Precise Orbits in Broadcast Ephemeris Form for GPS (G), Glonass (R), Galileo (E) and BeiDou (C)

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Precise combined solutions vs. merged BRDMs

- Within the MGEX project several IGS analysis centers publish post computed precise ephemeris in the SP3 format for a number of GNSSs: typically G+R+E (+ IGSO and MEO SVs of C, BDS2 only)
- These post computed ephemeris can be considered –to date- the most accurate multiGNSS realization of the ITRF2014 (IGS14, IGb14): same origin, orientation, scale and time for all the GNSSs
- **Question**: can one update the coefficients in the broadcast messages of the various GNSSs, and provide Helmert parameters so that the ECEF XYZT computed with this ‘extended broadcast message’ and the XYZT from the post computed (SP3) ephemeris agree with the smallest possible rms within the validity time of each ephemeris block?

GPS, GALILEO and BeiDou (MEO,IGSO)

Define 'Discrepancy' as follows: Δ(XYZT) = XYZT(SP3) – XYZT(BRDM) at 15 min

Method of analysis: minimize the sum of the squared discrepancies in X,Y,Z,T by selecting appropriate values of the 7 parameters on a daily basis (one set per day) **and** 12 sets of 18 orbital and clock parameters

$$\sum_{i=1}^{96} [\Delta(XYZT)]^2 =$$

$$f(Tx, Ty, Tz, Rx, Ry, Rz, Sc;$$

1 set per day

$$a0, a1, a2,$$

Clock corrections

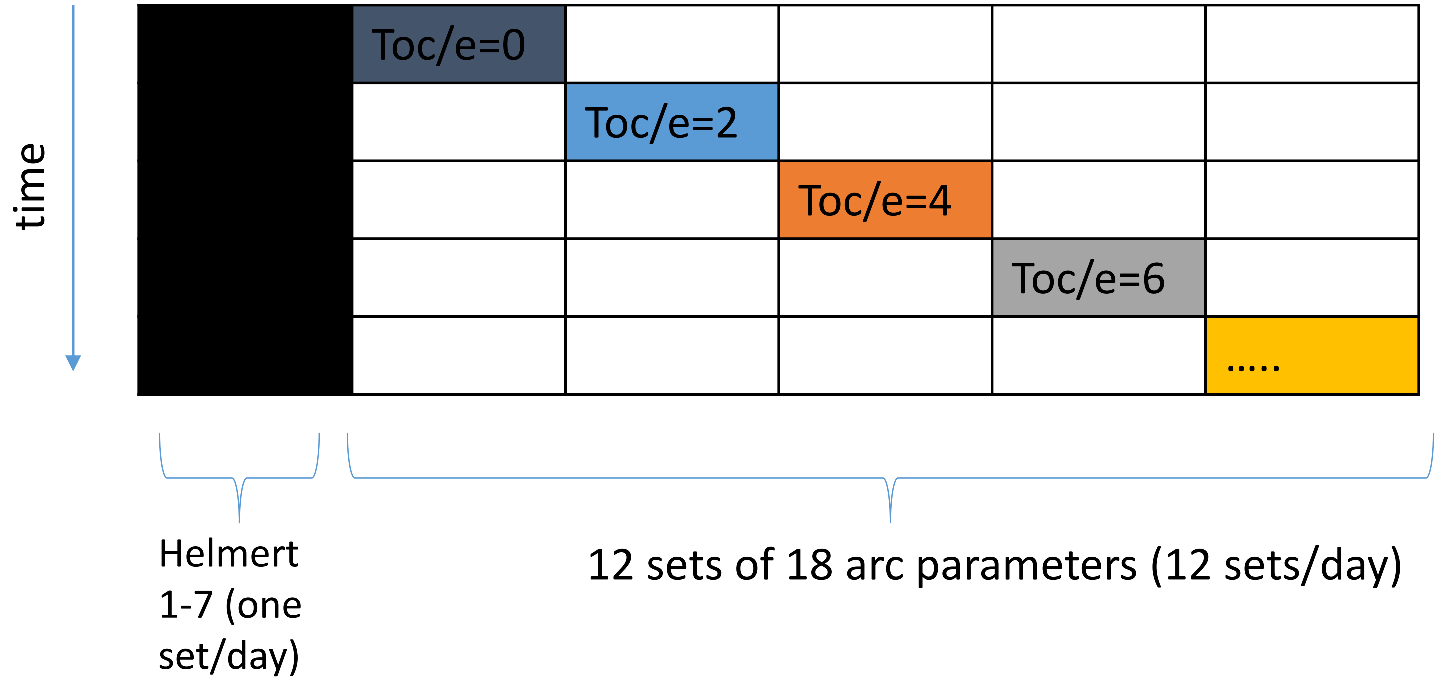
$$Crs, Delta_n, M0, Cuc, e, Cus, sqrt(a), Cic, Omega0, Cis, I0, Crc, omega, omdot, idot) = min$$

Orbit corrections

One set of 18 parameters every 2 hours (indexed with Toe or Toc): 18 x 12 + 7= 223 parameters; 96 x 4 = 384 observations
Some parameters are highly correlated (e.g. sqrt(a) with $Delta_n$, or the Helmert Rotations with the Inclination and node, or eccentricity with perigee) and are constrained to the a priori broadcast values; likewise for idot and omdot, there is insufficient dynamic range in 2 hrs to solve for them

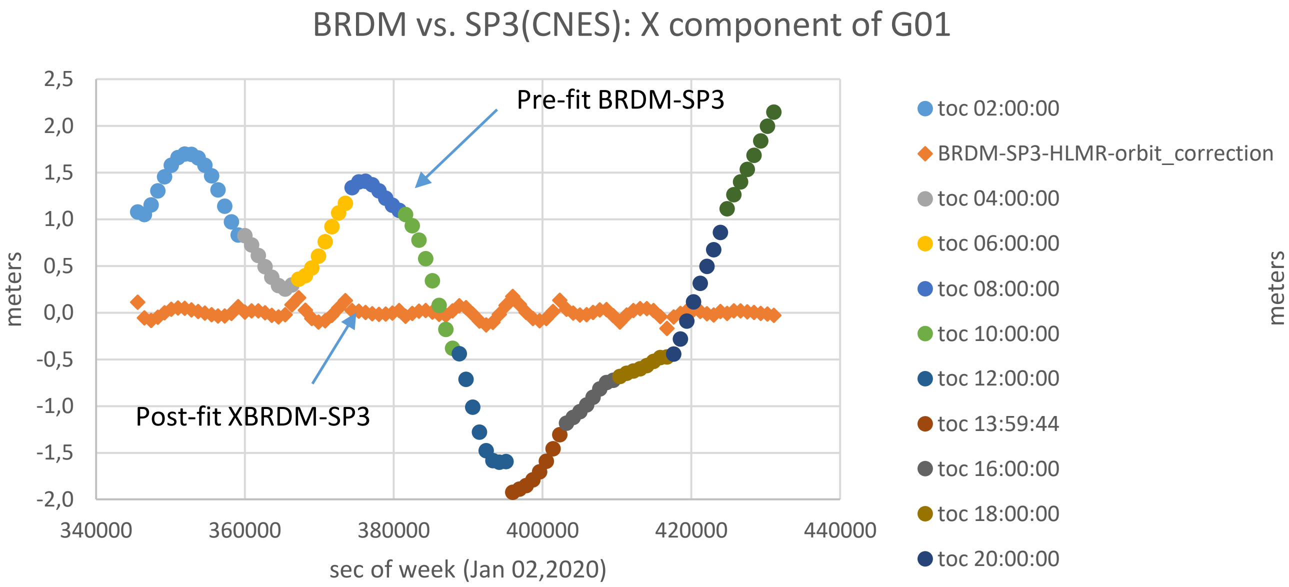
Structure of the Partial Derivative matrix H

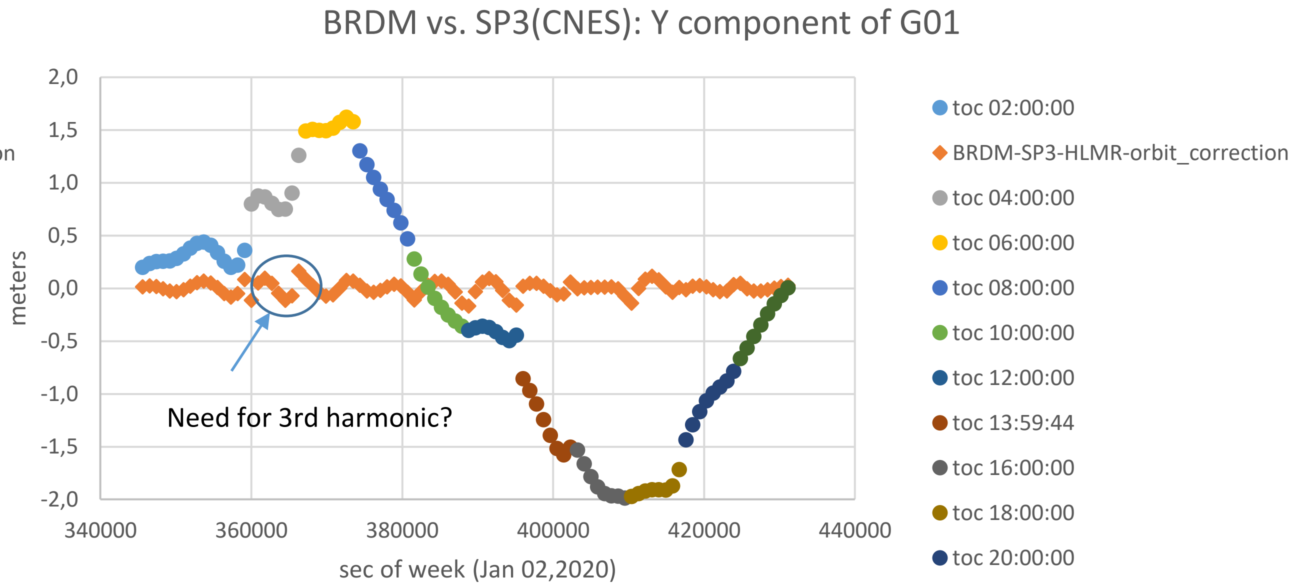
H =

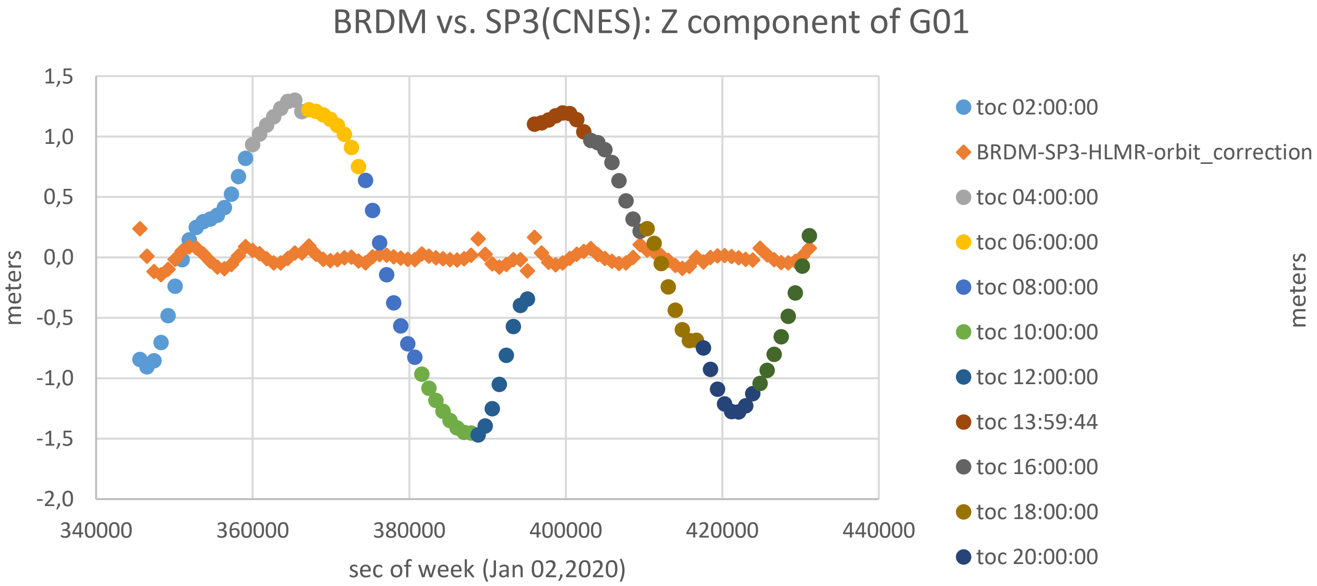


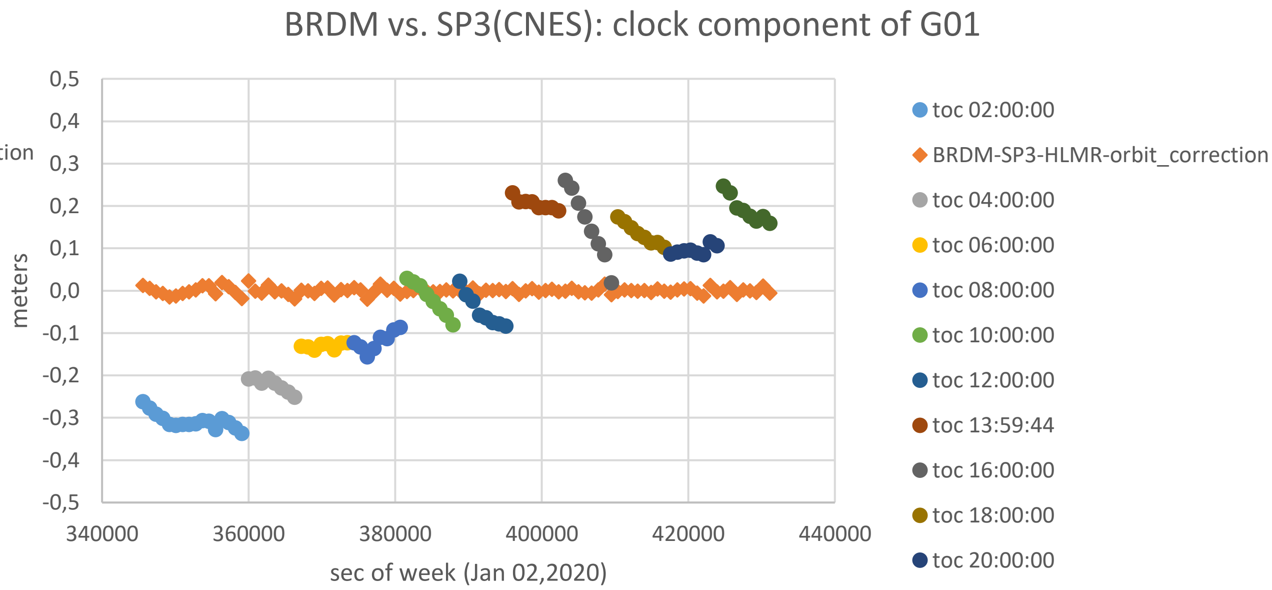
- No overlap of the 2 hour arcs
- Before inverting H^TH, Constraints are applied to the parameters (see previous page) kept fixed to the apriori (broadcast) values
- In summary we solve for 7 ‘global Helmert parameters’ (one set across the full day) and 12 sets of 7 arc parameters each, across 2 hrs arcs, representing one along track shift (M_0) and six sine and cosine amplitudes of 2nd harmonic perturbation along track, across track and radial

PRELIMINARY RESULTS with G01 on Jan 2, 2020









Numerical Results for G01 on Jan.2, 2020 with CNES SP3 orbit/clock

Prefit (SP3- BRDM): -0.013 ± 0.917 m; Postfit (SP3- XBRDM): 0.000 ± 0.051 m

Helmert parameters

	Tx (m)	Ty(m)	Tz(m)	Rx (mas)	Ry(mas)	Rz(mas)	Scale(*)
nominal	-0.09	0.20	-0.39	1.2	1.2	-11.1	3.86E-08
1 sigma formal error	0.04	0.04	0.02	0.5	0.4	0.1	8.48E-10

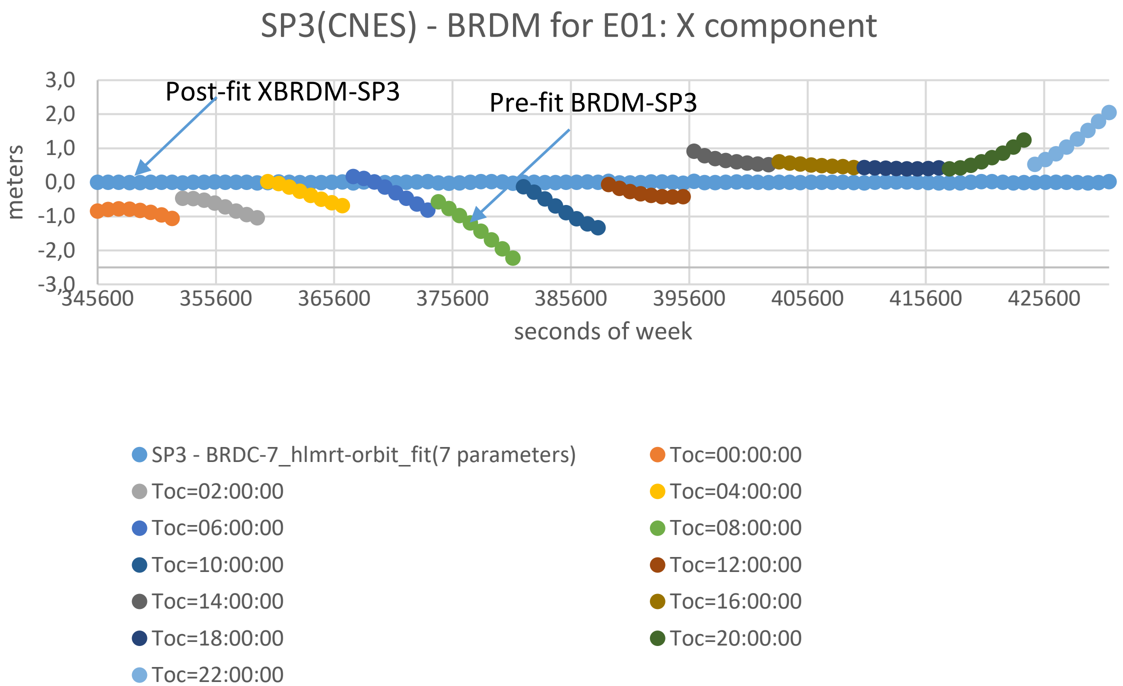
(*) scale accounts for CoM-APC radial bias

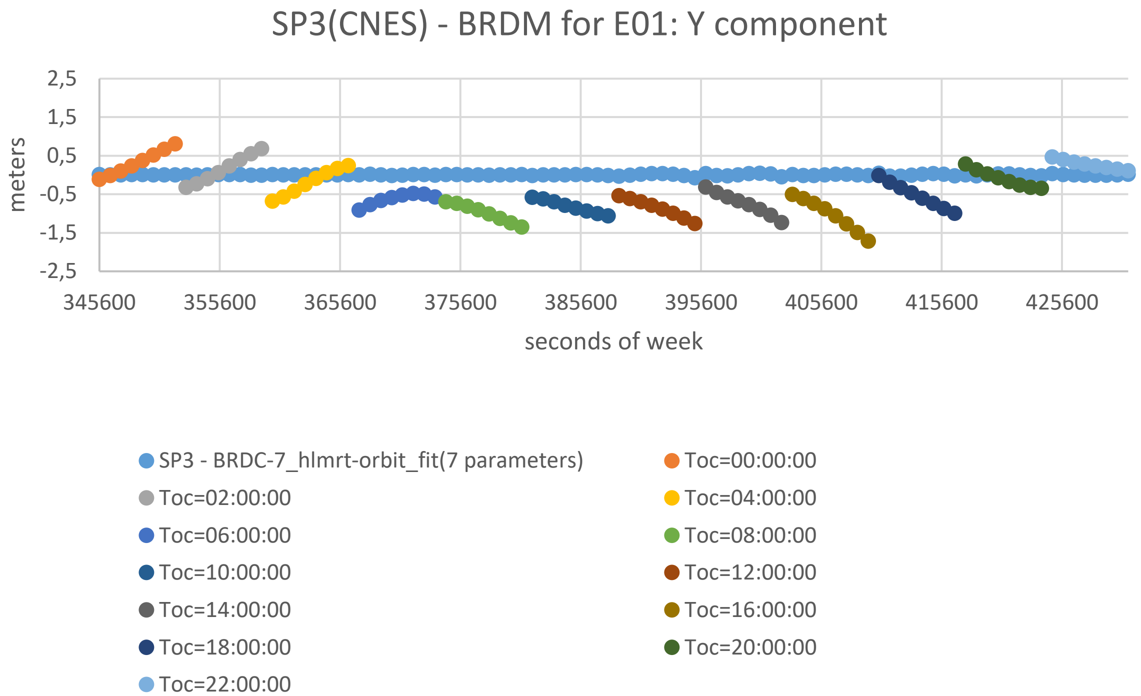
Arc parameters (corrections to a priori broadcast)

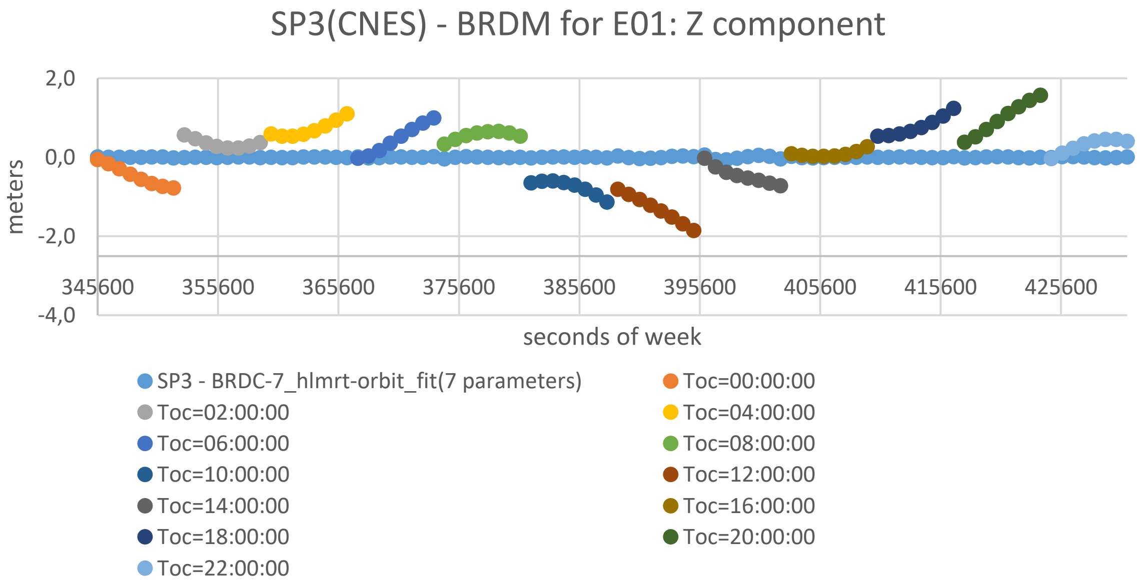
	a0 (sec)	a1(sec/sec)	a2(sec/sec^2)	M0 (m)	Crc (m)	Crs (m)	Cic (m)	Cis (m)	Cuc (m)	Cus (m)	
nominal	-1.1E-09	-9.5E-15	1.3E-18	-3.2E-01	-2.3E-01	2.0E-01	-1.9E-01	-1.7E-01	3.3E-01	3.5E-01	Toe=02:00 (4 hr arc)
1 sigma	6.4E-11	1.1E-14	2.8E-18	9.1E-02	2.7E-02	2.2E-02	4.7E-02	5.2E-02	2.7E-02	2.9E-02	
nominal	-6.5E-10	-3.8E-14	2.9E-18	-1.1E-01	-5.0E-01	-4.4E-01	-5.4E-02	-1.0E-01	4.7E-02	8.9E-01	Toe=04:00 (2 hr arc)
1 sigma	1.8E-10	7.2E-14	6.1E-18	1.3E-01	3.7E-02	4.7E-02	4.2E-02	4.5E-02	1.0E-01	7.6E-02	
…	…	…	…		…	…	…	…	…	…	Toe=06:00 (2 hr arc)
…	…	…	…		…	…	…	…	…	…	

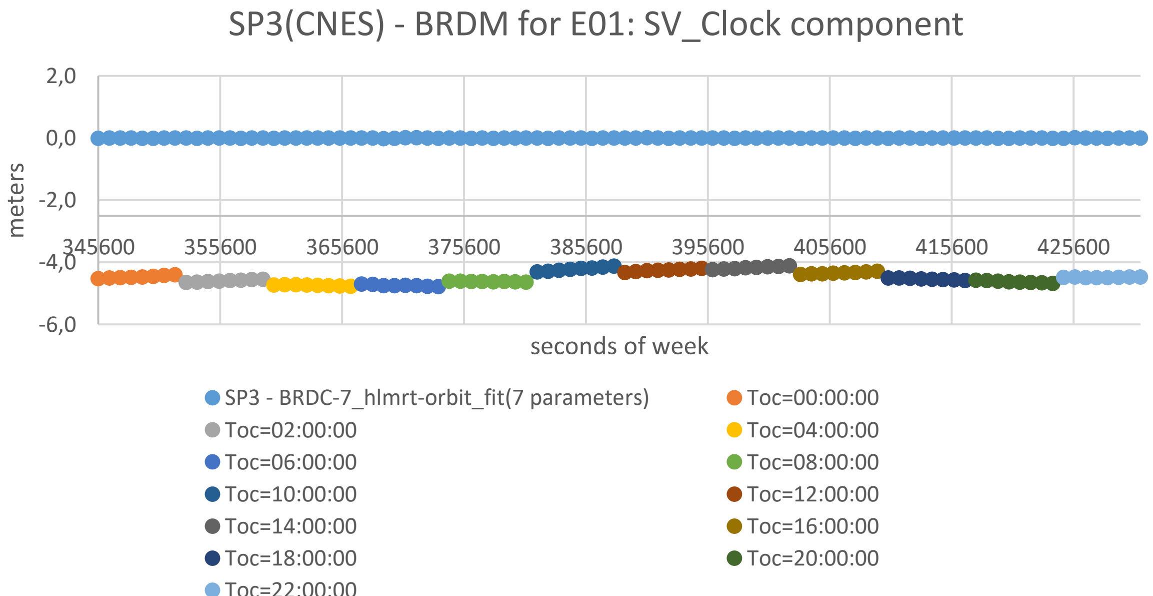
$\underbrace{\text{a0, a1, a2}}_{\text{Clock correction}}$ Along track shift (M0) $\underbrace{\text{Crc, Crs}}_{\text{2nd harmonic radial}}$ $\underbrace{\text{Cic, Cis}}_{\text{2nd harmonic x-track}}$ $\underbrace{\text{Cuc, Cus}}_{\text{2nd harmonic along track}}$

PRELIMINARY RESULTS with E01 on Jan 2, 2020









Numerical Results for E01 on Jan.2, 2020 with CNES SP3 orbit/clock

Prefit (SP3- BRDM): -1.224 ± 1.992 m; Postfit (SP3- XBRDM): 0.000 ± 0.014 m

Helmert parameters

	Tx (m)	Ty(m)	Tz(m)	Rx (mas)	Ry(mas)	Rz(mas)	Scale(*)
nominal	-0.05	-0.03	-0.04	0.3	-0.6	-0.8	2.46E-10
1 sigma formal error	0.02	0.02	0.02	0.29	0.17	0.06	4.68E-10

(*) CoM-APC coordinates from https://www.gsc-europa.eu/support-to-developers/galileo-satellite-metadata#5.1.2

Arc parameters (corrections to a priori broadcast)

a0 (sec)	a1(sec/sec)	a2(sec/sec^2)	M0 (m)	Crc (m)	Crs (m)	Cic (m)	Cis (m)	Cuc (m)	Cus (m)	Toe
-1.5E-08	4.3E-14	3.0E-18	-0.58	0.01	-0.13	-0.14	0.01	0.22	-0.93	02:00
-1.6E-08	6.0E-14	-6.5E-19	-0.39	0.05	-0.08	0.09	-0.05	0.81	0.49	04:00
-1.6E-08	1.5E-14	-5.9E-18	-0.40	0.03	0.05	0.02	0.01	-0.83	0.68	06:00
-1.6E-08	-4.8E-14	1.6E-18	-0.32	-0.07	0.07	0.16	0.05	-0.58	-0.97	08:00
-1.5E-08	-1.0E-15	-1.6E-18	-1.36	-0.06	0.10	0.18	0.09	1.07	-0.38	10:00
-1.4E-08	1.1E-13	-1.2E-18	-0.70	-0.13	-0.01	0.03	0.10	0.27	1.07	12:00
-1.5E-08	1.1E-13	-5.4E-18	-1.10	0.06	-0.16	-0.19	0.13	-0.98	-0.07	14:00
-1.4E-08	7.7E-14	-1.6E-18	-0.21	0.19	-0.09	0.03	0.02	-0.06	-1.00	16:00
-1.5E-08	1.9E-14	3.6E-18	-1.14	0.08	0.00	-0.07	0.01	0.63	0.70	18:00
-1.5E-08	-4.3E-14	-4.6E-19	-0.90	-0.03	0.08	-0.01	-0.06	-0.74	0.62	20:00
-1.5E-08	-6.1E-14	1.0E-18	-0.98	0.00	0.02	0.14	-0.01	-0.77	-0.90	22:00

Clock correction

Along track shift

2nd harmonic radial

2nd harmonic x-track

2nd harmonic along track

Uncertainties not given but similar or smaller than G01

Glonass

Define ‘Discrepancy’ as follows: Δ(XYZT) = XYZT(SP3) – XYZT(BRDM) at 15 min

Minimize the sum of the squared discrepancies in X,Y,Z,T by selecting appropriate values of the 9 (orbit) +3 (clock) parameters in 2hr or 1hr contiguous arcs

$$\sum_{i=1}^{96}[\Delta(XYZT)]^2 =$$

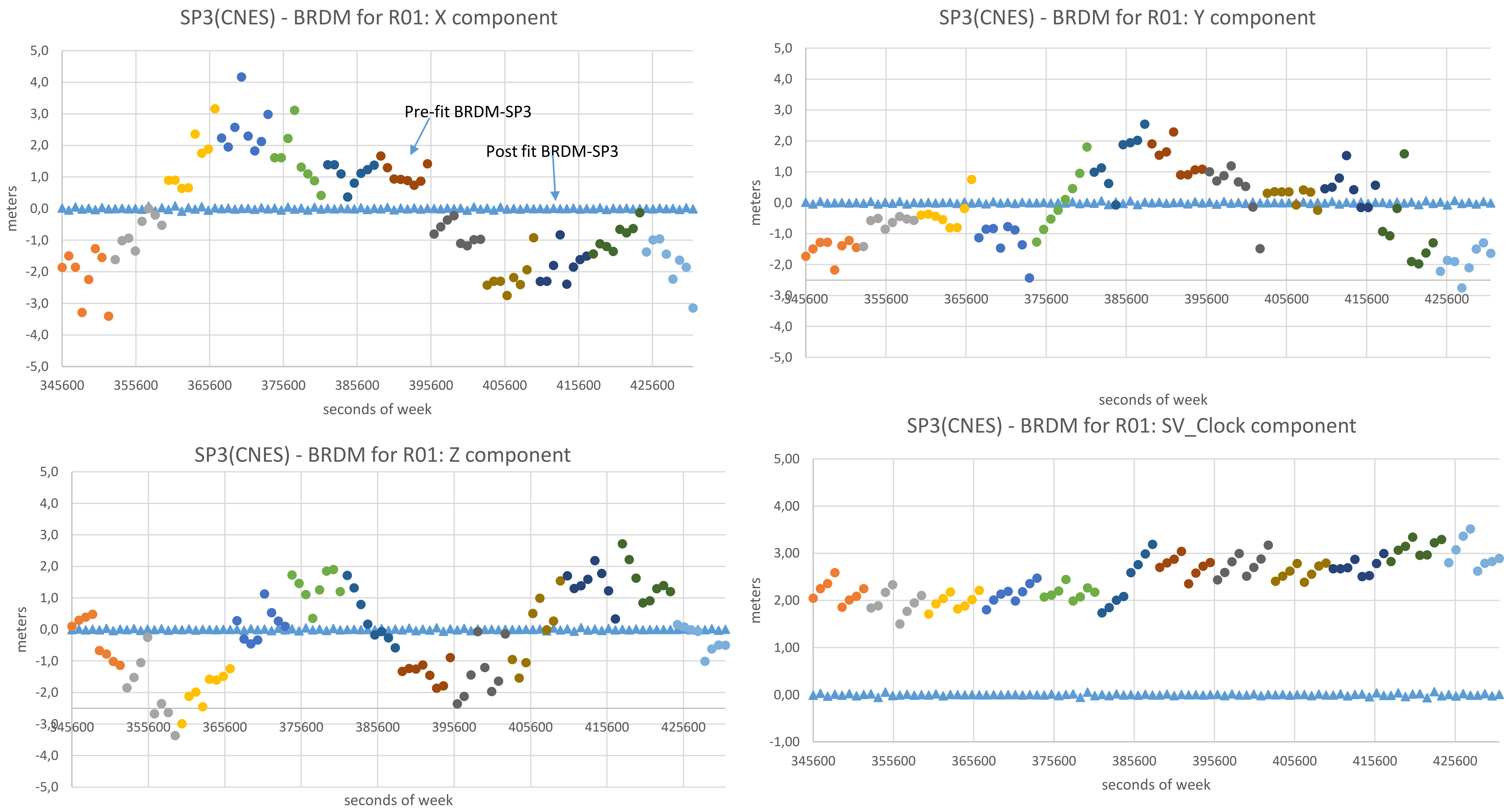
$$f(a0, a1, a2,\ \underbrace{X_0, \dot{X}_{0,}, \ddot{X}_{LS}, Y_0, \dot{Y}_{0,}, \ddot{Y}_{LS}, Z_0, \dot{Z}_{0,}, \ddot{Z}_{LS},}_{\text{Initial values for the numerical integration of the equations of motion in a rotating ECEF system}}) = min$$

Clock corrections

Orbit corrections (1 set every one or two hours)

One set of 9+3 parameters every 2 or 1 hours (indexed with Toe or Toc): 12 x 12 = 144 parameters (2 hour arc); 12 x24 = 288 (1 hour arc); 96 x 4 = 384 observations

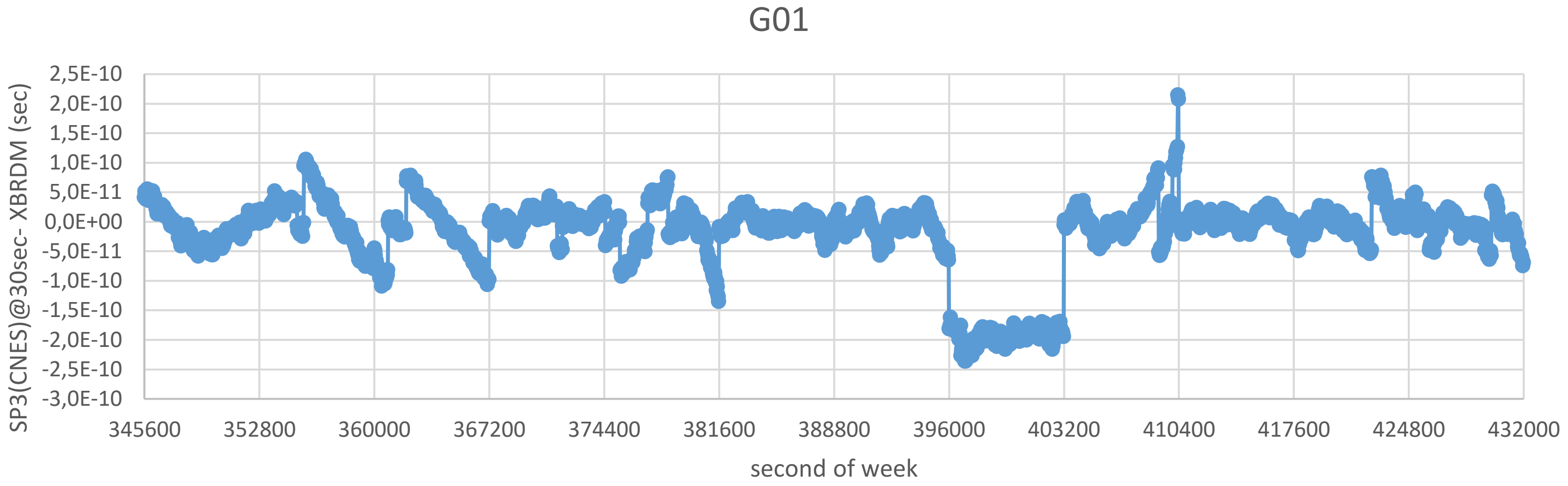
PRELIMINARY RESULTS with R01 on Jan 2, 2020 1hr arcs

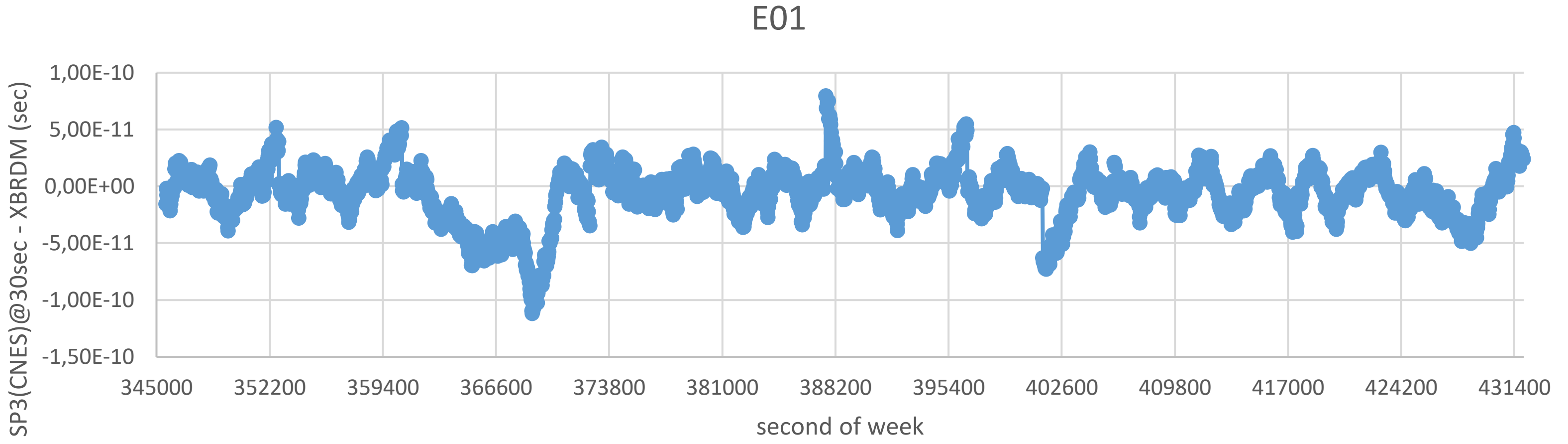


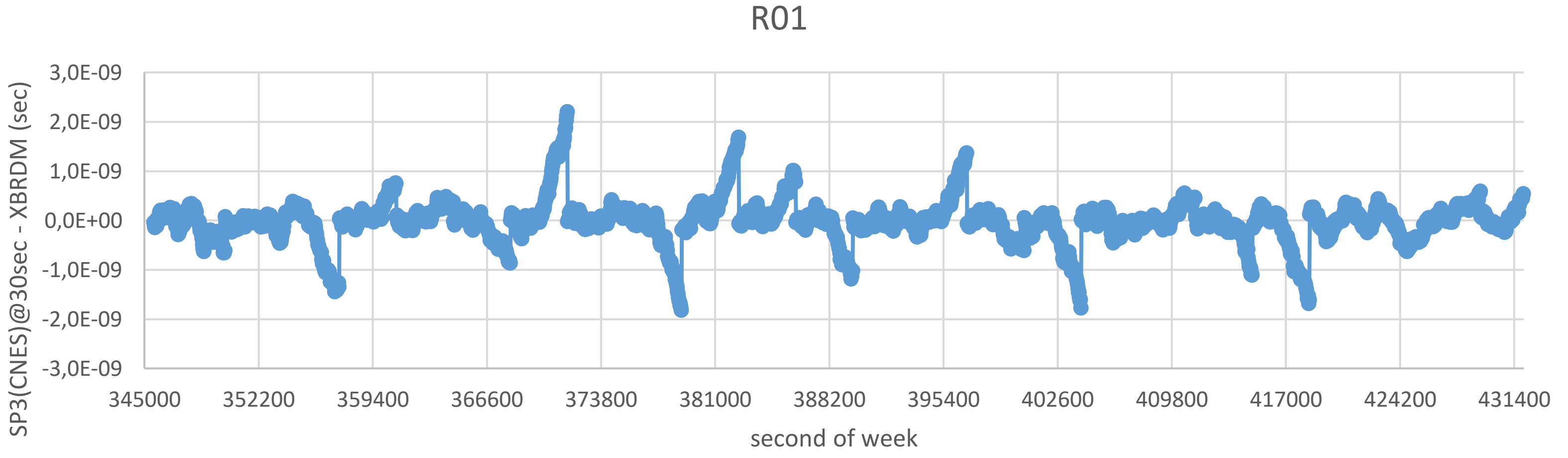
GPS Glonass and Galileo high rate (30 sec) clock vs XBRDM 2hr (1hr for Glonass) clock polynomials

When using a clock polynomial, the max discrepancy of the XBRDM polynomial model relative to the high rate clock data is

- < 0.25 ns for GPS,
- < 0.1 ns for Galileo
- < 2.0 ns for Glonass







Improving the gravity model in the navigation message: J_3 perturbations are required for centimetric accuracy modeling at MEO altitudes: $(J_3/J_2)*200$ m ≈ 0.47 m

Amplitude of Cuc, Cus

Amplitude of 3° harmonic along track perturb.

- GPS – like message: 12/3 = 4 hr periodic perturbations need to be modeled

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uk = vk + long_perigee

s2uk = Sin(2 * uk)

c2uk = Cos(2 * uk)

s3uk = Sin(3 * uk)

c3uk = Cos(3 * uk)

duk = cuc * c2uk + cus * s2uk+ c3uc * c3uk + c3us * s3uk

drk = crc * c2uk + crs * s2uk + c3rc * c3uk + c3rs * s3uk

dik = cic * c2uk + cis * s2uk + c3ic * c3uk + c3is * s3uk

….

JGM-3 zonals: J_2= -0.10826D-02; J_3= 0.25324D-05

- Glonass – like message: add the J_3 dependent force term to the J_2 term

$$F_x = -\frac{\mu x}{r^3} + J_2\frac{\mu a_e^2}{r^7}x\left[6z^2 - \frac{3}{2}(x^2+y^2)\right] + \omega_e^2 x + 2\omega_e y + \ddot{x}_{LS} + J_3\frac{\mu a_e^3}{r^9}xz\left[10z^2 - \frac{15}{2}(x^2+y^2)\right]$$

$$F_y = -\frac{\mu y}{r^3} + J_2\frac{\mu a_e^2}{r^7}y\left[6z^2 - \frac{3}{2}(x^2+y^2)\right] + \omega_e^2 y - 2\omega_e x + \ddot{y}_{LS} + J_3\frac{\mu a_e^3}{r^9}yz\left[10z^2 - \frac{15}{2}(x^2+y^2)\right]$$

$$F_z = -\frac{\mu z}{r^3} + J_2\frac{\mu a_e^2}{r^7}z\left[3z^2 - \frac{9}{2}(x^2+y^2)\right] + \ddot{z}_{LS} + J_3\frac{\mu a_e^3}{r^9}\left[\left\{4z^2[z^2 - 3(x^2+y^2)] + \frac{3}{2}(x^2+y^2)^2\right\}\right]$$

XBRDM and SSR

XBRDM concept:

- Modify the parameters in the navigation message by LSQ adjustment using SP3 (e.g. rapid, ultrarapid,predicted) as input for orbit and clock
- Estimate on a daily basis 7 Helmert parameters, one per SV: XYZT(BRDM)→ XYZT(SP3)
- Keep update rate unchanged wrt the original message
- Two hrs clock polynomials are very close to high rate (30 sec) clocks
- Applicable RTCM messages: RTCM messages 1019,1020,1042,1045/6 for GRCE respectively, and 1021 for Helmert parameters (one set per SV at daily/weekly intervals)
- Algorithm must incorporate Helmert rototranslation

SSR concept:

- Keep the original BRDM navigation message unchanged (as broadcast by the SV)
- Compute along track, cross track and radial corrections to BRDM in the form of linear functions of time
- Update rate of Real Time Products from IGS ACs (CLKnn): 60 sec or less for orbit and a few sec for clocks, with few sec latency (typical)
- Applicable RTCM messages: RTCM 1060 and 1066 for G and R respectively
- Algorithm must incorporate SSR corrections

Possible use of XBRDM in RT in support of SSR

- Compute XBRDM using ultrarapid SP3 products: update rate 1-4 hours, depending on GNSS and improved gravity model (J3)
- Compute SSR based upon the XBRDM rather than the satellite broadcast BRDM