# Preliminary Results of Test Computations as a First Step to a Kinematic Height Network

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## 1. Introduction

In resolution No. 5 of the EUREF Symposium 1999 in Prague it was accepted the concept of an integrated kinematic height network for Europe as combination of the European GPS permanent station network, repeated levellings, tide gauge observations and repeated gravity measurements.

In May 1999 a special working group was founded to determine and coordinate the direction of the future work. At the first meeting of this working group the following first tasks were established:

- analysis of the available repeated levelling measurements and storage of the data in the UELN data base
- development of software as base for test computations
- test of the software in a suitable test area

The treatment of these tasks will be reported here.

## 2. Description of the software – program KRUNA

The program KRUNA was developed at the BKG (SCHOCH 2000) to adjust height differences for more than one epoch in order to determine movements of the earth's crust.

### 2.1 Solution of the problem

The program assumes that the velocities are constant. Two kinds of observations can be processed:

- measured height differences
- adjusted vertical velocities

The observation equations of these two cases are the following:

*a)* $\Delta H_{ji,k} = H_j - H_i + v_j\,(t_k - t_{oj}) - v_i\,(t_k - t_{oi})$

*b)* $v_m = v_i$

with

| | | |
|---|---|---|
| $\Delta H_{ji,k}$ | ... | measured gravity related height difference between the points $i$ and $j$ at the epoch $k$ |
| $H_i, H_j$ | ... | heights of the points $i$ resp. $j$ at the reference epoch |
| $v_i$ , $v_j$ | ... | velocities of the points $i$ resp. $j$ |
| $t_k$ | ... | year of the measurement $k$ |
| $t_{oj}, t_{oi}$ | ... | reference epochs of the points $i$ resp. $j$ |
| $v_m$ | ... | measured velocity |

The unknowns for each point are

- the levelling height $H$ (gravity related height) at the reference epoch $t_o$
- if desired the vertical velocity $v$.

The required input data are for the case

*a)* – the measured height difference
– the variance of the measurement

It is assumed that the measured height differences from levelling observations are independent from each other.

*b)* – the "measured" vertical velocity derived from repeated GPS observations
– Because the vertical velocities are derived from GPS preprocessing these observations are correlated and the full variance-covariance-matrix is necessary.

In addition for datum fixing of the network the height of one point and the velocity of this or another point is needed.

If for one point two unknowns (height and velocity) are to be determined it is necessary that this point is contained at least in measurements of 2 different epochs.

The configuration of the epochs can be different. That's why it is important to consider gravity corrections of the levelling observations.

### 2.2 Parameters of the program

The program is written in FORTRAN77 and can be performed on UNIX operating systems. With appropriately reduced dimension conventions also MSDOS computers can be used.

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The number of the unknowns which can be determined with the adjustment program KRUNA depends on the available memory capacity.

The current version of the program works with the following dimension conventions:

- maximum number of points: 10 000
- maximum number of unknowns: 20 000
- maximum number of measured vertical velocities: 100
- maximum number of relations of one point to other points: 120

The whole number of measurements and the number of epochs are not restricted unless by the maximum size of 3 million elements for the profile of the system of the normal equations.

The program computes the local redundancies and the normalized improvements and carries out an outlier test according to Baarda. In addition it makes possible a variance component estimation.

## 3. Test computations

At a first step the test computations included only measured height differences as kind of observation.

In order to include adjusted vertical velocities further investigations are needed about the form of the variance-covariance-matrix of this kind of observations.

As test areas Denmark and the Netherlands have been selected because the data editing of the repeated levellings of these areas were most advanced.

### 3.1. The Netherlands

*3.1.1 Data material*

The used data material of the Netherlands contains the following 4 levelling epochs:

1) 2. Primary Levelling 1926 – 1940
   1401 points, 1696 measurements
2) 3. Primary Levelling 1950 – 1964
   1585 points, 1876 measurements
3) 4. Primary Levelling 1965 – 1978
   1994 points, 2117 measurements
4) Secondary Levelling 1986 – 1996
   5870 points, 8728 measurements

Before delivering the data the number of points of all epochs was reduced to about 30 % by the Survey Department of the Netherlands (see figure 1).

The individual measurement years of the epochs 1 – 3 are not known but only an average year for the whole epoch.

The observations are measured height differences without any corrections for gravity. The coordinates of almost all points are known apart of an area in the 4$^{th}$ epoch. These points couldn't be presented neither in the plan of the network nor in the drawing of the significant velocities.

*3.1.2 Results of the test computations*

At first each epoch was adjusted separately in order to check the data material. As a test the normal orthometric corrections were computed for all measurements for which the coordinates of the initial and end points are known. Most of them were smaller than 0.1 mm, only 8 corrections in the whole networks exceeded 0.5 mm. The reason is the small order of the height differences in the whole country. Although the gravity anomalies are not given the small heights of the points allow to the assumption that also the other part of the normal height correction could be ignored.

Each epoch was put into a separate observation group of the adjustment. The group variances were corrected by the variances which were computed in the separate adjustments of the epochs. There would be also the possibility of a variance component estimation but problems will arise in those cases if there are more than two epochs and their redundancies differ considerably. Than discrepancies between the epochs caused especially by variable velocities will result in very big variances of the epoch with the smallest redundancy.

The results of the adjustments of the various epoch combinations are shown in table 1.

It is noticeable that the number of identical points in the different epochs is very small compared to the total number of points; in the combination of all 4 epochs the share of identical points is 9 %. 40 % of them show a significant movement. (An adjusted velocity was looked on as significant if it has at least twice the size of the standard deviation.) The epoch combinations containing the fourth epoch achieve the best results concerning the precision. The reason is on the one hand the big length of time between the epochs and on the other hand especially the greatest mean redundancy of the measurements of that epoch. Looking at the graphics with the vertical movements of the Netherlands (see figure 2) one finds negative velocities (subsidencies) between −0.3 and −0.6 mm/year. The greatest velocities are found along the coast in the Southwest and in the Northeast of the country.

Table 1: Preliminary results of vertical movement adjustment – Test area – The Netherlands –

| epoch combination | 1 | 2 | 3 | 4 | 12 | 23 | 34 | 13 | 24 | 14 | 123 | 234 | 124 | 134 | 1234 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| standard deviation of the weight unit in mm ($S_o$) | 1.25 | 1.27 | 1.10 | **0.77** | 1.26 | 1.22 | **0.79** | 1.21 | 0.84 | 0.83 | 2.07 | **1.04** | 1.08 | 1.10 | 1.5 |
| standard deviation of the heights in mm ($S_H$) | 7.91 | 8.43 | 7.55 | **2.82** | 8.25 | 7.92 | **3.90** | 7.69 | 4.04 | 3.97 | 11.42 | **4.36** | **4.36** | 4.69 | 5.39 |
| standard deviation of the height difference in mm ($S_{Dh}$) | 1.93 | 1.85 | 1.65 | **1.04** | 1.89 | 1.75 | **1.20** | 1.78 | 1.23 | 1.23 | 2.98 | **1.44** | 1.47 | 1.55 | 1.92 |
| standard deviation of the vertical movements in mm/y ($S_v$) | | | | | 0.39 | 0.79 | 0.30 | 0.25 | 0.21 | **0.12** | 0.53 | 0.28 | **0.18** | **0.19** | 0.25 |
| number of points | 1404 | 1585 | 1994 | 5870 | 2611 | 3283 | 7580 | 3197 | 7221 | 7088 | 4268 | 8764 | 8176 | 8695 | 9685 |
| number of points in more than one epoch | | | | | **376** | 296 | 284 | 202 | 234 | 186 | 555 | 556 | 568 | 475 | **848** |
| share of points in more than one epoch | | | | | **14 %** | 9 % | 4 % | 6 % | 3 % | 3 % | **13 %** | 6 % | 7 % | 5 % | 9 % |
| number of points with significant movements | | | | | 44 | 87 | 69 | 44 | **108** | 102 | 85 | 200 | 284 | 205 | **342** |
| share of points with significant movements | | | | | 12 % | 29 % | 24 % | 22 % | 46 % | **55 %** | 15 % | 36 % | **50 %** | 43 % | 40 % |
| number of unknowns | 1403 | 1584 | 1993 | 5869 | 2987 | 3577 | 7862 | 3396 | 7453 | 7272 | 4821 | 9318 | 8742 | 9168 | 10531 |
| number of observations | 1696 | 1876 | 2117 | 8632 | 3572 | 3993 | 10749 | 3813 | 10508 | 10328 | 5689 | 12625 | 12204 | 12445 | 14321 |
| number of degrees of freedom | 293 | 292 | 124 | 2763 | 585 | 416 | 2887 | 417 | 3055 | 3056 | 868 | 3307 | 3462 | 3277 | 3790 |
| mean redundance | 0.173 | 0.156 | 0.059 | **0.320** | 0.164 | 0.104 | 0.269 | 0.109 | 0.291 | **0.296** | 0.153 | 0.262 | **0.284** | 0.263 | 0.265 |
| average length of time between 2 epochs in years | | | | | 29 | 20 | 15 | 49 | 35 | **64** | 49 | 35 | 64 | 64 | 64 |

Table 2: Preliminary results of vertical movement adjustment – Test area – Denmark –

| epoch combination | 1 | 2 | 3 | 12 | 23 | 13 | 123 |
|---|---|---|---|---|---|---|---|
| standard deviation of the weight unit in mm ($S_o$) | 1.29 | 1.08 | **0.63** | 1.09 | 0.99 | **0.82** | 1.82 |
| standard deviation of the heights in mm ($S_H$) | 12.64 | 10.43 | **6.17** | 9.88 | **6.02** | 6.86 | 9.29 |
| standard deviation of the height difference in mm ($S_{Dh}$) | 1.87 | 0.71 | **0.54** | 0.94 | **0.65** | 0.96 | 1.38 |
| standard deviation of the vertical movements in mm/y ($S_v$) | - | - | - | 0.26 | **0.13** | 0.15 | 0.15 |
| number of points | 975 | 6175 | 4498 | 6357 | 7919 | 4870 | 7944 |
| number of points in more than one epoch | - | - | - | 792 | 2753 | 602 | **3257** |
| share of points in more than one epoch | - | - | - | 12 % | 35 % | 12 % | **41 %** |
| number of points with significant movements | - | - | - | 293 | **759** | 255 | **759** |
| share of significant movements | - | - | - | 37 % | 28 % | **42 %** | 23 % |
| number of unknowns | 974 | 6174 | 4497 | 7148 | 10671 | 5471 | 11206 |
| number of observations | 1012 | 7186 | 4810 | 8198 | 11996 | 5822 | 13008 |
| number of degrees of freedom | 38 | 1012 | 313 | 1050 | 1325 | 351 | 1808 |
| mean redundance | 0.04 | **0.141** | 0.07 | 0.128 | 0.11 | 0.06 | **0.139** |
| average length of time between 2 epochs in years | - | - | - | 57 | 36 | 93 | 93 |

### 3.2 Denmark

*3.2.1 Data material*

The used data material of Denmark contains the following 3 levelling epochs:

1.)First epoch 1885 – 1904
1503 points, 1541 measurements

2.)Second epoch 1943 – 1961
9231 points, 10247 measurements

3.)Third epoch 1984 – 1992
7672 points, 7990 measurements

The numbers of the measurements are the number of the mean values of all sets within every epoch.

The measurements were summarized in such a way that only these points remained which are either nodal points or which are available in more than one epoch. There is a better correspondence of the various epochs than in the Netherlands (see figure 3).

The observations are measured height differences without gravity corrections.

*3.2.2 Results of the test computations*

The test computations were performed in the same way as these of the Netherlands: At first each epoch was adjusted separately. The computed variances were used as corrections in the adjustments of the epoch combinations. The results are shown in table 2.

The share of the identical points in the combination of all 3 epochs is 41 %, but only 23 % of them show a significant movement, because the adjusted velocities are smaller than those in the Netherlands. There are discrepancies between the velocities computed from the epochs 1 and 2 and those computed from the epochs 2 and 3. In some areas the sign of the velocity is inverted. These discrepancies are expressed in the big value of the a-posteriori standard deviation of the adjustment of all 3 epochs.

In the figures 4 – 6 the velocities are shown which are computed from the different epoch combinations. The comparison of the different results suggests some systematic errors in one of the epochs.

Apart from that the results show elevations in the Northeast and subsidences in the Southeast. Some obvious outliers should be removed from the data material.

## 4. Conclusions

The adjustment program KRUNA which developed at the data centre for computing vertical movements from repeated levelling observations is fit for work.

However it is important to interpret the results in connection with geologic data and with data of the stability of the points. To this end representatives of the respective country have to be consulted.

The part of the program which processes vertical velocities has still to be tested with suitable test data derived from permanent GPS observations. Therefore further investigations are needed about the form of the variance-covariance-matrix of this kind of observations.

## References

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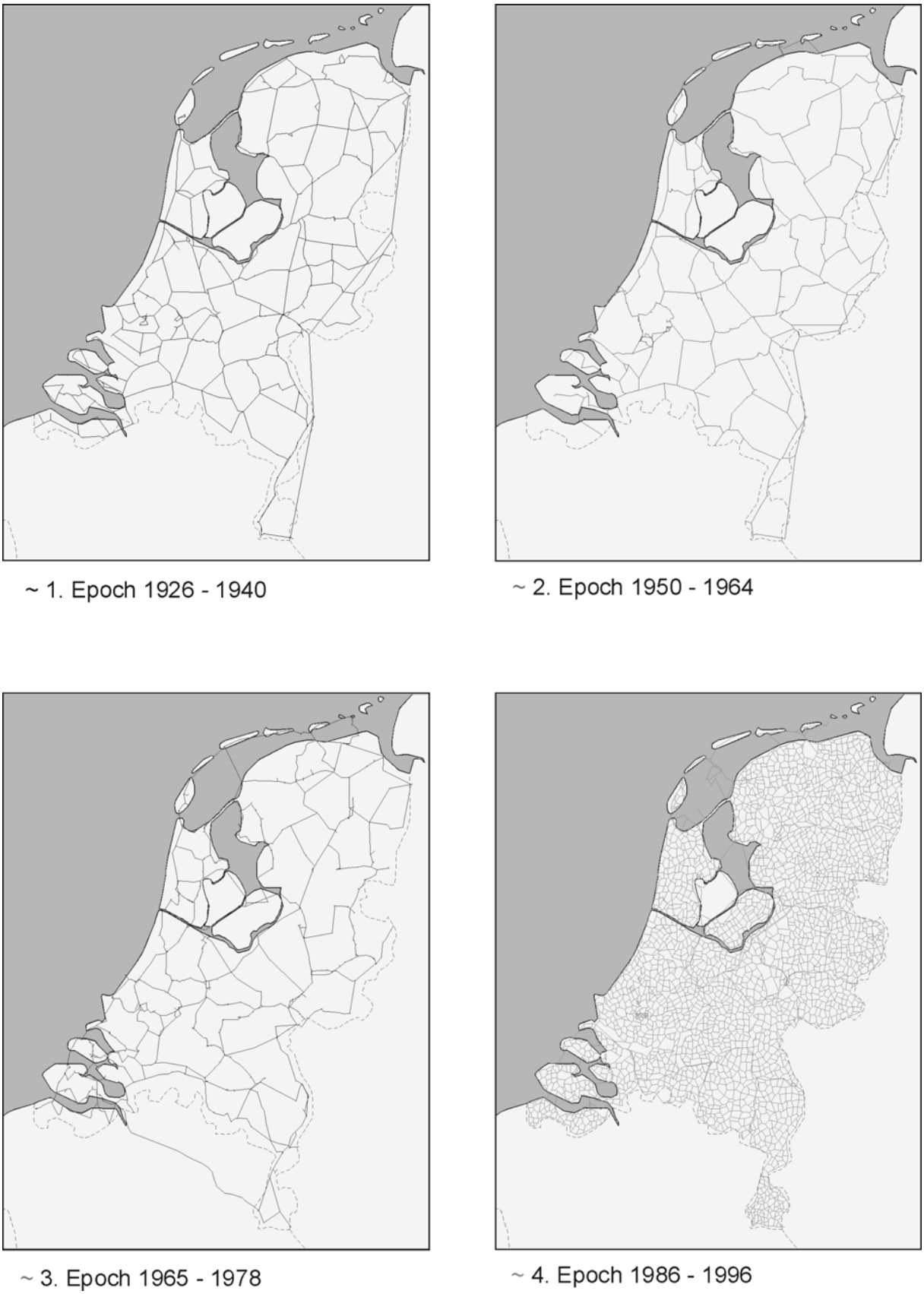



Figure 1: Levelling epochs of the Netherlands


Abb. 2 quer

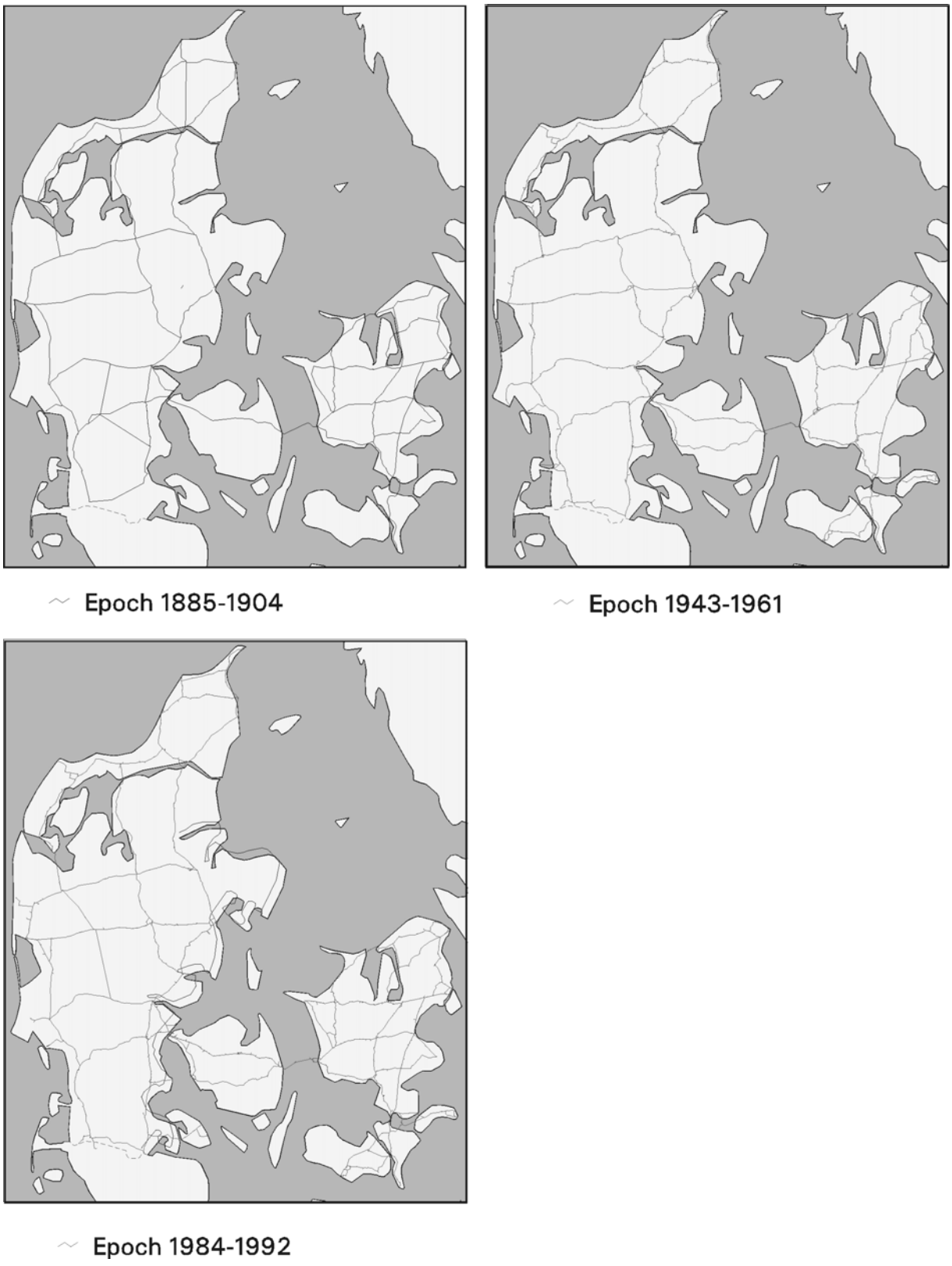
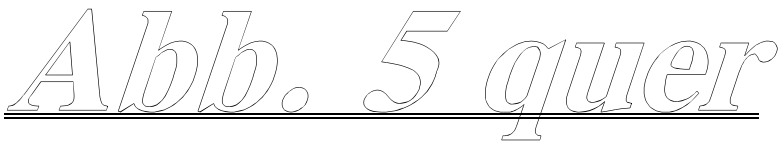



Figure 3: Levelling epochs of the Netherlands


Abb. 4 quer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