Resuming Zeniths

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Abstract

The use of zenithal distances to get height differences has practically disappeared with GPS measurements combined with precise geoid models. Not withstanding this technique is not precise enough, at least not yet, to allow the substitution of precise levelling. In this paper the authors describe a technique based upon a conjugation of EDM, GPS and zenithal distances, that allows a very precise and economic height difference determination.

1. Introduction

The precise levelling technique is most time consuming and expensive. There is an urgent need to improve this technique or to find some other way to obtain equivalent results, if not for first order levelled lines, at least for secondary ones.

This is a theme that concerns the authors due to theirs charges in the Geodetic Department of IPCC. In the last IUGG meeting in Birmingham 1999, the authors presented a poster on an alternative model based purely on GPS and zenithal observations. Here the authors will resume a paper published in Revista do Instituto Geográfico e Cadastral, nº 11, 1992, on the use of Fermat principle to get the precise height difference.

2. The principle

Suppose that a medium range distance is known very accurately, let us say with 1 ppm or less. Nowadays the GPS technique can provide such a precision and accuracy. Suppose also that you get an EDM instrument with equivalent or greater range of application. Finally suppose you can perform simultaneously and reciprocal zenithal observations between the end points of this distance.

In such conditions we can reverse the classical problem of estimating the average refraction over the baseline, in order to correct the observed distance, and use the GPS distance value combined with the EDM observed range to get this average refraction.

Once this average value obtained we can compare it with the mean refraction value for the ends of baseline and, using the proportion between them, cut the baseline into two proportional parts. These two parts will represent the equivalent path of the light beam joining both ends, in a two layer atmosphere, according Fermat's principle.

3. Fermat's principle

According Fermat the path of a light beam crossing a variable atmosphere will be minimum:

$$\int_A^B n\,ds = \text{minimum} \quad \text{or} \quad c\int_A^B 1/v\,ds = \text{minimum} \quad \text{or} \quad \int_A^B dt = \text{minimum}$$

since $n=c/v$ and $c=constant$.

3. The method

Remarking that the EDM observed distance is based upon a pre-defined refraction, the so-called standard refraction, and this observed distance is the product of the "measured" flight time times this standard refraction, we can achieve the following relations:

$$\rightarrow D_0 = v_s x Dt \rightarrow \rightarrow$$ with D_0 the observed distance and v_s the standard speed

$$D_0 = c/n_s x \int_A^B dt \qquad \int_A^B dt = n_s x D_0/c$$

Finally $n_s x D_0$ will be the minimum we search for.

Now if we know the true value of the distance we can have the average refraction value, n, over the baseline:

$n = (n_s x D_0)/D$ → → with D the true distance value

Suppose now that instead of the true atmosphere we have a two layer atmosphere. Let n_a and n_b be the refraction values of this layers. The equivalence of the two atmospheres, from Fermat's principle point of view, will be guaranteed if we divide the two layer atmosphere in such a manner that we have:

$n_a x D_a + n_b x D_b = n_s x D_0$ → → or $n_a x D_a + n_b x D_b = nxD$ → *[A]*

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with n_a and n_b the refraction values of layers *A* and *B*, and D_a and D_b the distances values on layer A and B.

In equation *[A]* we have two unknowns, the distances D_a and D_b. The refraction values will be measured on both stations.

For solving *[A]* we need another equation. This will be the following:

$D_a + D_b = D$ ➜ ➜ ➜ ➜ *[B]*

We know that *[B]* is not totally true, but the slight difference does not matter. If is needed we can estimate, using the zeniths, the value of this difference.

So, once we have the distance values, we can solve the zenithal triangle:

Abb. (verkleinert!)

Ränder weglassen!

$W = (Za+Zb)-(p+g)$ ➜ *and* $W= wa+wb$

$wa=\arcsin(\sin c x D_b/D)$ ➜ ➜ with $c=p-W$

Once *wa* and *wb* obtained, the zenithals will be corrected and the height difference computed.

4. Achievable accuracy

From *[A]* and *[B]* we get:

$D_b = (n_s x D_0 - n_a x D)/(n_b - n_a)$ *[C]* and, obviously, $D_a = D-D_b$

The evaluation of the right proportion of D_b/D_a, between the two sides of the zenithal triangle, is critical for an accurate result for the height difference.

The total differentiation formula of D_b reads as follow:

$d D_b =$

$(n_s x dD_0 - n_a x dD - D x dn_a)/Dn - (n_s x D_0 - n_a x D) x\, dDn / Dn^2$

considering $Dn = n_b - n_a$

Since, usually, $Dn<0$, the two middle terms will be positive and the two others negative, tending to cancel out. The error budget depend mainly in the last term, and consequently, the method requires, to be performant, *Dn* differences as great as possible and *a d Dn* error as little as possible.

The first request, on *Dn*, is better fulfilled over great height differences; the second request requires great care in calibrating the meteorological sensors against each other, i.e., it is very important that both thermometers and both barometers be compared before and after the observation in order to know the relative errors between them. The absolute errors are not so important.

Some experiences carried out in Geobase, a high precision test network, and in the CERN control network, reported in the paper above quoted, shows that this method can ensure accuracies of 2nd order levelling.

References

PINTO J.: *Uma aplicação do princípio de Fermat ao nivelamento trigonométrico*. Revista do Instituto Geográfico e Cadastral, nº 11, 1992.