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Performances of Modernized GPS and Galileo in Relative Positioning with weighted ionosphere Delays

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Presentation Plan

INRODUCTION

QUANTIFYING THE SPATIAL CORRELATION

OBSERVATIONS EQUATION FOR SINGLE EPOCH

Stochastic model

Ambiguity resolution (review of LAMBDA method)

Results and analysis

The topic developed in this paper tries by means of a simulation to perform a comparative study between the performances of two multi-frequency GNSS (Global Navigation Satellite System)

which are not yet in full capability:

- **Modernized GPS**

  and
- **Galileo.**

Performance include:

Time to fix ambiguities → **High success rate**

Precision of coordinates → **static relative positioning**

Case of long baselines (>100 Km):

Estimation of atmospheric delay makes the observation model weak

Considering the double-differenced (DD):

Atmospheric delays as quantities of stochastic nature, the number of unknowns in this case decreases and leads to the reduction of time of fixing ambiguities

<u>To reach our purpose:</u>

Focus on the stochastic model → takes into account the following effects:

- **Mathematical correlation**
- **Spatial correlation (depends on baseline length)**
- **Dependency of noise with satellite elevation angle.**

Quantifying the spatial correlation

Spatially correlated errors are specifically the followings:

- **Orbital errors**

  and

- **Atmospheric delays (Ionospheric and tropospheric delays).**

Approximately* (DD) orbital errors ($\sigma_{DD\delta\rho}$) with respect to the length of baseline b for GPS GALILEO are:

$$\sigma_{DD\delta\rho}(cm) = 0{,}0071\ b(Km) \text{ ......GPS} \quad (1)$$

$$\sigma_{\mathrm{DD}\delta\rho}(cm) = 0{,}0061\ b(Km) \text{ ......GALLILEO} \quad (2)$$

*Establishment of these formulas is based on Zero-Differenced (ZD or un-differenced) orbit error $\sigma_{\delta\rho}$=1m. In fact, this value corresponds to the broadcast precision of GPS constellation published by IGS (International GNSS Service)

Assumptions

Formulas (1) and (2) denote that for DD and for GPS:

Orbital error standard deviation amounts is about 1.5 mm for 20 Km baseline

Standard deviation of ZD orbital errors reaches about 1 m

Due to:

Spatial correlation among the ZD orbital errors.

We try to estimate approximately the amount of this correlation.

$$DD\delta\rho^{s} = \delta\rho_2^{s} - \delta\rho_1^{s} - \delta\rho_2^{r} + \delta\rho_1^{r} \quad (3)$$

$$\sigma_{DD\delta\rho}^2 = var(DD\delta\rho^{s}) = var(\delta\rho_2^{s}) + var(\delta\rho_1^{s}) + var(\delta\rho_2^{r}) + var(\delta\rho_1^{r}) + 2[-cov(\delta\rho_2^{s},\delta\rho_1^{s}) - cov(\delta\rho_2^{s},\delta\rho_2^{r}) + cov(\delta\rho_2^{s},\delta\rho_1^{r}) + cov(\delta\rho_1^{s},\delta\rho_2^{r}) - cov(\delta\rho_1^{s},\delta\rho_1^{r}) - cov(\delta\rho_2^{r},\delta\rho_1^{r})] \quad (4)$$

We make these approximations

-*The variances for ZD orbital error are assumed equal*:

$$var(\delta\rho_2^s) = var(\delta\rho_1^s) = var(\delta\rho_2^r) = var(\delta\rho_1^r) = \sigma_{\delta\rho}^2 \quad (5)$$

-The covariances between orbital errors belonging to different satellites is assumed null, this means

$$cov(\delta\rho_2^s, \delta\rho_2^r) = cov(\delta\rho_2^s, \delta\rho_1^r) = cov(\delta\rho_1^s, \delta\rho_2^r) = cov(\delta\rho_1^s, \delta\rho_1^r) = 0 \quad (6)$$

Also, we consider that

$$cov(\delta\rho_2^s, \delta\rho_1^s) \approx cov(\delta\rho_2^r, \delta\rho_1^r) = \sigma_{(\delta\rho_2, \delta\rho_1)} \quad (7)$$

Equation (7) is so called spatial correlation model

We consider the covariance $\sigma_{(\delta\rho_2,\delta\rho_1)}$ constant whatever the satellite s and r

After making these approximations, the variance of DD error becomes then:

$$\sigma^2_{\mathrm{DD}\delta\rho} = 4\left(\sigma^2_{\delta\rho} - \sigma_{(\delta\rho_2,\delta\rho_1)}\right) \quad (8)$$

from which we can conclude that the covariance $\sigma_{(\delta\rho_2,\delta\rho_1)}$ is positive

Thus the covariance between $\delta\rho_2$ and $\delta\rho_1$ can be determined by:

$$\sigma_{(\delta\rho_2,\delta\rho_1)} = \sigma^2_{\delta\rho} - \frac{\sigma^2_{DD\delta\rho}}{4} \quad (9)$$

(Spatial correlation model)

This model:

Knowing → *Variance of the ZD and DD error,*

Estimate → *Approximately the value of covariance*

Or

Derive → *The value of covariance* ***that exists between individual errors.***

By the same model we can estimate approximately the spatial correlation values relative to the tropospheric and ionospheric delays (using values provided in table 1) in order to build the stochastic model.

Equation (9), we adopt a simple spatial correlation formulation for the stochastic model. We assume also that all the DD errors which are of the same type having the same standard deviation. The temporal correlation and the elevation angle dependency have not been considered in this simple formulation.

The following table provides the values of standard deviations of DD tropospheric and ionosphere delays for different baselines.

		Baseline		
		Short (0-20 Km)	**Medium (20-100 Km)**	**Long (100-500 Km)**
Errors in DD	σ_{DDT} **(0,2–0,5 ppm)**	**<1 cm**	**~2.5 cm**	**<20 cm**
	σ_{DDI} **1^{st} order on L1**	**<10 cm**	**<40 cm**	**<100 cm**
	σ_{DDI} **2^{nd} order on L1**	**<0.5 cm**	**<1 cm**	**<2 cm**

Tab 1*: Standard deviations of atmospheric delays in DD for short, medium and long baseline [Feng 2008].*

Observations equation for single epoch

Before describing the observations equation, we start by noting that the ionospheric delay can be approximated by these models respectively [Datta-Barua et al 2008], [Alizadeh 2013]

$$I_{p,k} = \alpha_k^2 I^{(1)} + \alpha_k^3 I^{(2)}, \qquad k = 1,2,5 \qquad (10)$$

$$I_{\Phi,k} = -\alpha_k^2 I^{(1)} - \frac{1}{2}\alpha_k^3 I^{(2)} \;, \; k = 1,2,5 \qquad (11)$$

with: $\alpha_k = \frac{f_1}{f_k}$ and f_k the frequency k. $I_{p,k}$, $I_{\Phi,k}$ vector of DD ionospheric delays on frequency k, approximated to the second order on code and phase respectively. $I^{(1)}, I^{(2)}$ vector containing respectively the first and second order terms of the ionospheric delay on L1.

The observations equation at a single epoch i for triple-frequency GNSS, can be written as

$$\begin{bmatrix} p_i \\ \Phi_i \end{bmatrix} = \begin{bmatrix} -(e_3 \otimes F_i) & 0 \\ -(e_3 \otimes F_i) & (\Lambda \otimes I_{m-1}) \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \begin{bmatrix} \varepsilon_{p,i} \\ \varepsilon_{\Phi,\mathrm{i}} \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix}\varepsilon_{p,i}\\ \varepsilon_{\Phi,i}\end{bmatrix} = \begin{bmatrix}(e_3 \otimes I_{m-1})\\ (e_3 \otimes I_{m-1})\end{bmatrix} T + \begin{bmatrix}(f \otimes I_{m-1})\\ -(f \otimes I_{m-1})\end{bmatrix} I^{(1)} + \frac{1}{2}\begin{bmatrix}(h \otimes I_{m-1})\\ -(h \otimes I_{m-1})\end{bmatrix} I^{(2)} + \begin{bmatrix}\nu_{i,p}\\ \nu_{i,\Phi}\end{bmatrix} \quad (13)$$

T denotes respectively: DD orbital error, tropospheric delay. $\nu_{i,p}$, $\nu_{i,\Phi}$ noise (including multipath) on code and phase respectively.

$$f = [1 \quad \alpha_2^2 \quad \alpha_5^2]^T$$

$$h = [1 \quad \alpha_2^3 \quad \alpha_5^3]^T$$

α_2, α_5 as defined previously.

Stochastic model

In the stochastic model described below by equation (15), the following assumptions are made

- *The mapping function for code noise is assumed identical to carrier phase noise. The mapping function is chosen to be exponential that takes the form*

$$m(e) = p + qe^{-re} \quad (14)$$

with e : elevation angle. p, q, r constants.

- *In addition, we consider no correlation between different types of errors since they come from independent physical processes. For example, there is no need to consider that the ionosphere and troposphere effects are correlated.*
- *Model given by (9) is implemented to account for spatial correlations between individual errors,*
- *The temporal correlation is neglected in the stochastic model to avoid inverting a large and fully populated variance covariance matrix of measurements,*
- *No cross-correlation between signals, since we assume that the signals in either modernized GPS or Galileo will be likely non-correlated.*

Applying the variance propagation law, then the variance covariance matrix of observations at epoch i takes the form

$$Q_{\varepsilon_i} = \begin{bmatrix} Q_{p,i} & Q_{p\Phi,i} \\ Q_{p\Phi,i}^T & Q_{\Phi,i} \end{bmatrix} \quad (15)$$

where: $\varepsilon_i = \begin{bmatrix} \varepsilon_{p,i} \\ \varepsilon_{\Phi,i} \end{bmatrix}$ ***denotes the vector of unmodelled errors. The blocks*** $Q_{p,i}$, $Q_{p\Phi,i}$, $Q_{\Phi,i}$ ***are expressed as:***

$$Q_{p,i} = \frac{1}{2}\left(S_p D_{DD} S_p^T\right) \otimes T_{m-1} + D_p \otimes \left(\Omega M_i^2 \Omega^T\right) \quad (16)$$

$$Q_{p\Phi,i} = \frac{1}{2}\left(S_p D_{DD} S_\Phi^T\right) \otimes T_{m-1} \quad (17)$$

$$Q_{\Phi,i} = \frac{1}{2}\left(S_\Phi D_{DD} S_\Phi^T\right) \otimes T_{m-1} + D_\Phi \otimes \left(\Omega M_i^2 \Omega^T\right) \quad (18)$$

with: $S_p = [e_3 \quad e_3 \quad f \quad h]$, $S_\Phi = \left[e_3 \quad e_3 \quad -f \quad -\frac{1}{2}h\right]$

We note that the ZD variances of spatially correlated errors are cancelled out in the development of the stochastic model.

Matrix D_{DD} uses the values given by the formulas and table 1 above. The table 2 below is used to construct the matrices D_p and D_{Φ}.

	GPS			Galileo		
Frequency	L1	L2	L5	E2L1E1	E5b	E5a
σ code (cm)	20	20	15	20	20	15
σ phase (mm)	2	2	1.5	2	2	1.5

Tab 2 : Standard deviations for ZD noise at zenith for high-end receiver [Nardo 2015].

Ambiguity resolution (review of LAMBDA method)

After obtaining the float solution i.e. ambiguities and other estimates by least squares method, the ambiguities need to be fixed on the correct integers: this step is what we call *ambiguity resolution;* an advantage of this technique is it allows an improvement of the precision of the other estimates of interest. In this work, the ambiguity resolution method LAMBDA (Least-squares Ambiguity Decorrelation Adjustment) known among other methods by its high success probability of resolution is implemented, see the advantages of LAMBDA method in [Teunissen et al. 2002]. The LAMBDA method was first introduced in 1993 by P. J. G Teunissen in his paper [Teunissen 1993] and discussed in detail in [Teunissen 1995]. The LAMBDA method is performed into two steps: the first step is, the reduction of correlation of ambiguities by Z-transformation i.e. find the matrix Z that minimizes the product of diagonal elements of the transformed covariance matrix $Q_{\hat{z}}$

$$Q_{\hat{z}} = Z^T Q_{\hat{a}} Z \quad (19)$$

Ambiguity search space defined by

$$(\hat{a} - a)^T Q_{\hat{a}}^{-1} (\hat{a} - a) \leq \chi^2 \ (20)$$

a : vector of unknown integer ambiguities. χ^2 : positive number sufficiently small.

Once the ambiguities are fixed (denoted $\breve{a}$), the final or fixed solution $\breve{b}$ reads

$$\breve{b} = \hat{b} - Q_{\hat{b},\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \breve{a}) \ (21)$$

$$Q_{\breve{b}} = Q_{\hat{b}} - Q_{\hat{b},\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{b},\hat{a}}^{T} \ (22)$$

Measure the success of ambiguity resolution → ***Success rate*** →

Bootstrapping (Lower bound of integer least squares success rate, difficult to compute)

The bootstrapping success rate is given by

$$P_s = \prod_{i=1}^{n} \left(2\Phi\left(\frac{1}{2\sigma_i}\right) - 1 \right) \ (23)$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt \ (24)$$

where σ_i : the standard deviation of conditional ambiguity i. n : number of ambiguities

Results

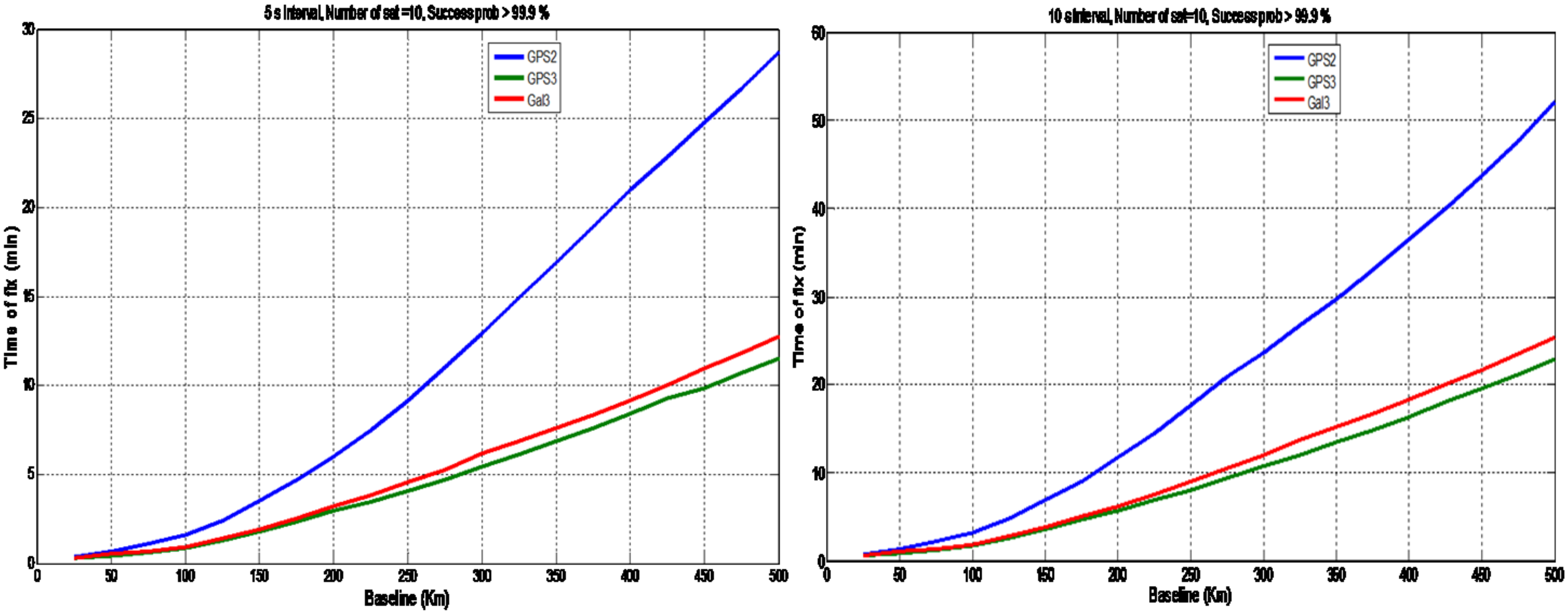

Fig.1 : *Time of ambiguity fixation with baseline variation for 5s left and 10 s right*

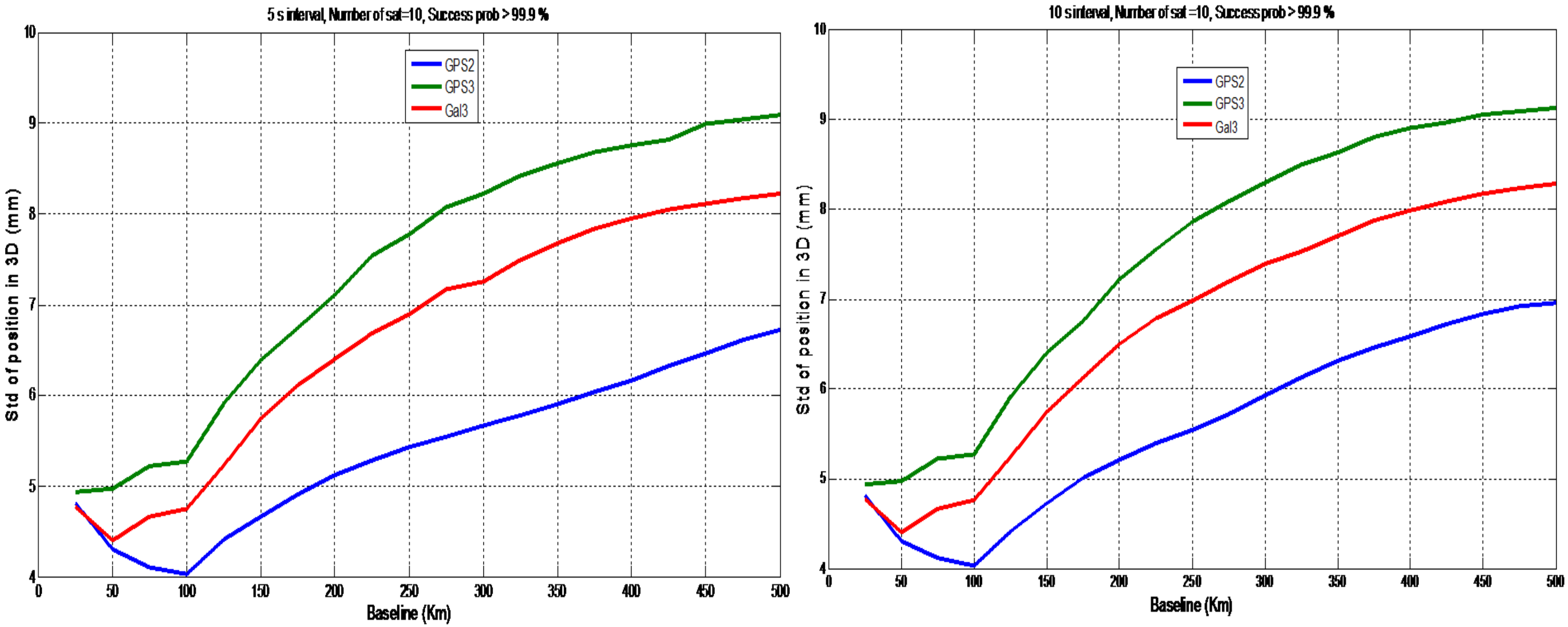

Fig 2: *Variation of time of fix of ambiguities with baseline length for triple-frequency GPS (top) and triple-frequency Galileo (bottom). Success probability: 99.9 %. 10 satellites continuously tracked.*

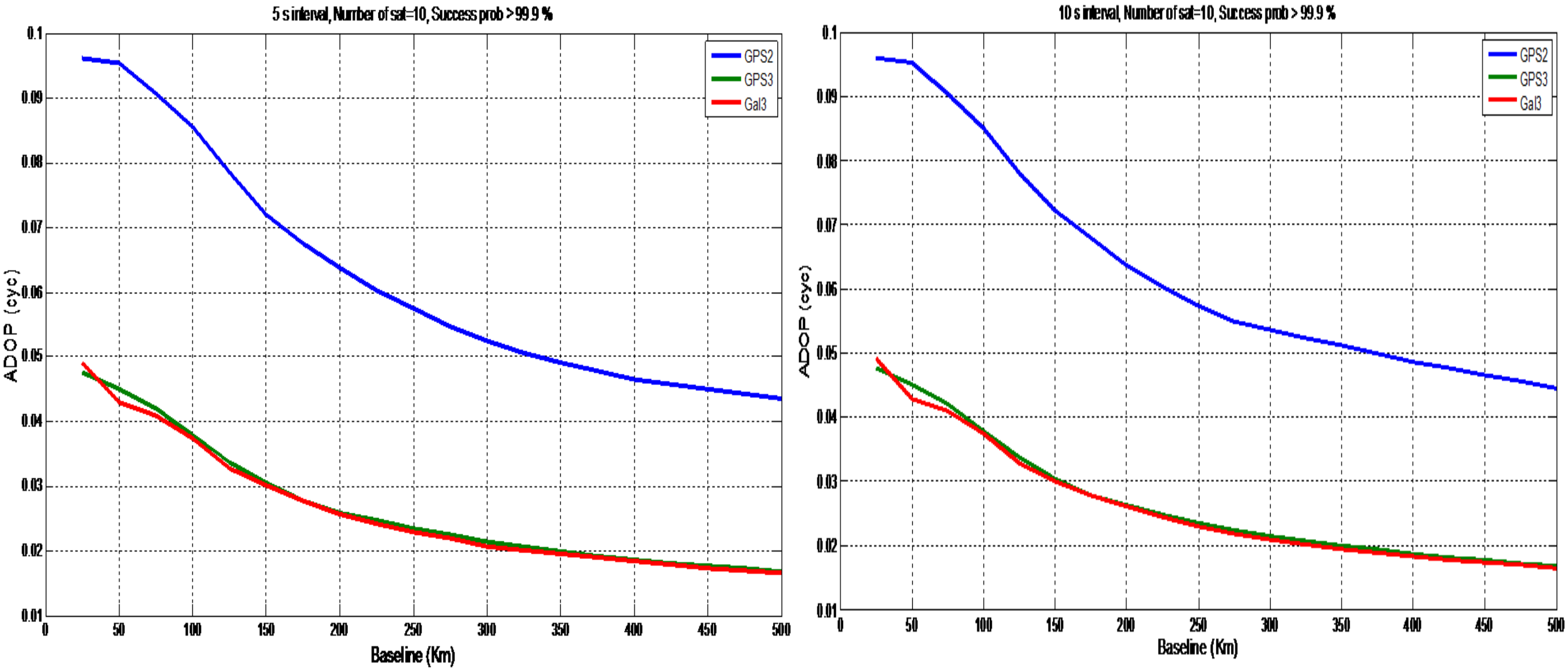

***Fig 3* :** Variation of the DOP (cycles) relevant of baseline length for double and triple-frequency GPS , triple-frequency Galileo for 5 s and 10 s interval. Success probability: 99.9 %. 10 satellites continuously tracked**.**

Analysis

- ***The GNSS performance that we focus on in this study, is essentially the time required to fix ambiguities for different baseline lengths***
- ***Integer ambiguities in the simulation are fixed by a success probability that exceeds 99.9%***
- ***GNSS observations are accumulated until a success probability of 99.9% is reached for each baseline***

In addition to the mentioned performance that we focus on, we investigate the effect of varying the sampling time on the results.

- ***Figure 1 below shows the plot of the time required to fix ambiguities against the baseline length.***
- ***We can remark from figure 1 that there is no significant difference in behavior between the two triple-frequency GNSS in terms of time required to fix ambiguities***
- ***When observations are sampled at 5 s interval with a baseline of 500 km, the time to fix ambiguities with a success probability of 99.9 % reaches about 13 min, whereas when choosing a sampling time of 10 s, the time to fix ambiguities becomes 25 min***
- ***The two triple-frequency GNSS behave almost by the same manner i.e. having fairly the same time of fix for the same baseline and same sampling time***
- ***Furthermore, it is clear from the figure 1 that the time of fix of ambiguities increases as the sample time increases. Although the significant observation time (25 min) allows to change the satellite geometry which is beneficial in decorrelating ambiguities, it appears not sufficient here to achieve this.***

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Thank you شكرا Merci

Gracias Dank je

СПАСИБО Grazie Danke