# Accuracy of Geometric Levelling Networks at the Territory of Croatia

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#### Abstract

In the period between 1945 - 1990 there were numerous works of geometric levelling carried out at the present territory of the Republic of Croatia. These works are the basis for the new height system of the Republic of Croatia. The accuracy assessment of these works has been determined, with respect to the measuring accuracy, as well as to the benchmark height accuracy in accordance with the new definition of vertical datum and height system of the Republic of Croatia.

### 1. Introduction

At the territory of the Republic of Croatia the extensive works on general geometric levelling were made in the period between 1945 and 1990. These are the following works: high accuracy levelling (NVT), precise levelling (PN), city levelling (GN), technical levelling of higher accuracy (TNPT) and technical levelling (TN). Within the scope of these works there were numerous levelling lines made and extensive bench mark fields established that cover, mostly adequately, the entire territory of Croatia pursuant to: geographic, traffic, economic and other characteristics of single areas of regions, and to financial, technical, material and organisational circumstances stipulating their execution. Bench mark fields and levelling lines are unified completely in a coherent system of state levelling network established on the classic hierarchy principle, i.e. following the orders of accuracy.

The extensive works of general geometric levelling are well defined by the data contained in the archives of the State Geodetic Administration of Croatia. Following various orders of accuracy there were altogether 468 levelling lines made at the territory of Croatia encompassing about 23000 benchmarks. Individual orders of levelling contain:

- 36 levelling lines of the First High Accuracy Levelling (INVT) and 45 levelling lines of the Second High Accuracy Levelling (IINVT) – I. order,
- 77 levelling lines of precise levelling II.order,
- 49 levelling lines of the city levelling II. order,
- 149 levelling lines of the technical levelling of higher accuracy III. order,
- 112 levelling lines of technical levelling IV. order.

The above stated levelling works were made practically in the period between 1946 and 1973. After the year 1973 there have been no systematic general levelling surveying. The original data, the adjustment results, operational documentation (network sketches, levelling figure sketches, positional bench mark descriptions, lists of heights) and other data are stored in the archives of the State Geodetic Administration.

At the beginning of the nineties and after the Republic of Croatia has been recognised as an independent and sovereign state, the work started on updating the inherited levelling networks and their usage in new circumstances. These works, carried out during the nineties, did not include relevelling and the renewal of benchmark fields due to high fieldwork costs (stabilisation, survey, etc.), but it is based on maximum usage of the archive data considering economic, financial and organisational infractructure (instruments, equipment, experts). Within the scope of these works a great amount of jobs has been carried out:

- study on Updating the Levelling at the Territory of the Republic of Croatia has been made with the assessment of the inherited situation in state levelling networks given in it, as well as the proposal for improving this situation (KLAK et al. 1992),
- the proposal of the Book of Ordinances about the Levelling Works has been made (KLAK et al. 1993) that has been used as a normative and technical basis for carrying out the works on levelling in general, i.e. making the revision of benchmark fields and of levelling networks,
- the extensive and systematic field revision of benchmark fields at the territory of the Republic of Croatia has been made that are encompassed by the state levelling networks, and it has been defined for every single bench mark in the field and the field as a whole, how far it has been preserved or destroyed,
- the operational documentation referring to benchmark fields has been systematically revised, including: modernisation, updating in accordance with the results of field revision, production of new levelling line and network maps and of benchmark positional descriptions,
- extensive computing processing and measuring data adjustment of levelling has been made along with their resystemising and with the reference to the new vertical datum and fundamental levelling network, and new heights for all bench marks have been determined,
- operational data for benchmark fields have started to be published in special publications in edition of the State Geodetic Administration (Klak et al. 1994, Klak et al. 1998a, ROŽI, et al. 2001),
- three new levelling lines of height accuracy levelling have been designed and surveyed (Koprivnica – Gorièan, Virovitica – Terezino polje, Batina – Udvar) intended to connect the I. Order levelling networks between the

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Republic Hungary and the Republic of Croatia (KLAK et al. 1995, KLAK et al. 1997a, KLAK et al. 2000),

- geopotential and normal system of the Republic of Croatia has been defined and the height system of the Republic of Croatia has been connected with the United European Levelling Network – UELN (Klak et al. 1997, KLAK et al. 1997b, KLAK et al. 1998, FEIL and ROŽI, 2000),
- the proposal has been made to have a new vertical datum introduced into official usage – HRVD71 – Croatian Vertical Reference Datum 1971 (FEIL and RožI, 2000a) that has been evaluated by international advisers (BROCK-MAN et al. 2001),
- the Study on Renewal and Maintenance of the Height System of the Republic of Croatia has been made accompanied by the analysis of the results obtained in the works carried out in the period from 1992 – 2000 and by the proposals for the works to be executed inevitably in the forthcoming period (FEIL and ROŽI, 2001).

One of the most important and previously mentioned topics is processing and adjustment of levelling networks intended for the determination of bench mark heights. The integral part of this processing is also the determination of accuracy estimation criteria. Since the processing has been made for the levelling lines at the entire territory of the Republic of Croatia, concrete indicators of levelling network accuracy have been obtained.

### 2. Adjustment of Geometric Levelling Networks

The adjustment of geometric levelling lines and networks at the territory of the Republic of Croatia was carried out gradually in the period between 1994 and 2000. It followed the field revision of benchmark fields, and it was based on archive documentation kept in the State Geodetic Administration of the Republic of Croatia and on the archives kept in few larger geodetic firms that had formerly executed some of these works. Levelling networks were adjusted in accordance with the hierarchy principle, i.e. by lower order networks referring to higher orders. As the reference for including the lower order networks (PN, GN, TNPT and TN), a new vertical datum of the Republic of Croatia has been adopted, as well as IINVT network as the fundamental levelling network (RožI, 2001), Fig. 1.



Fig. 1. Levelling network IINVT at the territory of the Republic of Croatia

Applying the function model of indirect regular non-correlated measurements the adjustments of networks have been made by the least squares method. Levelling networks and lines have been adjusted individually in a single levelling figures of IINVT, Fig. 1. In this way the data processing has been divided into a few phases. The results of data processing have been

consequently published in a series of studies made at the Faculty of Geodesy, University of Zagreb. The works have been executed in the following phases:

- II. levelling loop IINVT 1994,
- I. levelling loop IINVT 1995
- III. levelling loop IINVT 1995,
- VIII. levelling loop IINVT 1996,
- IV. levelling loop IINVT the first part 1996,
- VI., XIV., XV. and XVI. levelling loop IINVT 1997,
- V. levelling loop IINVT 1998,
- IV. levelling loop IINVT the second part 1998,
- III. and VIII. levelling loop IINVT supplementations 1999,
- VIII. levelling loop IINVT supplementation 1999,
- IX. levelling loop IINVT 1999,
- XII levelling loop IINVT 1999,
- VII. and XI. levelling loop IINVT and island levelling networks - 2000.

# 3. Accuracy Estimation Results for Geometric Levelling

The accuracy of geometric levelling networks at the territory of the Republic of Croatia is given by the accuracy criteria "a posterior", i.e. by reference probable measurement errors and probable errors of network nodal benchmark heights determined on the basis of adjustment. They have been determined according to single levelling orders and levelling figures of IINVT. Apart from the adjustment of the fundamental levelling network, i.e. IINVT network (RožI, 1995), there have been altogether 127 adjustments of lower order geometric levelling networks being of various complexity (number of measurements and unknowns). The adjustment has been executed for individual orders as follows:

- 16 precise levelling networks,
- 58 city levelling networks,
- 19 networks of technical levellings of increased accuracy,
- 34 technical levelling networks.

The accuracy criteria "a posterior" are given in the table 1, divided in individual levelling figures of IINVT and individual levelling orders. The table contains probable reference errors of measurement  $u_o$  and probable errors of nodal benchmark heights  $u_H$ . There are always two probable errors of nodal benchmark height given. The probable error of nodal bench mark height that has got the lowest amount (min.-  $m_H$ ) and the probable errors of the bench mark height that has got the highest amount (max.-  $m_H$ ). The bench marks referring to these probable errors have the highest, i.e. the lowest height accuracy. These probable errors are indicators of height accuracies for all other benchmarks encompassed by the adjustment.

|        | IINVT                     |                          |                          | PN                        |                          |                          | GN                        |                          |                          | TNPT                      |                          |                          | TN                        |                          |                          |
|--------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|
| Figure | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn |
| I.     |                           |                          | ±11,77                   | $\pm 1,86$                | $\pm$ 1,41               | $\pm$ 7,09               | $\pm 2,92$                | $\pm 0,26$               | $\pm 5,21$               | $\pm 1,87$                | $\pm$ 3,08               | $\pm 7,77$               | $\pm 3,89$                | $\pm 2,\!49$             | $\pm$ 8,72               |
| П.     |                           |                          |                          | $\pm 4,59$                | $\pm 6,04$               | ±35,15                   | $\pm$ 1,25                | $\pm 0,23$               | $\pm 2,16$               | $\pm$ 3,20                | $\pm 2,43$               | $\pm 8,53$               |                           |                          |                          |
| III.   |                           |                          |                          | $\pm 2,56$                | $\pm 1,\!61$             | ±10,01                   | $\pm$ 3,25                | $\pm 0,\!47$             | $\pm 4,63$               | $\pm$ 3,97                | $\pm 3,73$               | ±14,25                   | $\pm$ 6,09                | $\pm$ 1,21               | ±14,31                   |
| IV.    |                           |                          |                          | $\pm 2,\!28$              | $\pm$ 3,67               | ±10,23                   | $\pm 2,10$                | $\pm$ 1,03               | $\pm 2,07$               | $\pm 6,55$                | ±12,09                   | ±12,09                   | $\pm 6,73$                | $\pm 6,59$               | ±16,53                   |
| V.     |                           |                          |                          | $\pm 2,\!14$              | $\pm 2,\!44$             | $\pm$ 9,20               | $\pm 2,31$                | $\pm 0,50$               | $\pm 2,37$               |                           |                          |                          |                           |                          |                          |
| VI.    |                           |                          |                          | ± 2,03                    | $\pm 1,\!14$             | $\pm 6,06$               | ± 3,03                    | $\pm 0,31$               | $\pm 3,86$               |                           |                          |                          | $\pm$ 8,18                | $\pm 0,89$               | ±19,20                   |
| VII.   | 0.70                      |                          |                          |                           |                          |                          |                           |                          |                          |                           |                          |                          | ± 3,57                    | $\pm$ 3,48               | $\pm 6,50$               |
| VIII.  | ± 0,79                    | ± 0,33                   |                          | $\pm 2,\!47$              | $\pm 2,29$               | $\pm 9,14$               | $\pm 2,75$                | $\pm 0,55$               | $\pm 4,11$               |                           |                          |                          | $\pm 5,60$                | $\pm 2,47$               | ±11,29                   |
| IX.    |                           |                          |                          | ± 1,15                    | $\pm 1,24$               | $\pm 9,59$               | $\pm 4,84$                | $\pm 0,57$               | $\pm 4,73$               | $\pm 2,27$                | ± 5,41                   | $\pm 5,52$               | $\pm 6,95$                | $\pm 4,10$               | $\pm$ 7,83               |
| XI.    |                           |                          |                          |                           |                          |                          |                           |                          |                          | $\pm 3,33$                | $\pm$ 4,48               | $\pm 7,51$               | $\pm$ 8,05                | $\pm$ 4,62               | ±23,67                   |
| XII.   |                           |                          |                          | $\pm 1,15$                | $\pm 1,24$               | $\pm 9,59$               |                           |                          |                          | $\pm$ 3,96                | $\pm 2,99$               | ±12,03                   |                           |                          |                          |
| XIV.   |                           |                          |                          | $\pm 1,65$                | $\pm 4,51$               | $\pm 4,77$               | $\pm 5,10$                | $\pm 0,51$               | $\pm$ 7,17               | $\pm$ 1,47                | $\pm 5,55$               | $\pm 6,35$               | ±11,85                    | ±16,41                   | ±27,93                   |
| XV.    |                           |                          |                          |                           |                          |                          | ± 1,22                    | ± 1,33                   | ± 3,35                   |                           |                          |                          |                           |                          |                          |
| ISLAND |                           |                          |                          | ± 3,33                    | ± 1,76                   | ±25,92                   |                           |                          |                          |                           |                          |                          | ± 3,41                    | $\pm$ 1,08               | $\pm 5,98$               |

On the basis of the data in the table 1 one can determine representative values of accuracy criteria "a posterior" referring to single geometric levelling orders for the entire territory of the Republic of Croatia. These accuracy criteria "a posterior" are given in the table 2.

Table 2. Accuracy estimation for measurements and benchmark heights

|         | IINVT                     |                          |                          | PN                        |                          |                          | GN                        |                          |                          | TNPT                      |                          |                          | TN                        |                          |                          |
|---------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|
| Figure  | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn | u <sub>o</sub><br>mn⁄i√km | min.u <sub>H</sub><br>mn | max.u <sub>H</sub><br>mn |
| Croatia | ± 0,79                    | ± 0,33                   | ±11,77                   | ± 2,53                    | ± 1,14                   | ±35,15                   | ± 2,70                    | ± 0,23                   | ± 7,17                   | ± 3,42                    | ± 2,43                   | ±14,25                   | ± 6,61                    | ± 0,89                   | ±27,93                   |

# 4. Conclusions

On the basis of the accuracy estimation data given in the previous chapter, several conclusions can be made. First of all, based on the table 2, it can be concluded that accuracies of measurement achieved at the entire territory of the Republic of Croatia in all orders of levelling mostly meet the accuracy criteria defined in advance. The measuring accuracy is also logically reduced from the higher order networks to lower orders. The allowed values of probable reference measurement errors for single geometric levelling orders are:

- high accuracy levelling,  $u_o = \pm 1mm / \sqrt{km}$ ,
- precise and city levelling,  $u_o = \pm 2mm / \sqrt{km}$ ,
- higher accuracy technical levelling,  $u_o = \pm 5mm / \sqrt{km}$ ,
- technical levelling,  $u_o = \pm 8mm / \sqrt{km}$ .

With regard to measuring accuracy, the precise and city levelling are partly exceptional, because their accuracy is somewhat lower than it is allowed by the accuracy criteria.

The accuracy estimation data given in the table 1 show that there was no completely homogeneous measurement accuracy achieved at the territory of the Republic of Croatia within the frame of single geometric levelling orders. It is obvious that in some parts of its territory, i.e. in single IINVT figures, the measuring accuracy is different within the frame of the same levelling orders, and sometimes even considerably lower than allowed. The origins of this unhomogeneity are connected primarily with the surveying quality, measuring procedures, instruments and other conditions. On the other hand, they are partly the consequence of various time epochs that individual levelling orders belong to.

The height benchmark accuracy can be considered satisfactory and logical related to the measuring accuracy. In the whole precise levelling, taking the entire state territory into account, the nodal bench mark with the lowest height accuracy has the probable error of  $\pm$  35 mm, in the city and higher accuracy technical levelling of  $\pm$  7mm, i.e. of  $\pm$  14 mm and in technical levelling of  $\pm$  28mm. A great number of benchmarks has got a considerably higher height accuracy which can be very well seen from the data in table 1. In this matter it should be taken into consideration that the accuracies of bench mark heights are partly stipulated by the manner of network adjustment and by the fact that bench marks in higher order levelling being fixed.

The achieved measuring accuracies and height accuracies of benchmarks are surprising to a certain extent. Namely, taking into consideration the fact that the observation material was gathered in long period of time (surveying in different time epochs), measured with different instruments, procedures and accessories, and mostly collected in very modest material and technical conditions (the period between 1945 and 1963), there has been a relatively satisfactory result obtained. Regardless of some deficiencies contained by all means in the works of the geometric levelling, they still have adequate use value, especially for practical purposes.

Anyway, at the moment the Republic of Croatia does not have higher quality levelling works at its disposal that could be used, and the new systematic works on the renewal of bench mark fields and updating of geometric levelling state networks has not yet started.

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