The Fast Constraints Transformation (FCT) method for rigorous and efficient SINEX files handling: The case of ETRS89

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Key points of the presentation

- Transforming constraints (MC <-> OC) handling
- Classical approach
- Drawbacks of the Classical Appproach
- The Fast Constraints Transformation (FCT) methodology
- FCT‘s implementations
- Fields of apllications

Constraints transformation

The transformation among different constraints solutions (MC to OC and vice versa) still plays crucial role for the success of a TRF realization:

- Global TRFs (e.g. applying OC to a GNSS network –constraining the scale and the origin not to the GNSS-inherited one, same holds for DORIS)

- Regional TRFs: Expressing the regional network (GNSS initial network with fixed orbits) to an ITRF solution, using No Net Conditions (NNC): NNT (EUREF), NNT+NNR /SIRGAS)

- Local TRFs: as discussed at the Regional TRF bullet

Classical Approach

Let us consider the case of weekly/daily or even long term solutions released in SINEX (solution level type) and one wants to transform a constrained solution to a different one:

1. Invert the CV matrix, estimate the solution's vector

2. Remove the initial constraints (**needed to know**)

3. Impose the new type of constraints

4. Apprise the right hand side of the NEQ

5. Solve the NEQ system

Classical Approach: Drawbacks

The so-called classical approach is the most rigorous one, however:

1. It needs at least 5 steps of implementation
2. The inversion of the NEQ system is 'sine qua non': The inversion of a matrix includes all the stations (or more parameters) and in the case of velocities the size of a matrix yields 6m X 6m (m stations)

If we have relatively small number of stations, it is somehow straightforward to invert the NEQ-system. But in the case of really large networks (e.g. IGS, US CORS networks, Chinese networks, APREF) the computation time is constantly increasing (see next slides)→ **much more computation effort (CPU time)**

What about solving everything to powerful servers (e.g. BKG, BEV, ASI, Bern etc server)? It could be a solution, but:

1. Do all the agencies and institutes have the opportunity computing everything to large servers?

2. The number of the GNSS stations is increasing. Thus, reaching the limit of the computational power is really challenging

Fast Constraints Transformation (FCT)

We developed a method on transforming the different contraints type, without interfering to :

1. **NEQs**

2. **The largest number of the inversion will be 14 X 14 (full static and dynamic NNCs: NNT+NNR+NNS)**

3. **A priori knowledge of the initial contraints**

This practically means the usage of relatively smaller computation time, without loosing the rigorous results of the classical approach

For more details regarding the FCT, one may consult:

Ampatzidis, D., Wang, L., Mouratidis, A. et al. Rigorous and fast constraints transformations at the solution level: case studies for regional and global GNSS networks. GPS Solut 26, 44 (2022). https://doi.org/10.1007/s10291-022-01225-3

Fast Constraints Transformation (FCT) : Formulas from MC to OC

1. New estimation:

$$\hat{\mathbf{x}}^{OC} = \left(\mathbf{I} - \mathbf{C}^{MC}\mathbf{G}^{T}\left(\mathbf{C}_{G} + \mathbf{G}\mathbf{C}^{MC}\mathbf{G}^{T}\right)^{-1}\mathbf{G}\right)\hat{\mathbf{x}}^{MC}$$

$$\Rightarrow \hat{\mathbf{x}}^{OC} = \mathbf{L}\hat{\mathbf{x}}^{MC}$$

C_G : CV matrix of the NNC conditions (max 14X 14, min 1X1), **G** Helmert parameters design matrix

→ inverting max. 14 X 14!!!!

→ not interfering with NEQS

2. New CV matrix:

$$\mathbf{C}^{OC} = \mathbf{L}\mathbf{C}^{MC}\mathbf{L}^{T}$$

The MC to OC transformation could also be applied of the EUREF case of using NNT to the initial NEQ system

3. New a posteriori variance:

$$\hat{\sigma}^2_{OC} = \frac{\hat{\varphi}_{MC} + \mathbf{u}^{T}(\mathbf{I}-\mathbf{L})\hat{\mathbf{x}}_{MC}}{f_{MC} + k} = \frac{\hat{\sigma}^2_{MC} f_{MC} + \mathbf{u}^{T}(\mathbf{I}-\mathbf{L})\hat{\mathbf{x}}_{MC}}{f_{MC} + k}$$

$$= \frac{\hat{\sigma}^2_{MC} f_{MC} + \left(\hat{\mathbf{x}}_{MC}\right)^{T}\mathbf{C}^{MC}(\mathbf{I}-\mathbf{L})\hat{\mathbf{x}}_{MC}}{f_{MC} + k}$$

Fast Constraints Transformation (FCT) : Formulas from OC to MC

1. New estimation:

$$\hat{\mathbf{x}}^{MC} = \left(\mathbf{I} + \mathbf{C}^{OC}\mathbf{G}^{T}\left(\mathbf{C}_{G} - \mathbf{G}\mathbf{C}^{OC}\mathbf{G}^{T}\right)^{-1}\mathbf{G}\right)\hat{\mathbf{x}}^{OC}$$

$$\hat{\mathbf{x}}^{MC} = \mathbf{M}\hat{\mathbf{x}}^{OC}$$

$\mathbf{C_G}$ **:** CV matrix of the NNC conditions (max 14X 14, min 1X1), **G** Helmert parameters design matrix

→ inverting max. 14 X 14!!!!

→ not interfering with NEQS

2. New CV matrix:

$$\mathbf{C}^{MC} = \mathbf{M}\mathbf{C}^{OC}\mathbf{M}^{T}$$

3. New a posteriori variance:

$$\hat{\sigma}^2_{MC} = \frac{\hat{\varphi}_{OC} + \mathbf{u}^T(\mathbf{I} - \mathbf{M})\hat{\mathbf{x}}_{OC}}{f_{OC} - k} = \frac{\hat{\sigma}^2_{OC} f_{OC} + \mathbf{u}^T(\mathbf{I} - \mathbf{M})\hat{\mathbf{x}}_{OC}}{f_{OC} - k}$$

$$= \frac{\hat{\sigma}^2_{OC} f_{OC} + \left(\hat{\mathbf{x}}_{OC}\right)^T \mathbf{C}^{OC}(\mathbf{I} - \mathbf{M})\hat{\mathbf{x}}_{OC}}{f_{OC} - k}$$

Numerical Tests

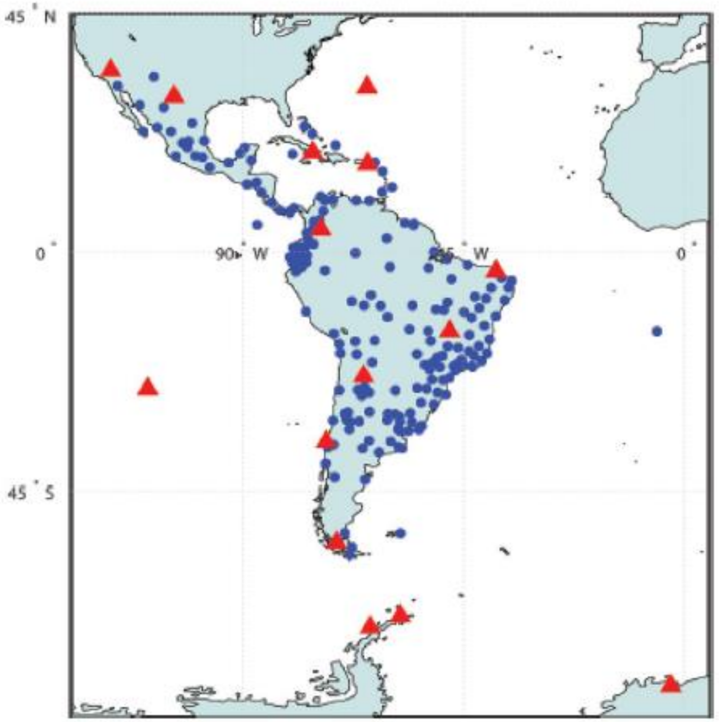
MC to OC

1. Weekly loose constrained SIRGAS solution to NNR/NNT (days 134-140, 2012)

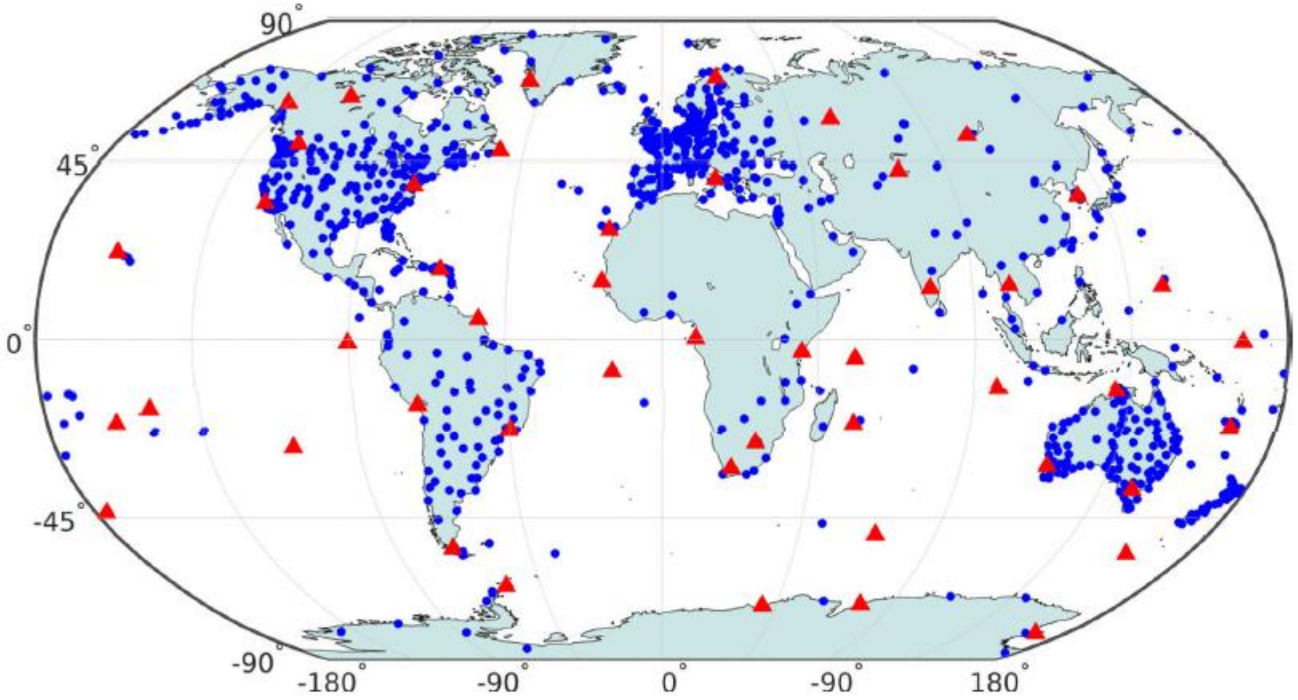
2. Weekly MC solution of IGS RERPRO 3 to OC NNR/NNT (week 1930, day 0)

3. Global GNSS network of 5000 (simulation)

Numerical tests

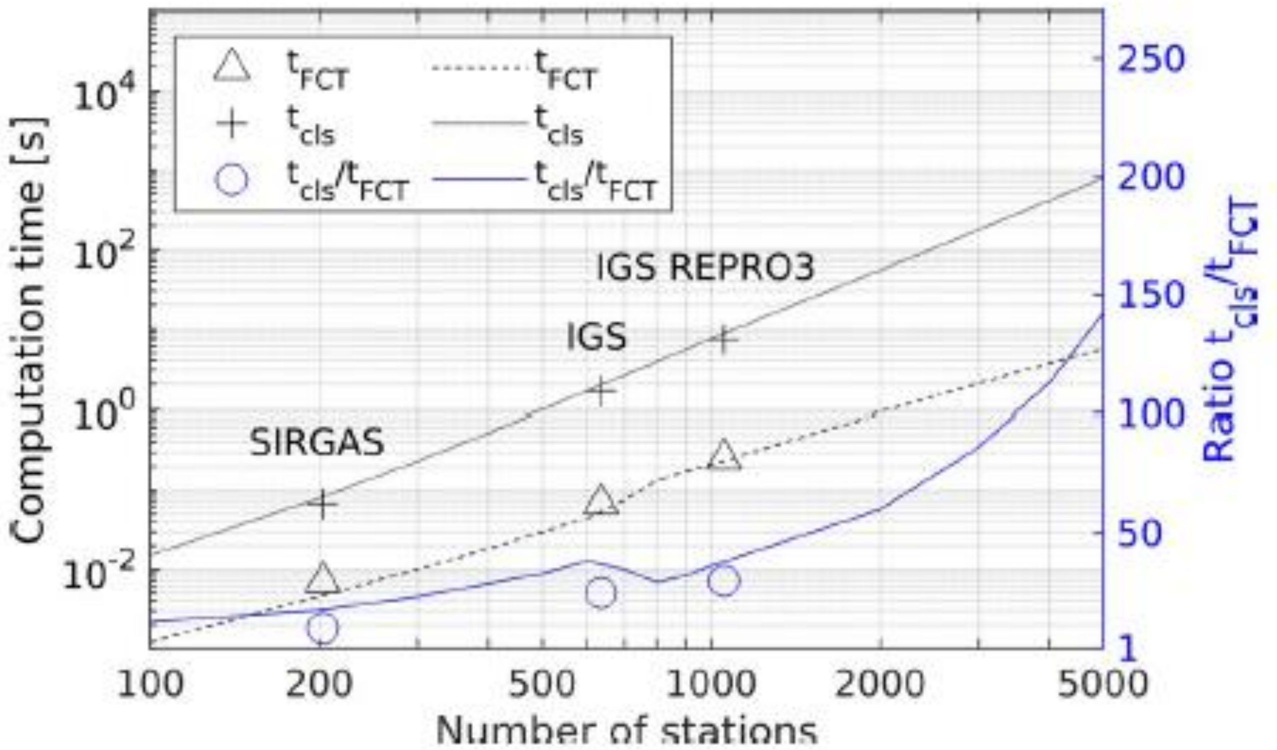


SIRGAS network consists of 203 stations. The red triangles are 15 fiducial stations; their coordinates participate in the no-net conditions. Blue dots are non-fiducial sites



IGS REPRO3 network consists of 1053 stations. The red triangles are 48 fiducial stations for NNT and NNR conditions, and blue dots are the non-fiducial stations

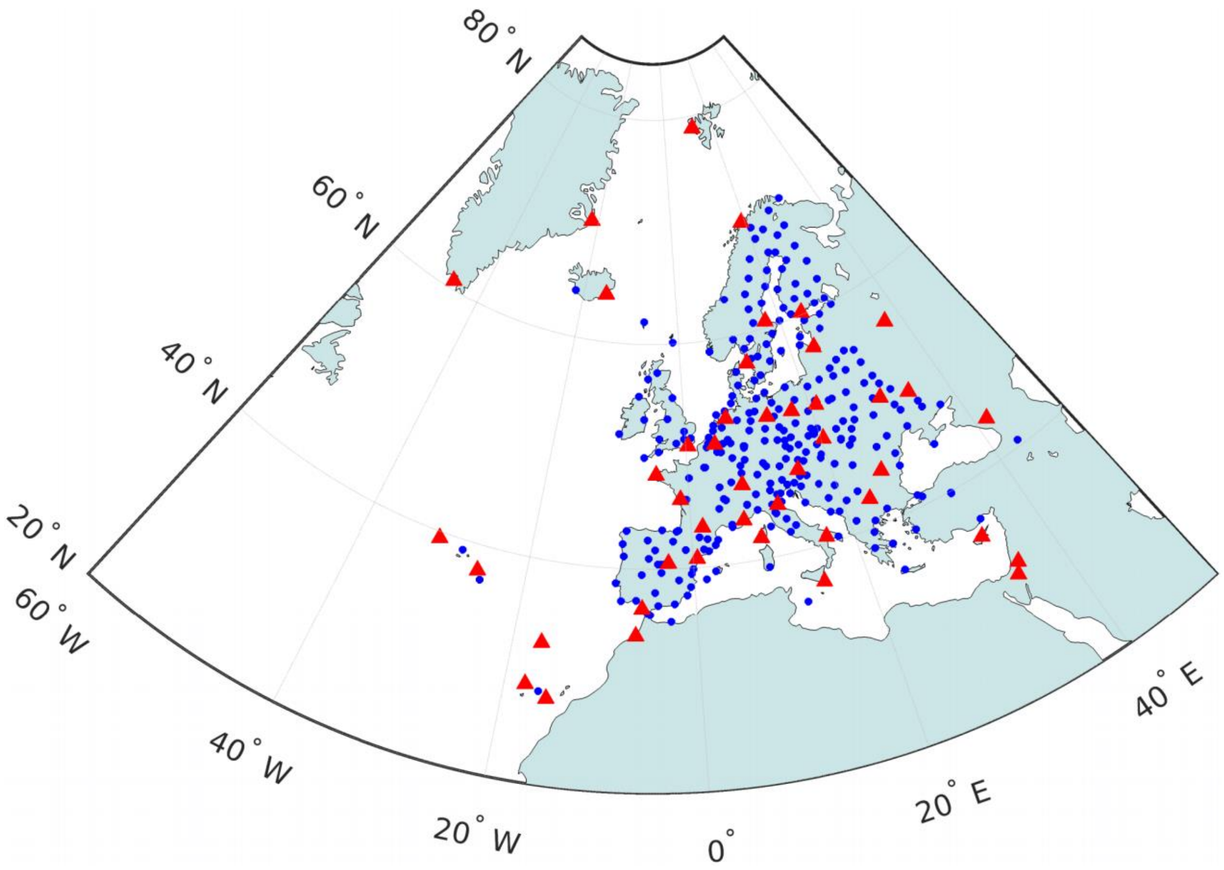
Computation time comparisons (classical approach vs FCT)



The ratio is increasing almost exponentially.

For the simulated network of 5000 stations, the FCT is **~ 140 faster than the classical approach**

ETRS89 case experiment

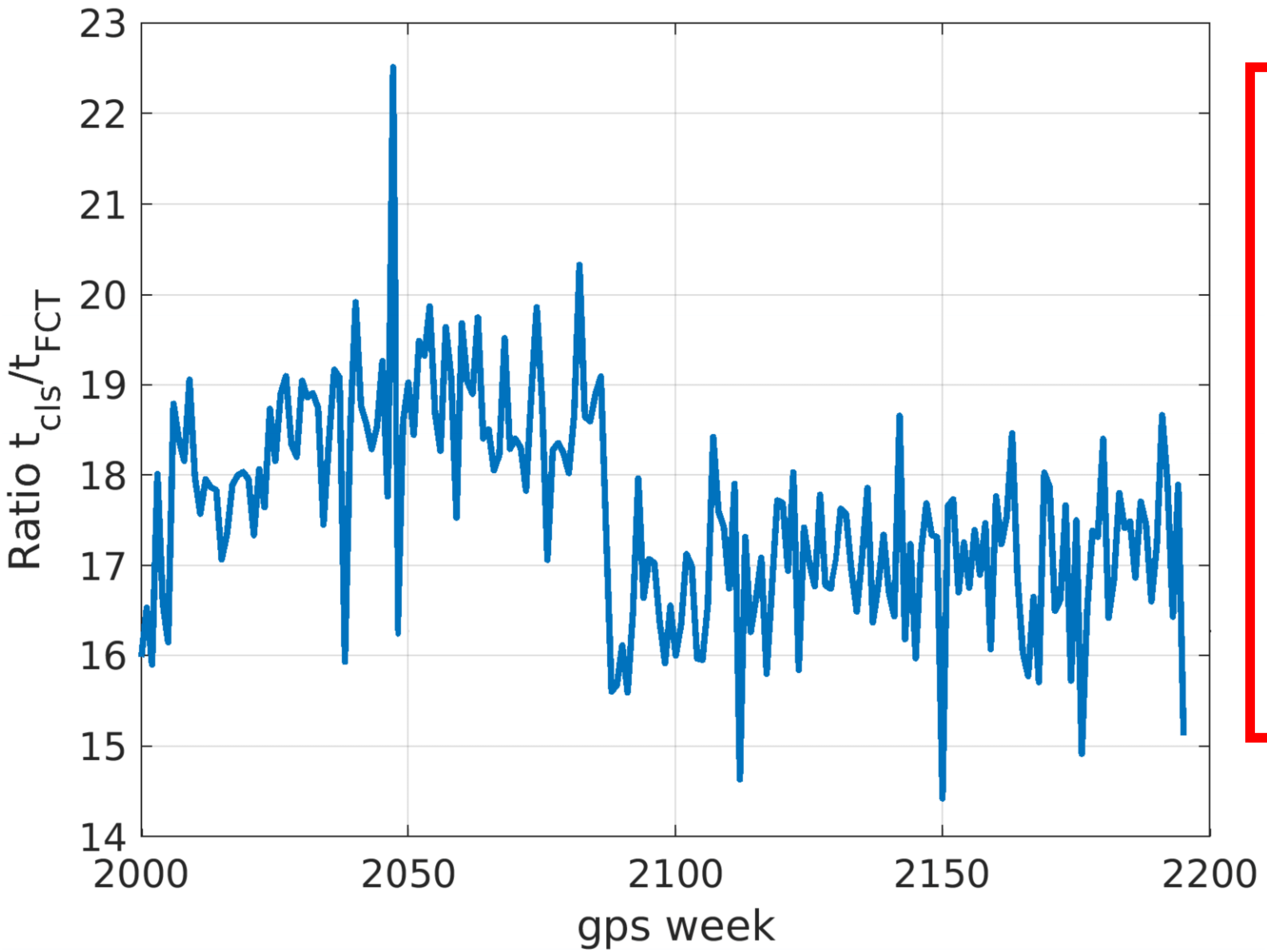


1. Weekly **solutions 2100-2195** (expressed to ITRF)

2. Experiment: Removal of the initial set of the MC constraints (see EUREF officially constraints sets) and then, implementing a new MC solution (randomly imposing a set with 1-2 less stations for the NNT than the initial official set)

3. Solved at the BKG server: Both for classical approach and FCT

Computational time ratio per week



1. 15-22 times faster using FCT than classical approach (per week!)

2. Using a strong and reliable Server (BKGs)

3. After 2080 week the ration is decreased; due to Server performance? Random?

4. The more stations, the larger ratio, as it is proven. Just consider future GNSS network expansion!

Conclusions and future work

1. FCT seems to present some crucial advantages regarding the computational time against to the classical approach

2. It is proved that the computational ratio between the classical approach and the FCT is increased exponentially

3. No need to know initial constraints set

4. Not interfering with NEQs

5. Applied in any TRF: Global , Regional Local

6. A general test for existing cases? E.g. ITRF, ETRS, SIRGAS, TRFs of big countries (USA, China, India etc.)

7. Applied in classical networks?

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THANK YOU FOR YOUR ATTENTION!!!
(and sorry for the maths!)

If you have any questions, comments, remarks, do not hesitate to contact us:

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