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Outline:

- Introduction & Aim
- Validation method
- Available data
 - Transformation grids
 - Local data
- Results interpretation and analysis
 - Adjustment results
 - Transformation grids based on local data: BG07-19 and BGBalt-19
 - Transformation grid validation: charts and statistics
- Conclusion and next steps



Introduction:

- EUREF Resolution No 3 (EUREF Symposium in Tallinn, 2019) recommends the adoption of the EVRF2019 as the new realisation of EVRS;
- The final results were provided to the participant countries in November, 2019;
- Request for validation of the EVRF19 to national height systems transformation grids.

Aim of the presentation:

To present a description of the validation results for two of the transformation grids:

- transformation between the EVRF2019 and the EVRF2007 (as the height component of BGS2005), both in zero tidal system – abbreviated as BKG07-19, and
- transformation between the EVRF2019 (in zero tidal system) and the Baltic Height System 1982 (in mean tidal system) - abbreviated as BKGBalt-EVRF19.



Validation method:

- data requirements
 - relevance;
 - accuracy;
 - spatial distribution/density over the territory of the country.
- validation method
 - ✓ an interpolation of values from the transformation grid at discrete points and a comparison with the relevant differences in the heights;
 - ✓ a comparison between two transformation grids.

- network adjustment
 - parametric least square adjustment;
 - differences in the heights between the two height system realisations as "unknowns";
 - precision/accuracy determination for the differences in the heights
 - available accuracy information for the control points – <u>fixed</u> vs <u>constrained</u> least square adjustment.



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Validation method:

 R_i (i = 1, 2) - different height system realisations, h_l (l = 1....L) - measured height differences between each two consecutive benchmarks (BMs); H_n (n = 1....N) - heights of the new BMs (i.e. BMs with heights in one height system realisation only), H_{inm} (m = 1...M) - heights of the BMs, which are common for both height system realisations.

	$\begin{bmatrix} \Delta h_1 \end{bmatrix}$		г—1	1	0	 0	0	0	ך 0	[Δ <i>H</i> ₁]
	Δh_2		0	-1	1	 0	0	0	0	ΔH_2
						 			•••	
$R_{-} - R_{-} \rightarrow$	Δn_L	_			•••	 •••	•••	•••		ΔH_N
$n_2 n_1$	ΔH_{in1}	_	0	0	0	 1	0		0	ΔH_{in1}
	ΔH_{in2}		0	0	0	 0	1		0	ΔH_{in2}
	ΔH_{inM}		L 0	0	0	 0	0		1^{I}	$\left[\Delta H_{inM}\right]$

D_i - lengths of the levelling sections,

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 σ_i designate the variances of the adjusted differences in the heights (upper left part of the matrix) and the common for the two realisations BMs (lower right part of the matrix).

The availability of the σ_{Hi} provides the opportunity for designing a **constrained LSA solution** in which the accuracy of the "known" points (BMs) is considered.

Alternatively, the heights of these BMs could be held fixed by specifying $\sigma_{Hi}^2 = 1$, i.e. obtaining a **fixed LSA solution**.

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		First Order levelling	Second Order
		lines	levelling lines
		3rd: 44, 48, 51 ÷ 54,	
notre	ork relevalling grale	part of line 55	3rd: all
netw	ork relevening cycle	4th: 1÷ 43, 45 ÷ 47,	
		49, 50, part of line 55	4th: none
		3rd: 1977 ÷ 1984	3rd: 1982 ÷ 1995
		4th: 1995 ÷ 2004	
ye	ars of measurement	4th: 2008 ÷ 2009	4th: none
		4th: 2015 ÷ 2016	
	RMS for 1 km		
acy	levelling based on	0.40 mm/km ^{1/2}	0.59 mm/km ^{1/2}
cur	measurements only		
acc	a posteriori RMS	1.26 mm/km ^{1/2}	2.12 mm/km ^{1/2}
	•		
total 1	number of benchmarks	4548	6289







Available Qa ransformation grids or



- Transformation grids provided by BKG:
- 4 transformation grids: EVRF19 to BGS2005/Baltic82 (ZTS and MTS);
- grid resolution: 0.15° (16.2 km);
- number of identical points: 59.
- Local data: First and Second Order
 State Levelling Network

transform.	BG(BGS2005) to EVRF2019	BG(Baltic82) to EVRF2019
abbr. as	BKG07-19	BKGBalt-EVRF19
identical BMs	59	58
min height difference	-0.051 m	+0.167 m
max height difference	+0.034 m	+0.277 m
mean height difference	-0.002 m	+0.228 m
standard deviation	±0.018 m	±0.024 m



• Adjustment results - example: LSA results related to BGS2005 (EVRF2007) to EVRF2019(z) transformation





• Adjustment results: fixed vs constrained LSA: statistics

	BGS2005-EVRF19 (fix)		BGS2005-E	VRF19 (con)	fix - con	
	$\delta\Delta H[m]$	$\sigma_{\Delta H}[m]$	$\delta\Delta H[m]$	$\sigma_{\Delta H}[m]$	differences [m]	
min	-0.0507	0.0000	-0.0183	0.0022	-0.0354	
max	0.0342	0.0107	0.0128	0.0044	0.0238	
ave	0.0031	0.0056	-0.0007	0.0029	0.0038	
std	0.0152	0.0015	0.0067	0.0004	0.0094	
σ_0	0.0015		0.3619			
	BG(Balt82)-EVRF19 (fix)		BG(Balt82)-EVRF19 (con)		fix - con	

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	$\delta\Delta H[m]$	$\sigma_{\Delta H}[m]$	$\delta\Delta H$ [m]	$\sigma_{\Delta H}[m]$	differences [m]
min	0.1867	0.0000	0.2052	0.0024	-0.0257
max	0.2654	0.0103	0.2425	0.0047	0.0259
ave	0.2309	0.0055	0.2267	0.0032	0.0042
std	0.0171	0.0015	0.0080	0.0004	0.0100
σ_0	0.0015		0.3873		

 $m_{\Delta H} = \mu \sqrt{\text{Qii} - \text{RMS}}$ of the adjusted differences in the heights

 μ – a posteriori RMS

Descriptive statistics of the differences [mm] between the results of the fixed adjustment and the BGS2005/BG(Balt82) to EVRF2019 transformation grid

	BGS2005	BG(Balt82)
Mean	0.6	-1.3
Standard Error	0	0.1
Median	0.4	-0.5
Mode	#N/A	#N/A
Standard Deviation	3	6.6
Sample Variance	8.9	43
Kurtosis	1.4	6.7
Skewness	0	-1.4
Range	22.8	73.6
Minimum	-12.5	-37.1
Maximum	10.3	36.5
Sum	6435.7	-14101.2
Count	10653	10653
Confidence Level (95.0%)	0.057	0.125

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Transformation grid BG07-19(fix)



- First and Second Order State Levelling Network data;
- biharmonic spline function;
- differences btw fixed LSA and grid;

Transformation grid BGBalt-19(fix)



- grid size 88 x 187 = 16456
- grid resolution 2'x2' (3.6 km)
- linear interpolation;
- max differences about ± 2.5 mm



Transformation grid BG07-19(con)



- First and Second Order State Levelling Network data;
- biharmonic spline function;
- differences btw fixed LSA and grid;

Transformation grid BGBalt-19(con)



- grid size 88 x 187 = 16456
- grid resolution 2'x2' (3.6 km)
- linear interpolation;
- max differences less than ± 1.0 mm



Results interpretation and analysis:

- Transformation grid validation: charts and statistics
- ✓ using independent data;
- ✓ using two methods: comparison at discrete points and comparison between two grids;
- ✓ results for both fixed and constrained solutions;
- ✓ validation of both BG/BGS2005 to EVRF19 and BG/Baltic82 to EVRF19 height transformations;
- ✓ results in form of statistics, charts, figures and histograms.



No	BMs in line	differences interpolated BG - BKG grid, [mm]					
line	(count)	mean	min	max	range	std	
1	251	2.0	0.1	3.5	3.4	1.0	
2	42	1.4	0.3	2.5	2.2	0.6	
3	71	1.3	-0.4	3.4	3.8	1.2	
4	30	-1.7	-3.3		differences (fir) [mm]	
5	26	0.7	0.2		ujjerences (
6	87	-0.4	-3.2	Mea	n	-2	2.5
7	30	0.3	-1.6	Star	dard Error	(0.1
8	56	-0.6	-3.0	Med	lian	-3	3.3
9	36	-1.3	-3.8	Mod	le Idard Daviatio	#N/A	5 3
10	41	-5.7	-9.3	Stat	inle Variance	23	5.2 5.4
11	92	-1.6	-4.8	Kur	tosis	-(0.1
12	114	1.3	-0.7	Ske	wness	(0.5
13	86	-2.0	-5.4	Ran	ge	83	3.6
14	106	-1.5	-8.3	Min	imum	-3	1.9
15	83	0.0	-6.2		limum	26250	1.7 6.7
				Cou	nt	106	5.7 53
				Con	fidence Level		
				(95.	0%)	0.2	91









Results interpretation and analysis – examples:

- Validation method: an interpolation of values from the transformation grid at discrete points and a comparison with the relevant differences in the heights
- Example: interpolated values from the BKG07-19 and the differences in the heights between BG(BGS2005) and EVRF2019, both obtained at the FO benchmarks (i.e. at discrete points);
- expected transformation differences are about 0.4 mm on average (range of ± 1 cm) with an average std of about 1.7 mm;
- In 56% of the levelling lines using either BKG07-19 or BG07-19, the expected range of the interpolated realisation differences is less than 5 mm;
- The std (of the interpolated transformation difference values) is higher than 1.5 cm for 49% of the lines and higher than 4.5 cm (or 3x1.5 cm) for 2% of the lines.

No	BMs in line	differences interpolated BG - BKG grid, [mm]				d, [mm]
line	(count)	mean	min	max	range	std
1	251	2.0	0.1	3.5	3.4	1.0
2	42	14	03	25	22	0.6



interpolated from BG07-19 and BKG07-19; difference



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VALIDATION OF TRANSFORMATION GRIDS FOR THE TERRITORY OF BULGARIA

Results interpretation and analysis – examples:

- Validation method: a comparison between two transformation grids
- Example: a comparison of the BG07-19 and the BKG07-19 transformation grid surfaces;
- For half of the Bulgarian territory using either the BKG07-19 or the BG07-19 (based on the fixed solution) would yield results that differ no more than 2 mm;
- However differences may reach values of 1 cm and more at particular locations;
- Similar patterns for the results based on the constrained solution;
- For the most of the Bulgarian territory the average difference in using BKG07-19 instead of the BG07-19 transformation grid is closer to 2 cm with a range of almost 6 cm.





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Results interpretation and analysis – examples:

- Validation method: an interpolation of values from the transformation grid at discrete points and a comparison with the relevant differences in the heights
- Example: interpolated values from the BKGBalt-EVRF19 and the differences in the heights between BG(Baltic82) and EVRF2019, both obtained at the FO benchmarks (i.e. at discrete points);
- Expected transformation differences are about 1.5 mm on average (range of ± 3.5 cm) with an average std of about 3 mm;
- In only 29% of the levelling lines using either BKGBalt-EVRF19 or BGBalt-19, the expected range of the interpolated realisation differences is less than 5 mm;
- The std (of the interpolated transformation difference values) is higher than 1.5 cm for 45% of the lines and higher than 4.5 cm for 24% of the lines.



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VALIDATION OF TRANSFORMATION GRIDS FOR THE TERRITORY OF BULGARIA

Mean

Mode

Range

Sum

Count

Results interpretation and analysis – examples:

- Validation method: a comparison between two grid surfaces
- Example: a comparison of the BGBalt-19 (based on the fixed solution) and the BKGBalt-EVRF19 transformation grid surfaces;
- For the most of the Bulgarian territory using either the BKGBalt-EVRF19 or the BGBalt-19 would yield results that differ insignificantly;
- Basic statistics of BG(Baltic82) to EVRF2019 transformation differences based on the BGBalt-19 (fix/con) and BKGBalt-EVRF19 transformation grids

statistic	BGBalt-19 (fix)	BGBalt-19 (con)
min	-0.0374 m	-0.0450 m
max	0.0363 m	0.0490 m
mean	8.4e-04 m	-0.0042 m
std	0.0063 m	0.0129 m





Results interpretation and analysis – conclusion:

BKG transformation grids

- LSA results (fixed solutions)
- BG transformation grids; results in [mm]

		BG07-19	BGBalt-19
	Mean	-2.5	-232.2
	Standard Error	0.1	0.2
[m]	Median	-3.3	-234.8
000	Standard Deviation	15.3	19.3
010 006	Sample Variance	235.4	372.3
002	Kurtosis	-0.1	-0.5
002	Skewness	0.5	0.4
	Range	83.6	100.6
58	Minimum	-31.9	-274.3
	Maximum	51.7	-173.7
	Sum	-26256.7	-2473804.7
	Count	10653	10653

transform.	BG(BGS2005) to EVRF2019	BG(Baltic82) to EVRF2019
abbr. as	BKG07-19	BKGBalt- EVRF19
identical BMs	59	58
min height difference	-0.051 m	+0.167 m
max height difference	+0.034 m	+0.277 m
mean height difference	-0.002 m	+0.228 m
standard deviation	±0.018 m	±0.024 m

	BGS2005	-EVRF19	BG(Balt82)- EVRF19		
	δΔΗ [m]	σΔΗ [m]	δΔΗ [m]	σΔΗ [m]	
min	-0.051	0.000	0.187	0.000	
max	0.034	0.011	0.265	0.010	
ave	0.003	0.006	0.231	0.006	
std	0.015	0.002	0.017	0.002	
identical BMs: 59					
identical BMs: 58					



Conclusion and next steps

- Based on the achieved results, the grids supplying transformation differences between EVRF2019 (ZT) and two height systems used in Bulgaria: BGS2005 (ZT) and Baltic height system (MT) are successfully validated.
- The validation process yield some interesting intermediate results.
- The transformation grids provided by BKG were successfully validated.
- The EVRF19-to-BGS2005 transformation and the EVRF19-to-Baltic82 transformation over the territory of Bulgaria are expected to have a mean value of about 2.5 mm and about 232 mm, respectively.
- The validation of the remaining two transformation grids is on-going.
- The computed transformation grids based on more than 10 800 BMs from the Bulgarian SLN (grid resolution: 2'x2') will be provided to the BKG and uploaded to the CRSeu website (<u>http://www.crs-geo.eu/</u>).





Thank you for your attention!



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