New height reference surfaces for Norway

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Abstract

In this article the computation of height reference surfaces at Statens kartverk is described. The difference between the geoid and the quasigeoid is shortly outlined together with a general method for adjusting gravimetric (quasi)geoids to GPS levelling measurements. Then the computation of the height reference surfaces is presented before comparing these models with independent GPS levelling data. Finally the necessary tasks that need to bed done in order to improve the geoids and the height reference surfaces in the future, are identified. The impact of the new satellite missions is also briefly discussed.

Introduction

Computation of high precision geoids has been a long lasting goal for geodesists. The advent of GPS has made this even more urgent, and the computation of such models has been a high priority task at Statens kartverk. To establish these surfaces a geoid model and GPS levelling data are needed. The first suggestions to adjust geoid models to GPS levelling data were made more than 10 years ago, but the amount of suitable data were at that time quite limited. In Norway systematic efforts in collecting these data first started in the mid 1990's. Dedicated GPS levelling campaigns like the European North - South traverse from the second half of the 1980's and the Scandinavian West East Traverse from 1992 were by this time already available as well as GPS levelling data from some minor local surveys. These datasets were established in order to test the computed geoids. They were however not particularly well suited for adjustment computations due to their geographical distribution so no fittings were performed based upon these data alone.

The geoid models have become so accurate now that the difference between the geoid and the quasigeoid no longer should be neglected. It is therefore necessary to know what kind of height system is being used in order to use the right type of "geoid" model. After a short introduction to this topic the general method that was used when computing these height reference surfaces, is presented. This is followed by a description of the Nordic Geodetic Commission (NKG) geoid NKG96n which was the fundamental initial model that was used in this work. Based upon this model several height reference surfaces have been computed in an iterative process. After presenting these models a comparison with independent GPS levelling data is performed in order to get an impression about the quality of the models. Before reaching the conclusions a few words about what can be done to get even better models in the future is presented including a short look at the new satellite gravity missions and their importance for high precision geoid determination.

Geoid, quasigeoid and height reference surfaces

Normally one does not explicitly specify which one of these surface one is talking about. In stead the word geoid is used as a common denominator for all of them. This despite the fact that what most people compute today are quasigeoids, and that almost all surfaces and models which are adjusted to a local vertical datum, are height reference surfaces. Considering only gravimetric geoids these surfaces are given by

- **Geoid.** An equipotential surface of the earths gravity potential W. Gravimetrical measurements are reduced from the earths surface down to the geoid. This requires knowledge about the variation of the earths density between the geoid and the surface. The geoid may in combination with GPS give orthometric heights.
- **Quasigeoid.** A geoid-like surface but no equipotential surface. Gravity observations refer to the earths surface so there is no need to make any assumptions about the earths density. This results in more complicated formulas where Stokes integral is the first term in an iterative solution. The quasigeoid may in combination with GPS give normal heights.
- **Height reference surface.** A surface adjusted to the local vertical datum. This surface is generally neither a geoid nor a quasigeoid, and it may include effects like the land uplift. In addition all the errors in the geoid, GPS and levelling are somehow assimilated into this surface. If the levelling is perfect and there is no land uplift, this surface is either a geoid or a quasi geoid depending on the chosen height system. Combined with GPS this surface will give heights in the desired vertical datum.

The height reference surfaces represents a more general class of surfaces than the geoid and the quasigeoid which are only special instances of this class. The geometrical relationship is shown in fig. 1 where H is the orthometric height, H^* the normal height, N the geoid height, ζ the height anomaly and $\bar{\gamma}$ is the average normal gravity value along the plumb line.

As a first order approximation the difference may be expressed like $N - \zeta = \Delta g \cdot H / \bar{\gamma}$ where Δg is the Bouguer-anomaly. See chapter 8 in HEISKANEN and MORITZ (1967) for further details. Due to this height dependency it is important for mountainous regions to be aware of this. Especially if heights are to be determined with a few cm's accuracy.

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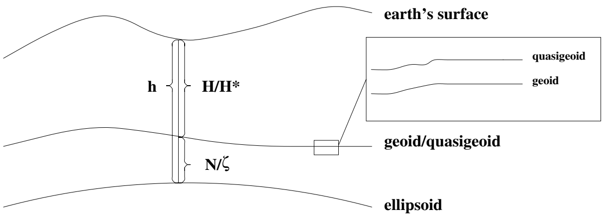

Fig. 1: The relationship between the geoid, the quasigeoid, geoid height, height-anomaly, orthometric height and normal height.

To compute the height reference surfaces a geoid model and GPS/levelling data are required. The actual computations can be done in an iterative way. Assuming that the "geoid" model G_n and a set S of GPS and levelling data are given, then the difference between the geoid height determined by this set and the model G_n can be identified. Assuming that the geoid models are given as grids then these differences can be gridded and added to the original model G_n giving the new model G_{n+1}. This new model is a height reference surface adjusted to the GPS levelling data set S based upon the model G_n. The initial model G_0 in this iterative procedure should be a geoid or a quasigeoid. Schematically and very simplified this iterative procedure may be written as

1. Given a model G_n (as a grid), geoid height N_n
2. Given a GPS/levelling set, geoid height $N_{GPS/lev}$
3. Find $\Delta N = N_n - N_{GPS/lev}$
4. Grid $N_{GPS/lev}$
5. Add this grid to G_n giving the new model G_{n+1}

This stepwise procedure has several advantages. The number of GPS/levelling measurements has until quite recently been quite limited. Although this situation is rapidly changing there are still some areas where such data are missing. A large part of the difference between the model G_0 and the GPS/levelling data is due to errors in the global geopotential models and effects like the land uplift. To eliminate these medium and long wavelength discrepancies only a limited number of adjustment points are needed. Having done this, for instance as the first step in the iterative procedure, by carefully selecting the data set S and the gridding procedure what is left for subsequent adjustments are the much smaller and short wavelength discrepancies. These later fittings can then be done when sufficient GPS/levelling data have been acquired for each local region. By properly selecting the gridding procedure the model can be adjusted to the local GPS and levelling data set while keeping the model unchanged outside this local area.

This method of fitting the geoid to GPS and levelling data assumes that both the GPS and the levelling are without errors. This is not true, but the purpose of computing these height reference surfaces is not to get a high precision geoid model. What is needed is a model which in combination with GPS gives the height in a system which as closely as possible coincides with the national height datum, and for many practical purposes this goal has been or will be reached with the existing or the new models of the future.

The land uplift has only been briefly mentioned so far, and provided that the GPS measurements are made over a not too long timespan then it is not necessary to have a model of the land uplift because the adjustment procedure will implicitly solve for and remove it's effect. It is however important to be aware of this time dependency, and if the models are to be valid not only for a limited timespan, then a model of the land uplift and a proper treatment of this effect is required.

Norwegian height reference surfaces

These surfaces were computed by using the iterative method just described. As initial model the NKG96n model by R. Forsberg et. al. (1996), was used. This model was computed by the well-known remove restore technique. See Sanso et. al. (1994) for further details. Compared to the earlier NKG89 geoid there are several improvements, more gravity data, erroneous gravity data have been removed, more detailed terrain models, new global geoptential-model EGM96 as opposed to OSU89B. The computational methods have also been improved thanks to the interest in FFT techniques and the work of Strang van Hees (1990), Forsberg and Sideris (1993) and de Min et al (1993). Based upon the NKG96n model and several GPS and levelling datasets the adjustment procedure has resulted in the following models.

Tab. 1: Height reference surfaces and the number of adjustment points.

n	Model	No. of adjust-mentpoints
0	NKG96n	0
1	VREF1996	39
2	VREF1998	124
3	VREF1999a	192
4	VREF1999b	206
5	VREF2000a	221
6	VREF2000b	223

The gridding was done with Forsbergs geogrid program using separate gridding parameters for each adjustment.

To get an impression of the magnitude of the fittings two examples are given in fig. 2 and 3. These figures also shows the locations of the adjustment points (white circles). Fig. 2 displays the difference between the VREF1996 and the original NKG96n models, while fig. 3 shows the difference between the VREF1998 and the VREF1996 models. Please observe that these figures are valid only for the mainland of Norway, so disregard the parts that are either in the ocean or in the neighbouring countries. This is just due to a numerical effect. As can be seen the figures are quite different. Fig. 2 displays quite smooth and long wavelength features while the corrections in fig. 3 are more irregular and high frequent. Another significant difference is the magnitude of the adjustments. The scales in the two figures are totally different and the adjustment values in fig. 2 are much larger than the ones in fig. 3 reconfirming what was mentioned earlier about the magnitude of the long wavelength compared to the short wavelength corrections.

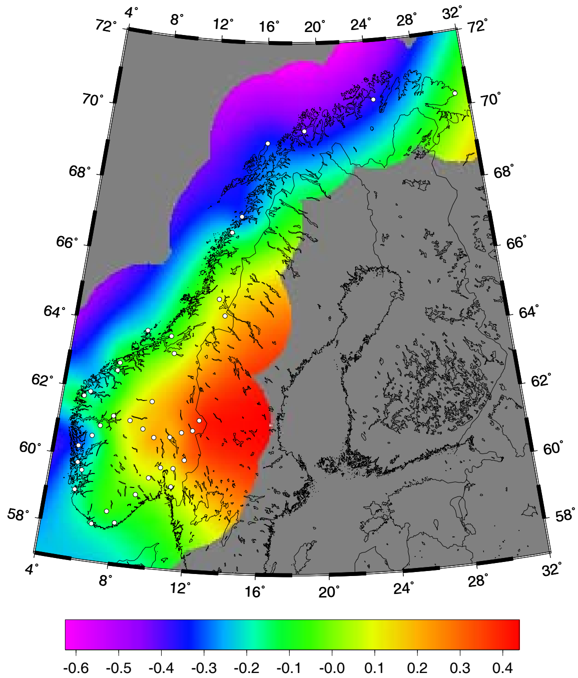

Fig. 2: The difference between the models VREF1996 and NKG96n (units meter) and the adjustmentpoints used when computing VREF1996.

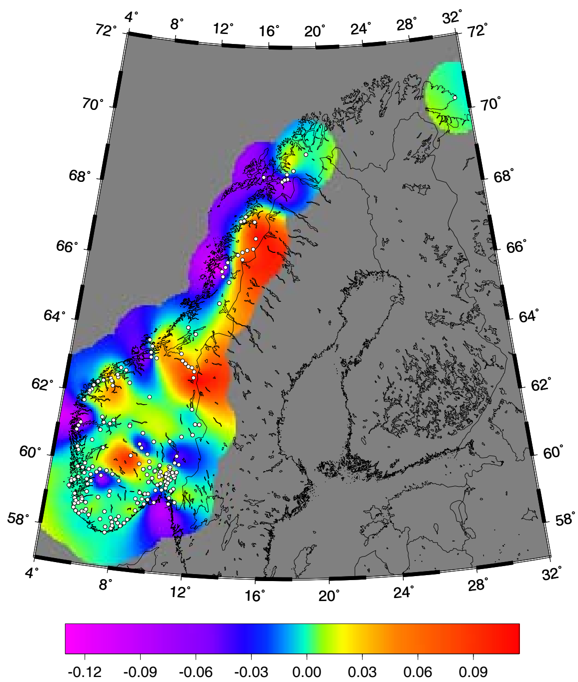

Fig. 3: The difference between the models VREF1998 and VREF1996 (units meter) and the adjustmentpoints used when computing VREF1998.

As has been mentioned earlier large parts of the correction in fig. 2 is due to long wavelength errors in the geopotential models, the land uplift and computational effects etc. In fact if the geoid model, the GPS and the levelling were without errors then fig. 2 would simply be the land uplift at least to a first order approximation when disregarding the corresponding change in the geoid.

What is also shown in these figures is the irregular distribution of the adjustment points. This is particularly prominent in fig. 3 showing the necessity to increase the number of GPS levelling points for those areas where the coverage at the moment is scarce if local fittings are to be done.

Quality control

To get an impression of the quality of the data all of the models have been compared with a test data set of GPS and levelling data. This data set has been extracted from the levelling database and consists of 915 points. The results of these comparisons are shown in table 2 which displays the number of points being compared, the mean value of the differences, the minimum, the maximum, the rms and the standard deviation all in the units of meters. Only those points which have not been used in the adjustment process are included, and note that the number of points in this table are not directly comparable to the numbers in table 1. As can be seen from this table the rms and the standard deviation generally diminishes from one model to the next. The large values for the NKG96n model is partly due to the fact that the data have not been corrected for the land uplift. If this had been done, then the numbers would have been smaller.

Table 2. Comparison of GPS/levelling data with height reference surfaces (units meter).

Model	N	mean	min	max	rms	stdv
NKG96n	915	-29	-467	378	161	158
VREF1996	877	2	-165	150	47	47
VREF1998	787	-1	-164	126	43	43
VREF1999a	718	-3	-157	114	33	33
VREF1999b	718	-4	-157	110	29	28
VREF2000a	718	-4	-156	110	28	28
VREF2000b	718	-4	-156	110	28	28

Fig. 4 shows the difference between the VREF2000b model and the GPS levelling data set. For large areas there is quite a good agreement, but there are still areas where there remains some work to be done.

These new models and their high quality have made it possible to use these models in controlling the levelling, at least to some degree, and several errors have been identified. It has also lead to a renewed investigation of the procedures used when computing fjord crossings. This work is however still in progress and not yet finished.

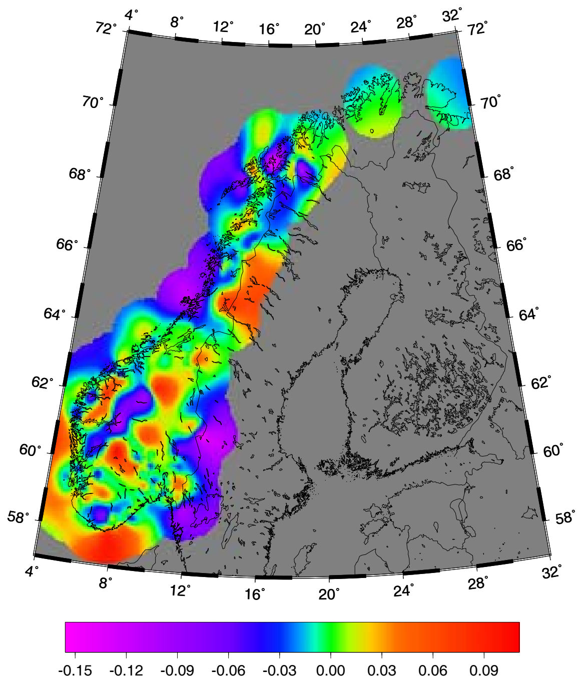

Fig. 4: The geoid difference between the GPS/levelling data set and VREF2000b (units meter)

Land uplift and time dependency

As has already been mentioned no model of the land uplift was used when computing the height reference surfaces. This is allowable as long as the GPS measurements are all done within a short timespan so that the variation of the land uplift within this time interval is small, but it represents a problem which should not be neglected. The determination of the land uplift has traditionally been done by tide gauge measurements and relevelling. Levelling is however a very expensive and time consuming method, and it will probably be replaced by satellite based techniques when determining the land uplift or subsidence. Among these new methods which are available now GPS is the most promising, but both SLR (Satellite Laser Ranging) and VLBI (Very Long Baseline Interferometry) can, at least in principle, be used. These satellite methods will determine the time dependency of the ellipsoidal height h ie. $\dot{h}$ where $h = H+N$, see fig. 1, while relevelling will give $\dot{H}$. An alternative method in determining the land uplift is to use the experiences gained from repeated gravity measurements along one of the Nordic "land uplift lines". Ekman and Mäkinen has derived a formula for the relationship between changes in gravity and height. Absolute gravity measurements may identify changes in gravity and this may then be converted to a corresponding change in height. Absolute gravity will also determine $\dot{g}$. A careful recomputation of the gravity network might then give a gravity network which is valid for a certain time or epoch. Together with post GRACE or GOCE satellites which will determine the time dependency of the long wavelength components of the earths gravity potential, this will lead to $\dot{N}$. Studies on determining this time dependency is only in its infancy and it represents one of the challenges that must be addressed in the future.

New improved models in the future

To improve our models in the future there are several tasks that need to be done and problems to be addressed. First of all the number of GPS and levelling measurements has to be increased. As can be seen from fig. 2 and 3, a rather limited data set has been used in these computations, and there are several areas where the distance to the nearest GPS point is quite large. Further studies need to be done on how to perform the adjustment of GPS levelling data to geoid models in an optimal way, possibly with the inclusion of other data like the deflection of the vertical. The gridding of the differences and the choice of covariance function also needs to be examined. To increase the number of GPS and levelling data, all suitable data should be used. In addition it may be necessary to have separate GPS geoid campaigns for some areas.

The traditional work of collecting gravity data, performing quality control etc. must be continued so that the gravimetric geoids, which are the fundamental initial models in the adjustment procedure, are as good as possible.

New alternative methods like the work being done by professor Grafarends group in Stuttgart, using geopotentials when computing geoids, Feistrizter (1998), is also interesting. The problems related to how the permanent part of the tidal signal is to be handled must also be addressed. Because, as described in Ekman (1988), there is not only one but 3 different geoids depending on how the tidal attraction from the moon and the sun is dealt with.

On a short time scale the focus will be on new improved height reference surfaces, but on a longer timescale the impact of several satellite missions will be tremendous. The first one the German CHAMP, CHAllenging Microsatellite Payload for geophysical research and application, has already been launched. GRACE, Gravity Recovery And Climate Experiment, primarily a US satellite, is planned to be launched in 2001 while the ESA satellite GOCE, Gravity field and steady state Ocean Circulation Explorer, is scheduled for launch in 2004. All of these satellites will contribute significantly to our knowledge about the earths gravity field, and especially GOCE will lead to vastly improved global geopotential models and geoids. As studies by van Onselen and van Gelderen have shown such new improved geopotential models are needed if a global vertical datum is to be achieved sometimes in the future. The new geoid models this will lead to, will not only make height determination by GPS much more accurate and replace levelling at least to some degree, but also lead to a much more optimal use of satellite altimetry data in oceanographic studies.

Conclusions

The difference between the geoid, the quasigeoid and the height reference surfaces was outlined and a general iterative procedure for adjusting the geoid or quasigeoid models to GPS levelling data was presented. The described procedure was used when computing the Norwegian height reference surfaces. These surfaces were then compared with an independent set of GPS levelling data, and the results showed that for some areas more adjustment data is needed while in other areas the models can already today be used when determining heights by GPS at least for some purposes.

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