Online and Postprocessed GPS-heighting based on the Concept of a Digital Height Reference Surface (DFHRS)

R. JÄGER, S. SCHNEID[1]

Summary

The DFHRS (**D**igital-**F**inite-Element-**H**eight-**R**eference-**S**urface) research and development project is funded by the German Ministry of Education and Research. It aims at the conversion of ellipsoidal GPS-heights h in an online or postprocessed GPS-heighting into the standard heights H, which refer to the height reference surface (HRS) of an orthometric, NN- or normal standard height system. The DFHRS is modelled as a continuous HRS in arbitrary large areas by bivariate polynomials over an irregular grid. Geoid information (geoid heights N, deflections of the vertical ξ,η) provided with a HRS datum adoption parametrization and identical points (h,H) as observations in a least squares procedure enable the statistically controlled DFHRS computation. Several geoid models may be introduced simultaneously and any geoid model may be splitted into different "geoid-patches" with individual datum-parameters and continuity requirements along the patch borders. The resulting DFHRS data-base provides a correction $\Delta=\Delta(B,L,h)$ to transform an ellipsoidal GPS-height h directly and online into a standard height H. Examples for the computation and use of DFHRS data-bases in DGPS-networks (e.g. SAPOS, Germany) are presented for different countries.

1 Introduction

With the trend towards replacing the former still present national datum systems in favour of ITRF-related datum systems and respective DGPS reference station systems (like e.g. SAPOS in Germany), the datum problem for the plan position component (B,L) in DGPS-based positioning applications will vanish by and by. For the reason of a physically different height reference surface HRS for the standard heights H (fig. 1) however, which are defined by geopotential numbers, the problem of a transition of the ellipsoidal GPS-heights h to the standard heights H referring to a HRS (geoid for an orthometric height system, quasi-geoid for a normal height system) will remain. Using directly geoid models such as EGG97 (Denker and Torge, 1997) or EGM96 (LEMOINE et al., 1998) the ideal formula

$$H = h - N_G(B,L) \quad , \qquad (1)$$

represented in fig.1, does not hold. The reason is, that geoid models have their own datum and additionally suffer from at least long-waved systematic effects (DINTER et al., 1997; JÄGER 1999, 2000; JÄGER and KÄLBER, 2000). Besides this the precision of the standard height H resulting from a GPS height is mostly restricted additionally by a poor short-waved accuracy of geoid models. The observation equation for the powerful standard approach for GPS-height integration, which was developed and implemented in the software package HEIDI2 some years ago (DINTER et al., 1997) and has meanwhile been applied by many DGPS users. The so called "geoid refinement approach" reads in the system of observation equations:

$$h + v = m \cdot H + N_G \qquad (2a)$$

$$N_G(B,L) + v = N_G + \partial N_G(\mathbf{d}) + NFEM(\mathbf{p},x,y) \qquad (2b)$$

$$H + v = H \qquad (2c)$$

$$C(\mathbf{p}) = 0 \quad . \qquad (2d)$$

The standard approach (2a-d) holds, if geoid heights $N_G(B,L)$ from a respective geoid model are available. The parametrization of a datum change $\partial N_G(\mathbf{d})$ reads (JÄGER, 1999, 2000):

$$\begin{aligned}\partial N_G(\mathbf{d}) = &\ [\cos(L)\cdot\cos(B)]\cdot u + [\cos(B)\cdot\sin(L)]\cdot v \\ &+[\sin(B)]\cdot w \\ &+[e^2\cdot N(B)\cdot\sin(B)\cdot\cos(B)\cdot\sin(L)]\cdot\varepsilon_x \\ &+[-e^2\cdot N(B)\cdot\sin(B)\cdot\cos(B)\cdot\cos(L)]\cdot\varepsilon_y \\ &+[-N_G]\cdot\Delta m_G \quad . \end{aligned} \qquad (2e)$$

In (2a-d) the height $N_G(B,L)$ of a geoid model is refined by a so called Finite Element Model NFEM(**p**,x,y) described in chap. 2. In (2e) N(B) means the radius of normal curvature of the ellipsoid at a point P(B,L). The datum parameters **d** comprise three translations (u,v,w), two rotations (ε_x, ε_y) and the scale change Δm_G of the geoid height N_G. The NFEM(**p**,x,y) acts as an additional overlay for a middle- and short-waved shape improvement of the geoid heights $N_G(B,L)$.

For more details concerning the datum transition problem (2b,e) and the refinement NFEM(**p**,x,y), as well as for the discussion of the special cases of the standard approach - namely the "pure geoid-approach" and the "pure FEM-approach" - it is referred to DINTER et al.

[1] Reiner Jäger, Sascha Schneid: Fachhochschule Karlsruhe - University of Applied Sciences, Moltkestrassse 30, D-76133 Karlsruhe,Germany. Tel: ++49 721 9252620, Fax: ++49 721 9252591.
EMail: dfhbf@fh-karlsruhe.de . URL: www.dfhbf.de .

(1997) and JÄGER (1999, 2000). The mathematical background of the powerful tool of the Finite Element Model NFEM(x,y,**p**), which will become also the central core of the DFHRS concept, is treated in chap. 2.

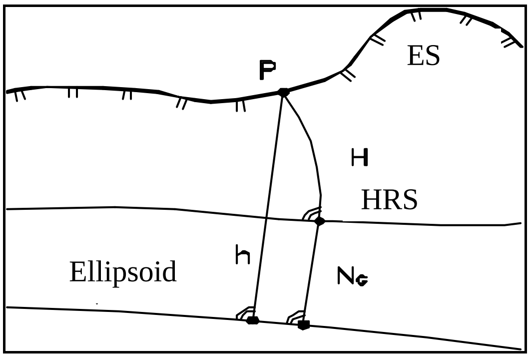

Fig. 1: Ellipsoidal GPS height h, standerd height H, height reference surface HRS, its ellipsoidal height N_G and earth surface ES at a point P(B,L)

A disadvantage of the above standard approach (2a-d) is, that it is in its full power a typical post-processing application. As identical points (H,h) are needed, the approach is not very economical for an online GPS heighting in DGPS networks (fig.3). Furtheron the geoid model heights N_G(B,L) are only used at discrete points (as "direct observations") and so the complete geoid height information N_G(B,L) is neglected and remains unused. Besides this also the vertical deflection information (ξ,η), e.g. available from geoid models, remains totally unused. The application of the standard approach (2a-d) still requires experts knowledge, so that it is not adequate for “any” DGPS user. Because of the above mentioned disadvantages the standard approach (2a-d) remains suboptimal compared to the new DFHRS-concept presented in chap. 3.

2 FEM Representation of Height ReferenceSurfaces

A powerful tool used already within the standard GPS height integration approach (chap. 1) and a central tool of the DFHRS approach (chap. 3) as well, consists in the representation of the height reference surface HRS or its additional refinement by a finite element surface called NFEM(**p**,x,y). NFEM(**p**,x,y) is carried by the base functions of bivariate polynomials which are set up in regular or irregular the meshes (fig. 2, fig. 5). If we describe with $\mathbf{p}^i$ the polynomial coefficients (a_{00}, a_{10}, a_{01}, a_{20}, a_{11}, a_{02},...)^i of the i-th mesh, we have for the height NFEM($\mathbf{p}^i$,x,y) of the HRS over the ellipsoid (fig. 1) in the i-th mesh:

$$\text{NFEM}(\mathbf{p}^i, x, y) = \mathbf{f}(x(B,L), y(B,L)) \cdot \mathbf{p}^i \quad ; i = 1, m \tag{3a}$$

$$\mathbf{p}^i = (a_{00}, a_{10}, a_{01}, ...)^i \tag{3b}$$

$$\mathbf{f}(x(B,L), y(B,L)) = (1, x, y, x^2, xy, y^2, ...) \tag{3c}$$

The vector **f** means the so called Vandermond vector and contains the different powers of the coordinates (x,y) according to the polynomial degree n. The total parameter vector **p** consists of the coefficient sets $\mathbf{p}^i = (a_{j,k})^i$, (j=0,n; k=0,n), of all m meshes. The plan position in (3a,c) is due to the metric ellipsoidal coordinates (y(B,L)=“East” and x(B,L) =“North”) introduced e.g. as UTM or Lambert coordinates, which are functions of the geographical coordinates (B,L).

To imply a continuous surface NFEM(**p**,x,y) one set of continuity conditions of different type $C_{0,1,2}$ has to be set up in the computation of NFEM(**p**,x,y) for each couple of neighbouring meshes. The continuity type C_0 implies the same functional values, the continuity type C_1 implies the same tangential planes and the continuity type C_2 the same curvature along common mesh borders of the DFHRS as represented by NFEM(**p**,x,y) (4a). The continuity conditions occur as additional observation equations C(**p**)=0 to be added to the parametrization of NFEM(**p**,x,y). The condition equations C(**p**)=0 are related to the polynomial sets of the coefficients $(a_{jk})^m$ and $(a_{jk})^n$ of each couple of neighbouring meshes m and n. To force e.g. C_0-continuity, the difference $\Delta N_{m,n}$ in the geoid height N_G of any point S at the common border SA–SE of two meshes m and n (see fig.2) has to become zero. So the basic condition equation for a polynomial representation of n-th degree reads (DINTER et. al., 1997) :

$$\begin{aligned} \Delta N_{m,n}(t) = \textstyle\sum_{j=0}^{n} \sum_{k=0}^{n-j} (a_{jk,n} - a_{jk,m}) \cdot \\ (y_{SA} + t \cdot (y_{SA} - y_{SE}))^j \cdot \\ (x_{SA} + t \cdot (x_{SA} - x_{SE}))^k \\ \equiv 0 \quad . \end{aligned} \tag{3d}$$

With $(y_{SA}, x_{SA}, y_{SE}, x_{SE})$ we introduce the plan metric coordinates of the nodal points SA and SE (fig. 2). Equation (3d) represents a polynomial of n-th degree parametrized in the border line parameter $t (t \in (0,1))$.

The subset of (n+1) C_0-continuity condition equations C(**p**)=0 for the border between mesh m and n results in case of C_0-continuity from (3d) by setting all (n+1) coefficients related to t to zero.

The mesh size and shape for the computation of the NFEM(**p**,x,y) representing the so called "geoid part" (see 5a,b,c) of the DFHRS data base by may be chosen arbitrary (fig. 2, fig. 5). The best approximation of a HRS by NFEM(**p**,x,y) results of course by introducing small meshes, e.g. in the range of 5 km in order to keep a 5 mm range for any HRS shape approximation by a polynomial degree up to n=3.

A special advantage and characteristic of the NFEM(**p**,x,y) representation consists in the fact, that the nodal points of the FEM grid are totally independent of the location of the geodetic observations and the geoid data points. Observation data which are presently used for the determination of the parameter vector **p** of NFEM(**p**,x,y) are height observations (h, H, ΔH, Δh), the geoid height observations $N_G(B,L)$ and the deflections of the vertical observations (ξ,η).

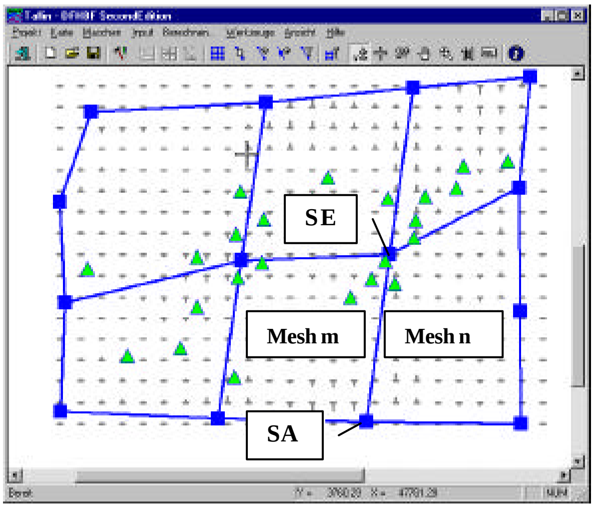

Fig. 2: Irregular mesh grid and different observation types (geoid data grid; triangles meaning identical points (H,h)). Example of DFHRS data base computation of Tallinn, Estonia with DFHRS production software.

3 Digital Finite Element Height Reference Surface (DFHRS) - Basic Ideas of the Concept

The DFHRS concept aims at a direct online or postprocessed GPS heighting with an optimum and simultaneous use of all available data sources. Within this aim the profile of a GPS-heighting is easy to formulate: An ellipsoidal GPS-height h, determined at a plan position x(B,L) and y(B,L) is to be made convertable directly to the height H of the standard height system. The converted height H should result online on applying a respective correction to h, and the resulting H should not suffer with a quality-decrease compared to the heights H resulting from a postprocessed GPS height integration.

In the following the general so called DFHRS concept is presented, which fulfils all above requests and shows besides this even some more positive aspects. The concept is to produce in a first step in a controlled way a new kind of data base product. This step is called the DFHRS data base <u>production step</u>.

The second step is to make this data base accessible for an online DGPS-heighting. This is called the <u>application step</u>. The use of the DFHRS in a postprocessing mode (e.g. in GIS) is of course included in the concept. In the application step either the DGPS user has the DFHRS at his disposal on his field equipment or the DGPS service exclusively uses the DFHRS for the evaluation of a correction $\Delta=\Delta(B,L,h)$ to convert a GPS height h into the height H of the standard height system (principle, see fig. 3).

3.1 DFHRS Data Base Production

The <u>DFHRS data base production step</u> reads in the system of observation equations as follows:

$$h + v = H + h \cdot \Delta m + \mathbf{f}(x,y) \cdot \mathbf{p}, \qquad \text{with } NFEM(\mathbf{p},x,y) =: \mathbf{f}(x,y)\cdot\mathbf{p} \tag{4a}$$

$$N_G(B,L)^j + v = \mathbf{f}(x,y) \cdot \mathbf{p} + \partial N_G(\mathbf{d}^j) \tag{4b}$$

$$\xi + v = -\mathbf{f}_B / M(B) \cdot \mathbf{p} + \partial B(\mathbf{d}_{\xi,\eta}) \tag{4c}$$

$$\eta + v = -\mathbf{f}_L /(N(B) \cdot \cos(B)) \cdot \mathbf{p} + \partial L(\mathbf{d}_{\xi,\eta}) \tag{4d}$$

$$H + v = H \tag{4e}$$

$$C + v = C(\mathbf{p}) \tag{4f}$$

With $\partial N_G(\mathbf{d}^j)$ (2d) and with $\partial B(\mathbf{d}_{\xi,\eta})$ and $\partial L(\mathbf{d}_{\xi,\eta})$ we introduce the datum part of the geoid heights of any geoid model or of single “geoid patches” (see chap. 4) and the datum parts of the deflections of the vertical (ξ,η) respectively. Explicit formulas for $\partial B(\mathbf{d}_{\xi,\eta})$ and $\partial L(\mathbf{d}_{\xi,\eta})$ are given in JÄGER (2000). With $\mathbf{f}_B$ and $\mathbf{f}_L$ we introduce the partial derivatives of the Vandermonds' vector $\mathbf{f}(x(B,L),y(B,L))$ (3c) with respect to the geographical coordinates B and L. M(B) and N(B) mean the radius of meridian and normal curvature at a point P(B,L) respectively.

Identical points (H, h) and if available, one or a number of geoid models $N_G(B,L)^j$ are used as observations to produce the DFHRS by a least squares estimation related to (4a-f). The DFHRS on the right side is, except of the scale part $\Delta m \cdot h$ (4a), represented completely by the finite element model NFEM($\mathbf{p}$,x,y)=$\mathbf{f}$(x,y)·$\mathbf{p}$, and the continuity is provided by the continuity equations C($\mathbf{p}$) (4f) below.

This means that the geoid model input of any geoid model $N_G(B,L)^j$ or “geoid patch” is “mapped” to the DFHRS by removing the datum part $\partial N_G(\mathbf{d}^j)$. An additional NFEM-refinement term may be set up in (4b). The production step of the DFHRS (4a-f) is embedded in a statistical quality control concept of a least squares estimation, so that any observation component - including the input of “mapped” and datum-adapted geoid-model - is well controlled (JÄGER and SCHNEID, 2001a). With respect to geoid models $N_G{}^j$ (4b) the DFHRS approach is to be regarded as the second step of a two step adjustment, which improves geoid models $N_G{}^j$ in terms of the new product NFEM($\mathbf{p}$,x,y) representing the "geoid part" (see 5a) of the DFHRS.

The most valuable way to check the external accuracy (“reproduction quality”) of the DFHRS data base parameters ($\mathbf{p}$, Δm) (4a-f) is to compute successively the DFHRS-height $H_{i,DFHRS}$ of each identical point H_i from h_i when using the individual data base $DFHRS_i$, where H_i was excluded from the respective production (4a-f). The reproduction quality measure ∇H_i is then simply given by the value of the difference

$$\begin{aligned} \nabla H_i &= H_i - H(B,L,h,DFHRS)_i \\ &= H_i - H_i(5a,b,c)\ . \end{aligned} \tag{4g}$$

The computation of all ∇H_i can be performed however in the unique production step, where all identical points (H_i,h_i) are used. The respective formula reads:

$$\nabla H_i = -\frac{v_{H_i}}{r_{H_i}} \quad . \qquad (4h)$$

With v_{H_i} and r_{H_i} we describe the correction and the redundancy part of the observation H_i in equation (4e).

3.2 DFHRS Data Base Application

The decisive components and formula parts of the production step, which are afterwards needed in the application step – namely in an online GPS-heighting - are contained in (4a).

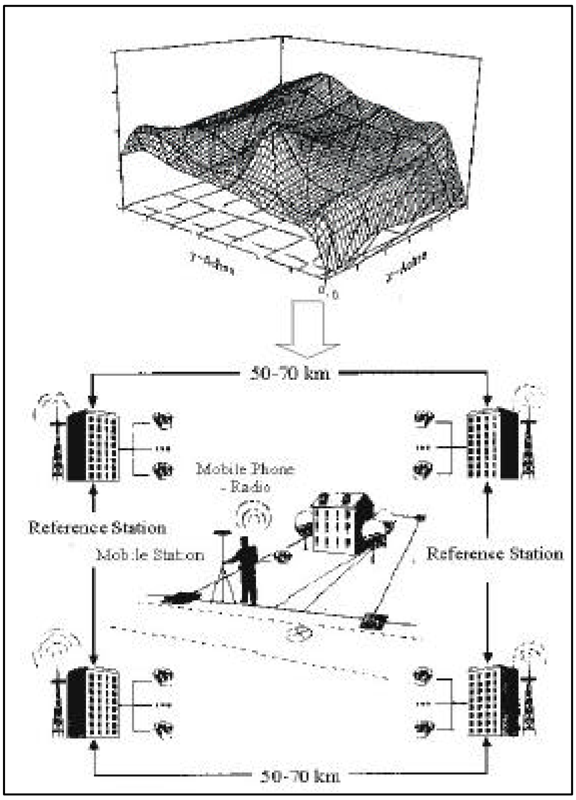

Fig. 3 : DFHRS data base symbolized by the Finite Element Model NFEM(**p**) of the HRS (above) and "application scenery" of an online GPS-heighting (below).

Equation (4a) leads to the following correction scheme, which has to be applied to the GPS height h in an online (or postprocessing) use of the DFHRS data base in order to convert h into the standard height H:

$$H = h - \Delta(B, L, h) = h - \text{corr1} - \text{corr2} \qquad (5a)$$

$$= h - \mathbf{f}(x(B, L) y(B, L)) \cdot \mathbf{p} \; - h \cdot \Delta m \qquad (5b)$$

$$= h - \text{NFEM}(\mathbf{p}, x(B, L), y(B, L)) \; - \; h \cdot \Delta m \, . \qquad (5c)$$

In opposite to the limitations concerning the use of conventional geoid models N_G by formula (1), the "DFHRS-correction" Δ(B,L,h) in the corresponding formula (5a) holds. The first correction part “corr1=corr1(B,L)” is due to the FEM part NFEM(**p**,x(B,L),y(B,L)) (fig. 3, above) of the DFHRS data base (“geoid correction”). The second correction “corr2 = corr2(h)” is due to the scale Δm between the GPS heights h and those of the standard height system H ("scale correction").

4 Special Requests and Advantages

To reduce the effect of long-waved systematic errors of geoid models as well as those of the standard HRS (DINTER et al., 1997; JÄGER, 1990) the mathematical model of the DFHRS-concept (4a-f), and the DFHRS software respectively, allow to subdivide any given geoid height model N_G(B,L) into a number of so called “geoid-patches” (fig. 5), each with an own set of datum-parameters $\mathbf{d}^j$ by $\partial N_G(\mathbf{d}^j)$ (4b). Fig. 4 shows the residuals of two different DFHRS adjustments for the country of Baden-Württemberg, Germany (see also fig. 5). Fig. 4 (left) shows long-waved systematic errors, which occur on introducing only one datum parameter set **d** for the whole area, meaning without "patching". The result of the “geoid-patching” on the right shows the benefit of the patching with respect to reduce the influence of systematic errors by the subdivision of the geoid model N_G(B,L) into a number of different patches with individual datum parameters $\mathbf{d}^j$ (JÄGER and KÄLBER, 2000). Geoid-patching significantly improves the accuracy of the resulting DFHRS .

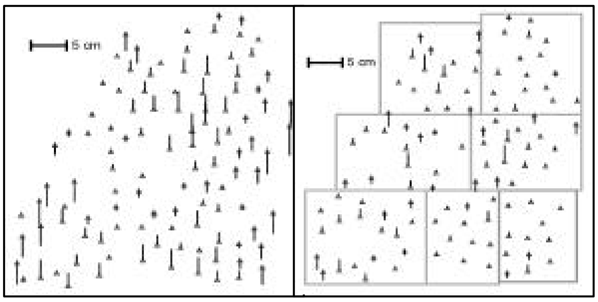

Fig. 4: Effect of a “geoid-patching” at the example of Baden-Württemberg: Large residuals in identical points with only one set of datum parameters (left) and much smaller residuals with 7 patches with individual datum sets $\partial N_G(\mathbf{d}^j)$ (right).

It is of course also possible to introduce by $N_G(B,L)^j$ (4b) different (patched or unpatched) geoid models, even concerning the same area.

The continuity equations (4f) are introduced as additional observation equations with variable weights. So the dimension of the normal equation matrix is not blown up due to the conditions C(**p**) and the approach is kept very flexible: The continuity conditions can be introduced "hard" or "soft", according to a prescribed weight, and they can also be statistically tested.

5 DFHRS Software and Examples

For the computation of Digital-Finite-Element Height Reference Surfaces a special C++ software (“DFHRS–production software”) has been developed at FH Karlsruhe - University of Applied Sciences within the running research and development project DFHRS (URL: www.dfhbf.de). Several functions for visualisation and utilities for an automatic and a manual meshing and a powerful least-squares adjustment have been implemented. The DFHRS production software enables the mathematical model (4a-f) including gross error detection and variance component estimation for all observations and observation groups respectively, and it sets up the DFHRS data base in a compressed format, enabling at the same time a copy protection key. The DFHRS data base consists of a block of mesh-grid information, a block with the DFHRS parameters **p** and Δm, and optionally a third block with their covariance matrix. The DFHRS data base access software is available as Dynamic Link Library (DLL) for an implementation in any DGPS online software.

A first DFHRS data base was computed for the (40 x 40) km area of Tallinn/Estonia (fig. 2) (JÄGER and SCHNEID, 2001b,c). Using the EGG97 geoid model (DENKER and TORGE, 1997) and 23 identical points, the average reproduction quality (4g,h) of the standard heights H evaluated from GPS heights h was in the range of 4 mm (max. 10 mm). The reproduction quality of the (250 x 350) km of DFHRS Baden-Württemberg (fig. 5) was also better than 1 cm using 192 identical points, 1013 meshes with a mesh size of 7 km, and 28 EGG97 patches.

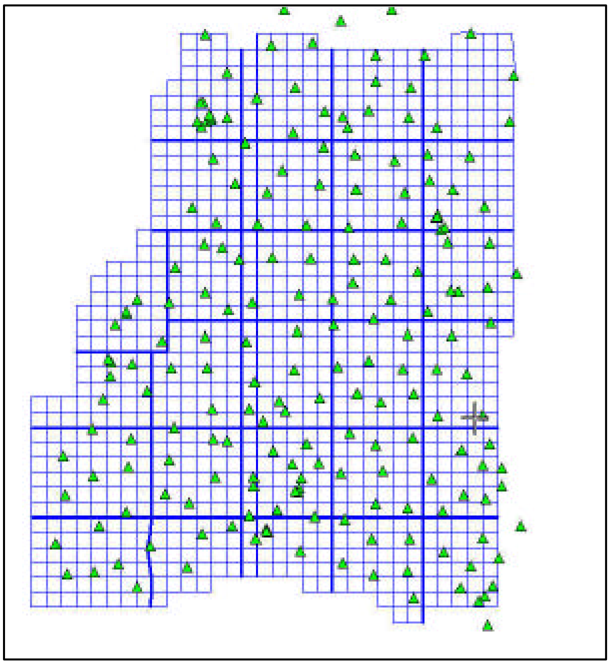

Fig. 5: DFHRS production for the country of Baden-Württemberg, Germany. Area size 250 km x 350 km. Almost regular meshes with an average size of 7 km, 28 geoid-patches, 1013 meshes and 192 identical points.

DFHRS data bases are already available and used respectively as standard in the practice of DGPS heighting in the SAPOS® DGPS networks of Saarland, Hessen, Baden-Württemberg and Bavaria.

A third series of test computations was performed for a (500 x 700) km area in Venezuela (JÄGER and SCHNEID, 2001c). Only 22 identical points and geoid heights N_G from the EGM96 were used for the DFHRS production. Even the large size of meshes (about 70-80 km) and the less accurate EGM96 provided a DFHRS with an average reproduction quality of 15 cm for the standard heights H.

6 Conclusions

The DFHRS (**D**igital **F**inite Element **H**eight **R**eference **S**urface) concept provides a new standard for an online GPS heighting in DGPS networks. The approach is based on the representation of the Height Reference Surface (HRS) by the base functions of bivariate polynomials. These are are set up in the area over a grid of arbitrary shaped finite element meshes. To imply a continuous HRS, a set of continuity conditions is introduced for each border of neighbouring meshes. With respect to geoid models N_G^j the DFHRS approach is to be regarded as second step of a two step adjustment, which improves any geoid model N_G^j in terms of the resulting DFHRS. The DFHRS data base, computed in the DFHRS production step, provides any DGPS user in the DFHRS application step with a correction $\Delta=\Delta(B,L,h)$, which converts the ellipsoidal height h directly to the standard height H. Identical points and transformations are not needed any more in the application step. So geodetic expert knowledge is reduced to the production step, while GPS heighting becomes as simple and economic as it can be.

The production step allows to use all above observation type simultaneously. The whole geometrical information, namely identical points and height differences (h, Δh, H, ΔH) as well as geoid model heights N_G and deflections of the vertical (ξ,η) are set up in a strict least squares adjustment, which becomes most efficient in this way with respect to the resulting DFHRS. The extension of the DFHRS approach (4a-f) with respect to gravity observations is intended. The production step simultaneously enables the statistical quality control of the DFHRS. In the sense of a two step adjustment the other observation groups enable the control and improvement of the geoid model input, which is simultaneously "mapped" to the new DFHRS product. To reduce the influence of long-waved systematic errors, geoid models may be devided into continuous “geoid-patches”. The resulting DFHRS data base is stored optionally together with its covariance matrix. The DFHRS data base access software is ready to be implemented as a DFHRS Dynamic Link Library (DLL) in any GPS-RTK-software for online GPS-heighting or in postprocessing software, e.g. for GIS.

7 Evaluation of the European Height Reference Surface (E_HRS)

The DFHRS concept has successfully been introduced in practice. DFHRS data bases have become an official sales product of state land survey agencies and also private companies in different countries. The approach (4a-f) is presently also used for the evaluation of the Height Reference Surfaces (HRS) of large scale areas such as Venezuela (JÄGER and SCHNEID, 2001c) and Germany. The results of DFHRS computations (see chap. 5) prove accuracies better than 1 cm. Another topic of interest and external requests is directed to the production of DFHRS data bases for "GIS and navigation" on a (5-50) cm accuracy level. Such "rapid" or "light" DFHRS data bases are easy to compute, simply by enlarging the mesh size without a need of change in the data blocks. So the DFHRS concept is all in all best prepared both for producing different accuracy levels of DFHRS data bases, as well as for using different data sources (e.g. several geoid models). The detection and reduction of systematic errors in big networks is enabled, and the DFHRS approach (4a-f) is implemented in a powerful software with graphic tools. So the authors offer the DFHRS concept and software as potential and flexible candidate for the controlled evaluation of the HRS of Europe (E_HRS) in the near future work of EUREF TWG.

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