The Influence of Ionospheric Refraction in the Computation of Large Densification Networks

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Abstract

The GPS signals travelling through the atmosphere are affected by free charged particles in the ionosphere. The degree of ionisation changing over the time is correlated with the solar activity having besides others the period about 11 years, known as Solar Cycle.

High degree of automation, low time consumption in data evaluation and possibilities to obtain reliable results in general made the use of so called "hands-off" software packages common in computation of large densification networks. But, the possibilities for modelling of different factors accompanying with GPS measurements in those packages are limited. The balanced, between the impact of the ionospheric refraction in case of L_1 alone and increase of noise level accompanying with introducing of linear combinations of GPS signals, solution is described. The changing accuracy characteristics during the last Solar Cycle in base of the measured 15 692 GPS vectors of the Densification Network of Estonia are outlined.

1. Introduction

The GPS signals travelling through the atmosphere are affected by free charged particles in the ionosphere. The impact of the ionosphere on the propagation of radio waves could be expressed by the total number of electrons in a rotation cylinder centred on the line of sight receiver-satellite with cross section of one square meter, known as Total Electron Content (TEC)

$$E = \int_{Receiver}^{Satellite} n_e(s)ds \qquad (1)$$

where n_e is electron density in units of electrons/m^2 and s is the signal path.

The impact of the ionosphere is directly related with degree of ionisation changing continually together with changes in solar radiation and Earth's magnetic field. It can be shown (LANGLEY 2000) that the changes are both in space and time. The maximum ionospheric activity occurs in ± 20/ from geomagnetic equator and in auroral and polar regions. During the diurnal variation the maximum ionisation occurs at about 14:00 of local time being in minimum at night times. If we take a look at the seasonal variation the higher activity could be observed in summer. Besides others the degree of ionisation is correlated with the solar activity having the period, known as Solar Cycle, about 11 years. The impact of the ionospheric refraction in case of single-frequency data to the differential phase measurements is a systematic baseline shortening proportional to the baseline length and ionospheric refractivity. For the mid-latitudes a horizontal scale bias of – 0.06 ± 0.08 ppm /TECU is estimated (BEUTLER et al., 1989, LANGLEY 2000). Degree of the impact is a function of zenith distance, i.e. depends on the antenna cut-off angle used in GPS solution. Among the systematic shortening of the baselines the rapid changes in the receiving phases of the carrier signals accompanies with high ionospheric activity. That could cause a temporary loss of lock resulting in the cycle slips and decrease the signal to noise ratio of receiving signals making the ambiguity resolution unstable (BEUTLER et al., 1989).

2. Data used: The Densification Network of Estonia

The Densification Network is the densification of the I and II order networks established within 1996 – 1998 by AS PLANSERK, working under contract with the Estonian Land Board (ELB). National Geodetic Network consist of 13 I order points submitted to the EUREF as class B standard and 199 II order points (RÜDJA 1998, RÜDJA 1999). For the data processing the Bernese GPS Software (ROTHACHER and MERVART 1996) and IGS final orbits were used giving a good opportunity to take the wide spectrum of different factors accompanying with GPS measurements, including the affect of ionosphere, into a consideration. The Densification Network consisting of 3922 points was established area by area during the period from 1992 up to 2001 (Fig. 1) by RAS REI, OÜ REIB and AS PLANSERK, working under contract with the ELB. Excluding the first two years the network was monumented with point pairs having distance between the points 500 m on average. The distance between point pairs and between the single network points established during two first years is 5 km on average.

In measurements the Ashtech P-12, Ashtech Z-12, Javad Regency and Legacy GPS receivers were used. Choke ring antennas were used starting from year 1999. The main characteristics of GPS measurements were as follows: average length of the measurement session $1^h 30^m$, sampling rate 15^s, antenna cut of angle 10/. The data was evaluated using single baseline concept software GPPS from package PRISM (Ashtech Inc.), in calculations the broadcast ephemerides and troposphere model with standard parameters were used. The use of more sophisticated multistation-multisession software like Bernese GPS Software was put aside because of time consumption of calculations taking the huge number of measured GPS vectors, totally 15 673, into account. In the process of data evaluation the antenna cut-off angle was raised to 15/. The network was adjusted with software Global XAPositioning System (Inpho Technology OY).

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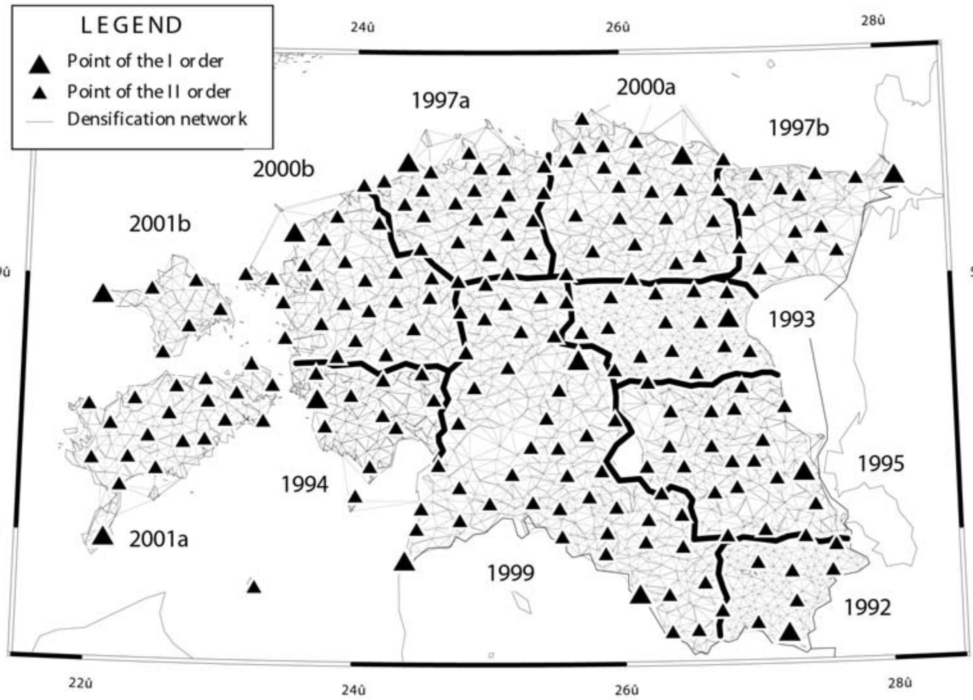

Figure 1. The Densification Network of Estonia and the years of the establishment.

3. The choice for bound between L_1 alone and ionosphere free solutions

Reduction of the ionospheric refraction can be done in numerous methods. Introduction of the ionosphere models into the L_1 solution is one, good, example (WILD 1994, SCHAER 1997, SCHAER et al. 1998, WOLF and PETIT 1999). The use of some from principally unlimited number of linear combinations of GPS signals L_1 and L_2 is another way. In selection of some particular combination criteria like ability to produce integer phase ambiguities, large wavelength to help ambiguity fixing, low ionospheric refraction and low observation noise are considered. In commercial softwares the possibilities are limited at once. In the PRISM package, used in our case, there are only two possibilities – the ionosphere free linear combination L1c or Widelane algorithm. L1c linear combination was put aside because of the inability of fixing phase ambiguities. The goal of the Widelane algorithm (Ashtech Inc. 1992) is to get L_1 ambiguities fixed, ionosphere free solution. First the widelane ($L_1 – L_2$) ambiguities will be resolved:

$$N_{1-2} = N_1 - N_2 \text{ or } N_2 = N_1 - N_{1-2} \qquad (2)$$

where N_1 and N_2 are the L_1 and L_2 ambiguities, respectively and N_{1-2} is the widelane ambiguity.

Resolved widelane ambiguities will be then used in the Lc solution for fixing narrow-lane (L_1+L_2) ambiguities:

$$N_{L1c} = N_1 - aN_2 \quad (3)$$

$$N_{L1c} = N_1 - a(N_1 - N_{1-2}) \quad (4)$$

where a is frequency ratio of L_1 and L_2 signals for reduction of ionospheric refraction.

Let us interpret the random errors like accompanying with noise level of the linear combinations used plus from other error sources and the systematic as shortening of the vector length concurring with neglecting the affect of ionospheric refraction. Determined by limited algorithms available in PRISM software and taking the amount of vectors making the choice vector by vector impossible into account the certain bound between L_1 alone and Widelane solutions was searched i.e. balanced solution between noise level and ionospheric refraction. The process overview in example of one area of the network (denoted as 2001a in Fig. 1) measured from July to September in 2001 is brought out in the following.

- First all the possible vectors were calculated using both the L_1 alone and Widelane algorithm. Changes in formal accuracies (RMS), in vector lengths and repeatabilities between those mentioned solutions were calculated and scattered in 1 km clusters. Determined by insufficient amount of longer vectors the length in comparisons was limited to 9 km. In Fig. 2 – 4 the corresponding differences are shown.
- In the next step several networks were combined including the vector solutions containing L_1, Widelane and combinations of them in 1 km steps. Selection of vectors was made beforehand to form a network shown in Fig. 1. Networks were adjusted using software Global XÅPositioning System (Inpho Technology OY) keeping the coordinates of I and II order net fixed. In Fig. 5 and 6 *a posteriori* relative and absolute accuracy estimation of different solutions are presented.
- The scales of the particular network solutions calculated as described earlier were compared with those of I and II order network. For that the so called minimally con-

strained network adjustments were performed taking the coordinates of only one point of the higher order net as constrained. The scale differences between fixed and minimally constrained solutions were estimated then using 7 parameter HELMERT transformation, in Fig. 7 the differences are shown. We note here that the scale differences between fixed and minimally constrained solutions were in case of I and II order network – 0.004 ppm ± 0.001 ppm and – 0.008 ppm ± 0.002 ppm, respectively.

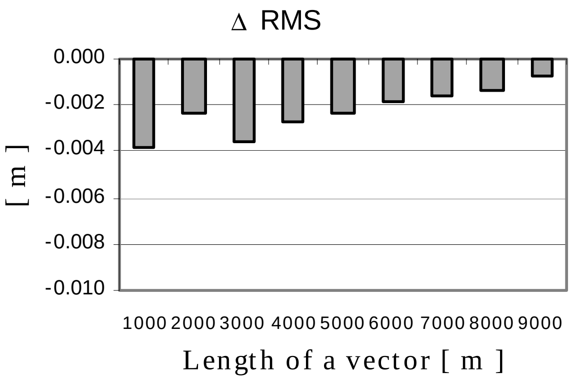

Fig. 2. Scatter of the formal accuracy as L_1 minus Widelane solution

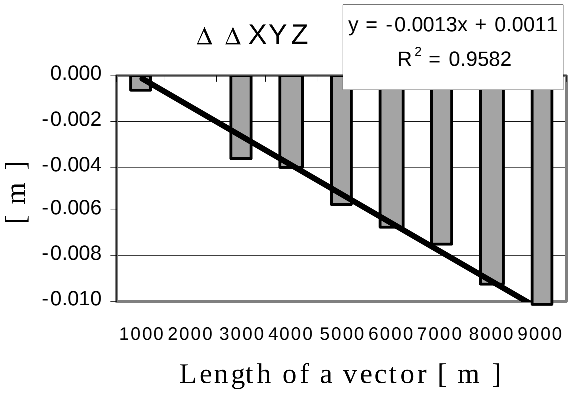

Fig. 3. Differences in length of vectors as L_1 minus Widelane solution.

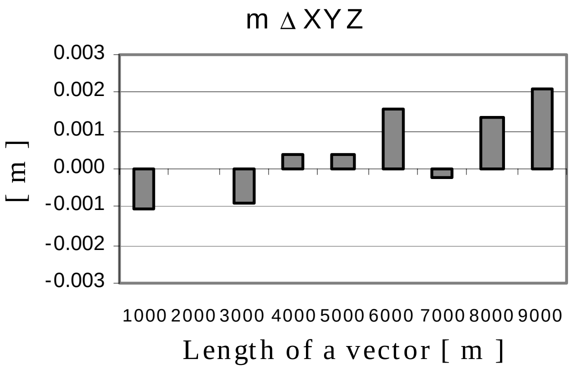

Fig. 4. Scatter of repeatability as L_1 minus Widelane solution.

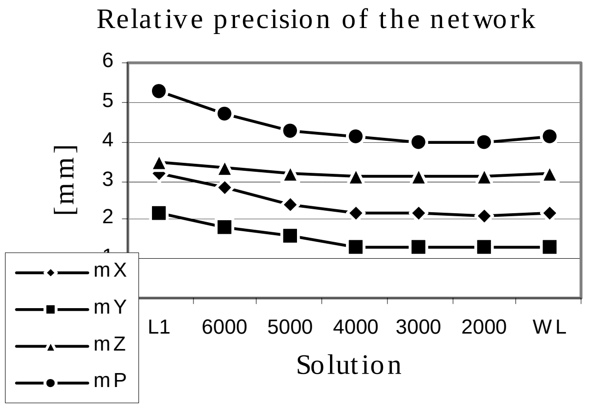

Fig. 5. Scatter of a posteriori relative accuracy including L_1, Widelane and balanced solutions.

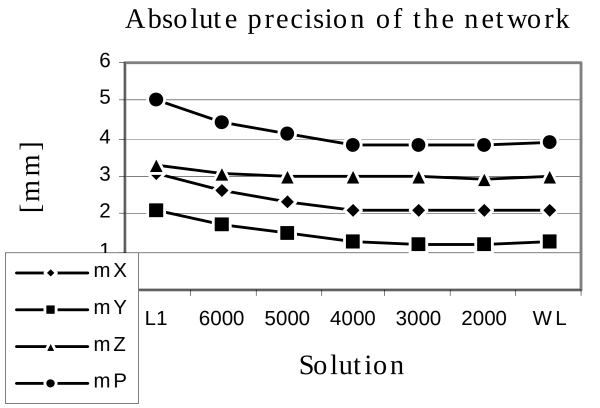

Fig. 6. Scatter of a posteriori absolute accuracy including L_1, Widelane and balanced solutions.

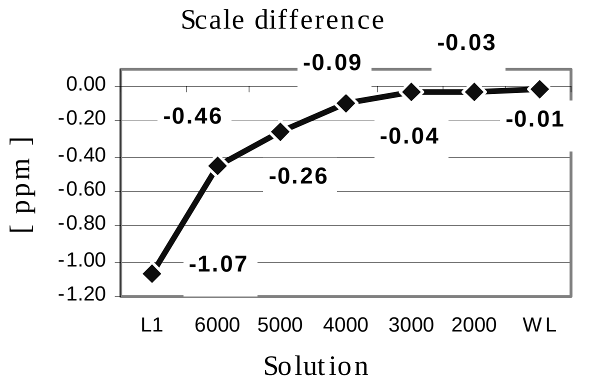

Fig. 7. Scatter of the scale differences compared with higher order network containing L_1, Widelane and balanced solutions.

For the shorter baselines the random errors are dominating, but starting from about 4 km the systematic errors became dominating as could be recognized from the figures. Comparisons made for *a posteriori* accuracy from the network adjustment show no significant improvement starting from 4 km as well. Based on comparisons made the bound 4 km between L_1 alone and Widelane solutions applicable for this part of the network was chosen.

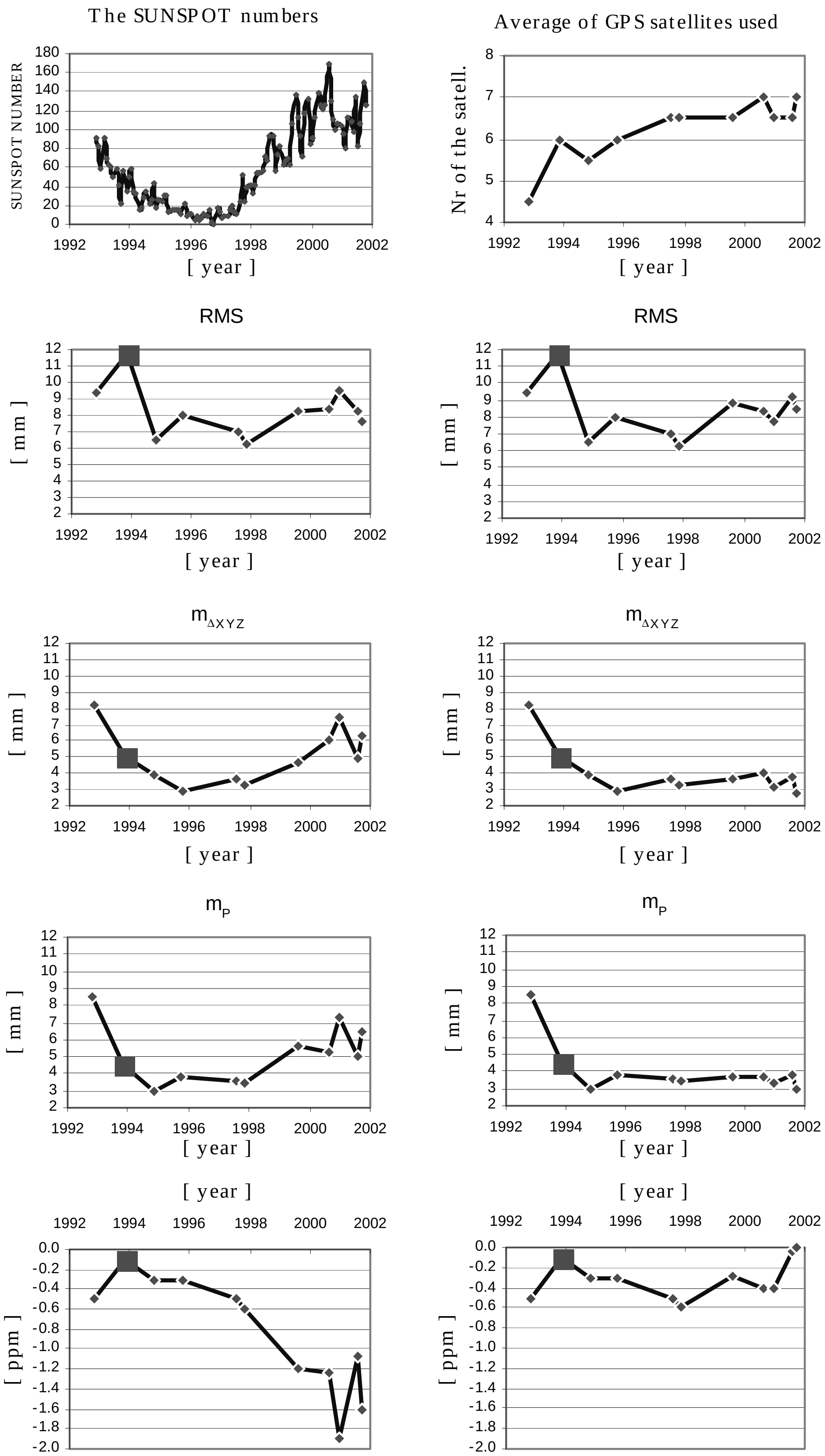

Fig. 8. The monthly average SUNSPOT numbers, average of GPS satellites used (right and left in the top), formal accuracies of the vector solutions (RMS), repeatability, a posteriori absolute accuracy from the network adjustment and the estimated scale difference between the Densification and I, II order networks (from top to bottom). L_1 solutions (left) and balanced solutions implemented from year 1999 upward (right). With square the L1c/Widelane solution is denoted.

4. The Solar Cycle and the accuracy characteristics of the network

The GPS measurements of the Densification Network cover almost all of the last Solar Cycle. The monthly average SUNSPOT numbers strongly correlated with solar flux and another changed factor, the average number of GPS satellites used in vector calculations are shown in the top of Fig. 8.

Up to year 1999 no special attention was paid to ionospheric refraction in calculation of GPS vectors of the Densification Network. All but one, area measured in 1993, were calculated using L_1 signal only. Reason to use L1c and Widelane algorithms in 1993 was arbitrary, not selected for reduction of the ionospheric refraction. Balanced solutions like described in previous chapter were implemented to the network areas established in year 2001. Limited by the time and economical resources available, GPS vectors of the network areas established earlier than 1999 were not recalculated, three areas measured during the period from 1999 up to 2000 were balanced partially taking the average scale distortion – 0.4 ppm of the earlier measured surrounding areas as a reference.

For all eleven network areas we have the L_1 alone solution with exception of year 1993 calculated using L1c and Widelane algorithms, the balanced solutions are available for the last five. In Fig. 8 the formal accuracies, repeatabilities of double measured vectors, *a posterior* accuracy estimations from the network adjustment and estimated scale differences comparing with scale of the I and II order net are shown. As it is commonly known the formal accuracies reflect only the inner consistency of the vector solution and the influence of many error sources correlated in time is not showing up. That can also be detected in Fig. 8 where the higher noise level associated with Lc and Widelane algorithms used in 1993 and the role of satellites available are clearly visible, but not the impact of ionospheric refraction. Correlation between the accuracy characteristics and solar activity is detectable on the others left hand side figures. In the same time the introduction of balanced solutions improve both the scale and accuracy estimation (up to about 60 %) as could be indicated in the right hand side of the figures. The number of GPS satellites available and the lower accuracy of the broadcast ephemerides appear to be dominating over the ionospheric refraction before the year 1995. For better presentation of Fig. 8 the bounds between L_1 alone and Widelane solutions are shown in Fig. 9.

The differences between the scale distortions as from comparision with higher order net minus from comparision between L_1 alone and Widelane vectors are shown in Fig. 10 for the network areas with corresponding solutions available. The agreement between the two scale distortion estimators is good in general. The reliability of the transformation, impact of formed network, the influence of other error sources such as orbital errors and tropospheric refraction cause the differences.

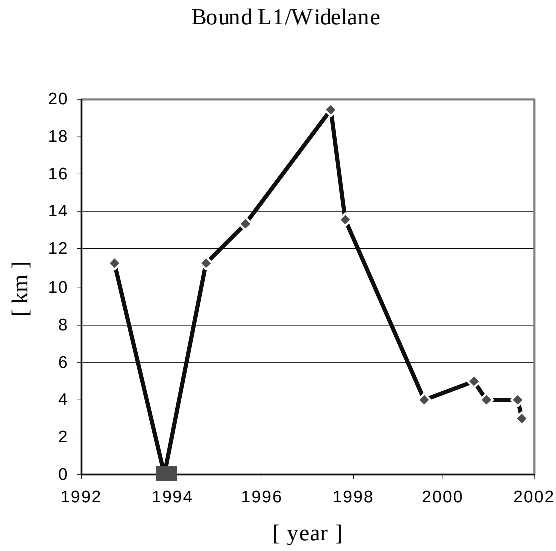

Fig. 9. Bounds between the L_1 alone and Widelane solutions.

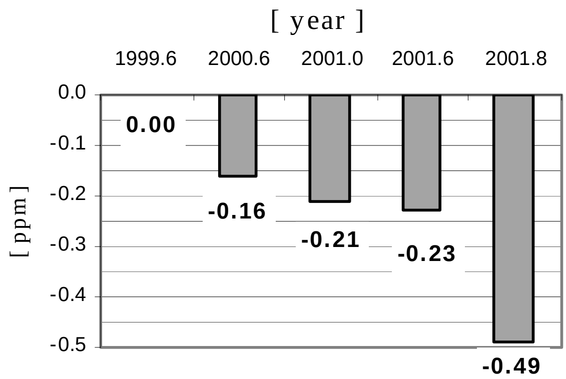

Fig. 10. The differences between the scale distortions as from comparision with higher order net minus from comparison between L_1 alone and Widelane vectors.

So far we were concentrated on a systematic shortening of the GPS vectors alone. During the adjustment procedure the scale error will be removed to some extent because of control by higher order net.

The Densification Network was adjusted in two groups, Estonian mainland and West-Estonian Islands, in 2001 (RÜDJA 2001a, RÜDJA 2001b). The residuals of the 6 parameters HELMERT transformation (no scale estimated) between the constrained and minimally constrained adjustments of network with included balanced solutions in Estonian mainland are presented in Fig. 11. The scale distortion, about 12 cm in gross country scale, between those two solutions as dominating shows up. The residuals after applying the 7 parameters transformation with the scale difference estimation (– 0.4 ppm) are shown in Fig. 12. Taking the changes in the residuals between 6 and 7 parameter transformations and presented in Fig. 10 the differences between two scale distortion estimators into account we can conclude, that the main part of the scale error is eliminated in constrained adjustment. That is in case of relatively dense higher order network in Estonia with average distance between the points 15 km.

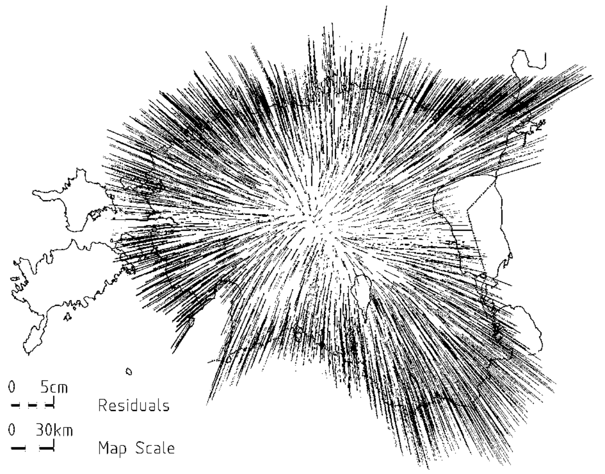

Fig. 11. Residuals of the 6 parameters HELMERT transformation, no scale difference estimated.

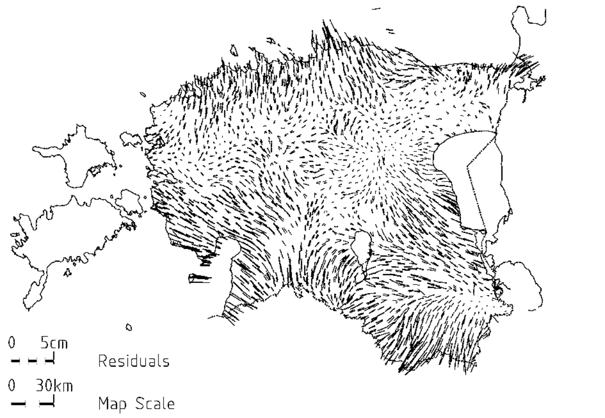

Fig. 12. Residuals of the 7 parameters HELMERT transformation, scale difference estimated.

In Fig. 13 and 14 the point displacements in plane between two constrained network adjustments with included L_1 alone and balanced solution are shown for two network areas measured in 2001. The changes in scales between L_1 alone and balanced solution in those network parts were - 1.1 ppm and - 2.1 ppm, respectively. In spite of the higher order control the changes, especially in Fig. 14, are still significant with respect of the precision estimation.

The scale distortion removal in a constrained adjustment is dependent on geometry of both, the higher order and densification network. In additional comparison, not shown here, with the same scale distortion (- 1.1 ppm) between L_1 alone and balanced solution for both network areas the differences were less but still bigger for the last.

The overall accuracy of the vector components of the Densification Network based on 6112 vectors measured twice or more was estimated. The differences were collected into a 1 km clusters and for each cluster the repeatability was calculated. The length in calculations was limited to 9 km due to small number of vectors above this limit. The repeatabilities are shown in Fig. 15, together with distance dependent increase of the repeatability estimated by linear regression. The change in accuracy figures after about the 5 km, the average bound between L_1 alone and balanced solution, is slightly detectible. Despite the limited spectrum of the length the values are relatively good base for *a priori* accuracy estimation in the Large Densification Networks.

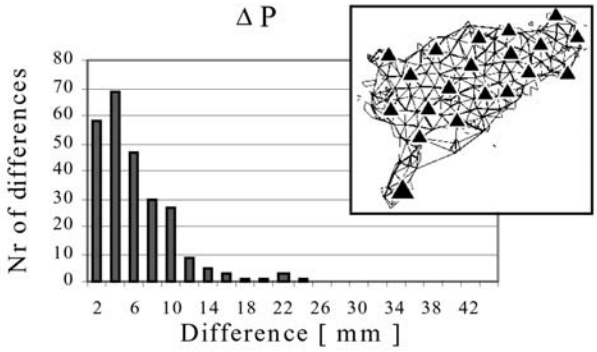

Fig. 13. The point displacements in plane between two constrained network adjustments with included L_1 alone and balanced solutions, respectively.

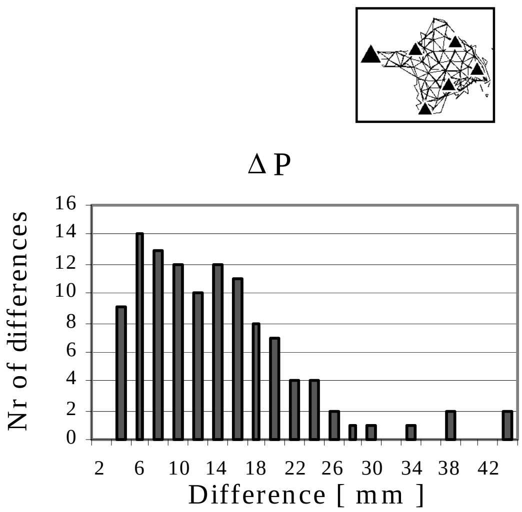

Fig. 14. The point displacements in plane between two constrained network adjustments with included L_1 alone and balanced solutions, respectively.

5. Conclusions

Taking the magnitude of the scale distortions discussed into account the amazing accuracy of the GPS technique should be pointed out.

High degree of automation, low time consumption in data evaluation and possibilities to obtain reliable results in general made the use of so called "hands-off" software packages common in computation of large densification networks. Because the limited possibilities for modelling of different factors accompanying with GPS measurements in those packages we recommend to investigate the possible systematic distortions at least in case of large networks. As was shown, the impact of the ionospheric refraction is clearly detectable even over the relatively short distances – 5 km in average in our study. Applying of some ionosphere free linear combination, starting from certain distance, could improve *a posterior* accuracy estimation up to 60 % and secure from changes in coordinates over 3F level in case of high ionospheric activity and weak higher order control. Together with increase in degree of ionisation the bound for mid-latitudes and the antenna cut-off angle of 15º (used in our calculations) between L_1 alone and reduced for ionospheric refraction solutions changes from about 15 to 3 km. The accuracy and the density of the higher order control in removing of scale bias are important.

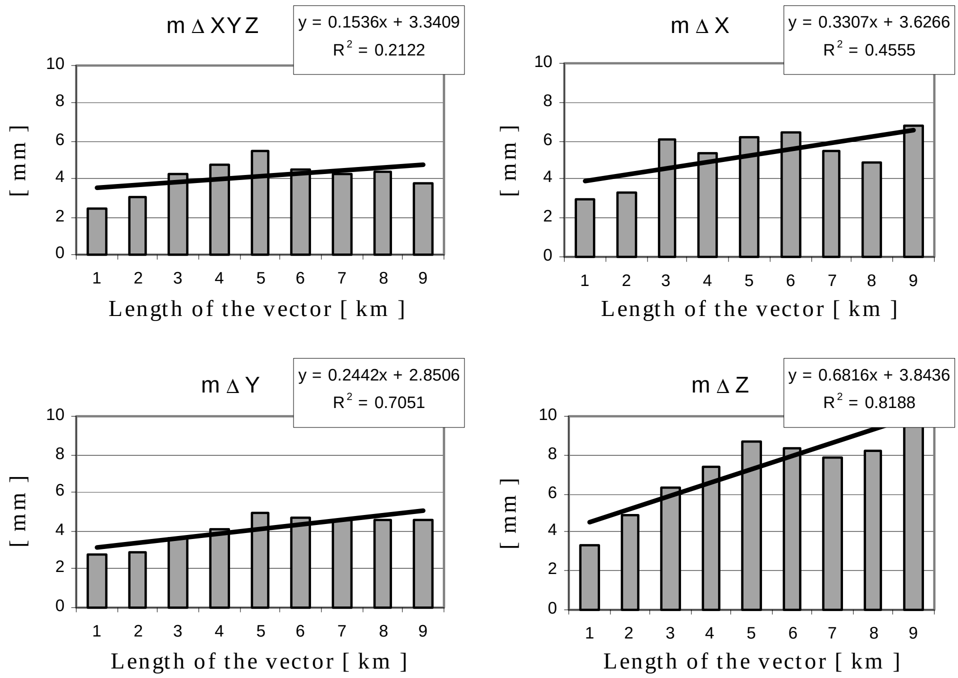

Fig. 15. The overall repeatability of vector components based on 6112 vectors measured twice or more.

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References

Ashtech Inc. (1992), PRISM, process user's manual.

BEUTLER, G., BAUERSIMA, I., GURTNER, W., ROTHACHER, M., SCHILDKNECHT, T., GEIGER, A. (1989), *Atmospheric refraction and other important biases in GPS carrier phase observations.* In (Brunner, F.K., ed.): Atmospheric Effects on Geodetic Space Measurements. School of Geomatic Engineering. The University of New South Wales, pp.15-44.

LANGLEY, R.B. (2000), *GPS, The Ionosphere, and the Solar Maximum.* GPS WORLD, July 2000.

ROTHACHER, M., MERVART, L. (edt) (1996), *Bernese GPS Software, Version 4*.0, Documentation. Astronomical Institute University of Berne.

RÜDJA, A. (1998, *manuscript), The final adjustment of the national geodetic network.* Technical report. AS PLANSERK, Tallinn. (In Estonian).

RÜDJA, A. (1999): A new ETRS89 system for Estonia. In E. Gubler, *Torres, J*.A., Hornik, H. (Eds): Report on the Symposium of the IAG Subcomission for the European Reference Frame (EUREF) held in Prague 2-5 June 1999. Veröff. Bayr. Komm. Intern. Erdmessung 60, EUREF Publ.No.8, München.

RÜDJA, A. (2001a, *manuscript): Adjustment of the densification network (Estonian mainland).* Technical report. AS PLANSERK, Tallinn 2001. (In Estonian).

RÜDJA, A. (2001c, *manuscript): Adjustment of the densification network (West-Estonian islands).* Technical report. AS PLANSERK, Tallinn 2001. (In Estonian).

SCHAER, S. (1997), *How to use CODE's Global Ionosphere Maps.* Astronomical Institute, University of Berne.

SCHAER, S., BEUTLER, G., ROTHACHER, M. (1998), *Mapping and predicting the ionosphere.* Proceedings of the IGS AC Workshop, Darmstadt, Germany.

WILD, U. (1994), *Ionosphere and Geodetic Satellite Systems, Permanent GPS Tracking Data for Modelling and Monitoring.* Geodätisch-geophysikalische Arbeiten in der Schweiz, Band 48.

WOLF, P., PETIT, G. (1999), *Use of IGS ionosphere products in Tai.* 31st Annual Precise Time and Time Interval (PTTI) Meeting. December 7-9, 1999 Laguna Cliffs Marriott, Dana Point, California.