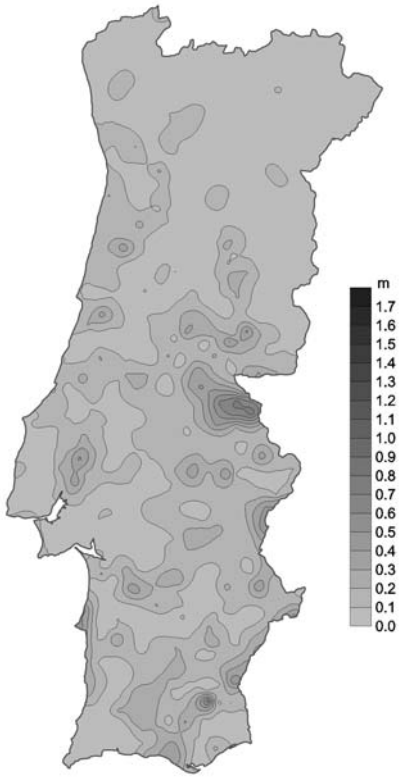


*Figure6: HGDLx to HGD73 transformation – residuals obtained for the 2nd order network using the Local Interpolation.*

From the TIN it is possible to form a Grid that, being a more regular representation, makes easier the interpolation over it.

## 3. Transformation between ETRS89 and HGD73

The IGP has started to observe the Portuguese geodetic network with GPS. Till the present day, the observation of the 1st order network was completed and the 2nd order is in course. So, there are co-ordinates in ETRS89 only for the trig points of 1st order. Therefore, we could only use these points to make the transformation from ETR89 to HGD73. The fictitious points were also computed in ETRS89 and used for this transformation. It was applied the UTM projection to the ETRS89 co-ordinates, as recommended by EUREF.

Since there were not many available trig points with co-ordinates both in HGD73 and ETRS89, we had to use a different procedure from the one employed on the previous transformation, concerning base and control points. However, it is important to use a large number of control points to accomplish more reliable results.

In the computation of Bursa-Wolf's 7 parameters and in the estimation of the coefficients of the polynomial functions, the number of base points is not very significant, as long as they are well distributed. On the contrary, the accuracy of the local interpolation is directly connected to the number of base points.

For **Bursa-Wolf's 7 parameters** method were used 12 base points and 92 control points. The residuals produced the following results:

r.m.s. = 0,6m;

highest residual = 1,3m.

The **Polynomial Functions** were computed with 64 base points (the fictitious and 13 more). Polynomial transformations from the 2nd till the 9th degree were tested. To evaluate the accuracy of each one, 92 trig points were used. The best results were achieved for the 5th degree polynomial functions:

– r.m.s. = 0,1m;

– highest residual = 0,3m.

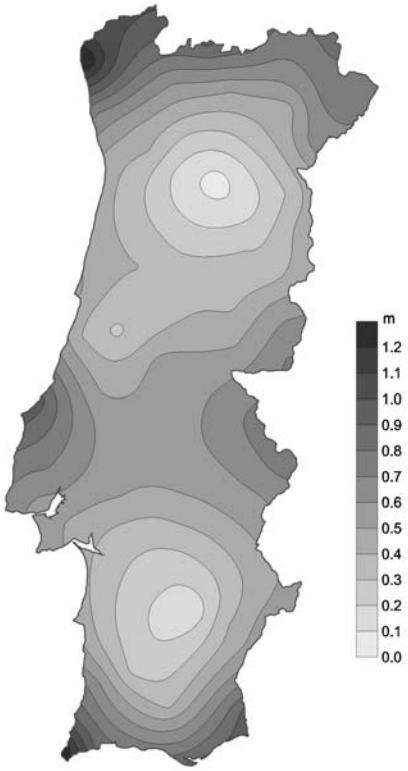


*Figure 7: ETRS89 to HGD73 transformation - residuals obtained .for the control network using Bursa-Wolf's 7 parameters*

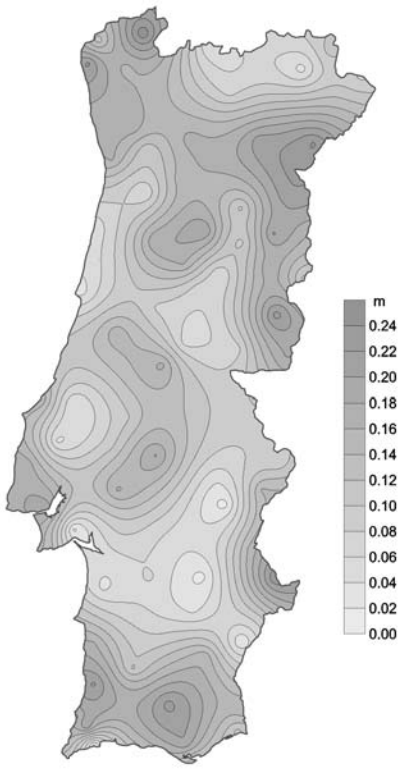


*Figure 8: ETRS89 to HGD73 transformation - residuals obtained .for the control network using Polynomial Functions of the 5th degree*

In the Local Interpolation procedure, the number of base points is very important, since smaller triangles give better results. So, for this technique, were used 144 (the fictitious and 93 of the 1st order network), leaving only 12 as control points. With the UTM projection applied to the ETRS89 co-ordinates, the residuals supplied the following results:

– r.m.s. = 0,6m;

– highest residual = 0,9m.

If we applied to the ETRS89 a Gauss projection similar to the one of the HGD73 reference system, the results came out much better:

– r.m.s. = 0,1m;

– highest residual = 0,1m.

## Some Considerations

The transformation between HGDLx and HGD73 is very useful since those two reference systems still co-exist in Portugal Mainland. Depending on the purpose, different techniques should be applied. However, for most cartographic applications the accuracy reached with the Local Interpolation could be enough. We are studying the areas where the worst results appeared, and we think that probably they proceed from local problems in the HGDLx system.

In the transformation from HGD73 to ETRS89 the results are better because HGD73 is a more consistent reference system than HGDLx. Polynomial functions produce results that can be considered enough for most applications. Applying to both reference systems similar cartographic projections produces more accurate results.

Polynomial functions have the advantage of their easy implementation and produce satisfactory results for both transformations. Another experience could be using incomplete polynomial functions, excluding the coefficients that don't have statistical significance, and also testing functions of the geographic co-ordinates.

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