The way for an optimal reference frame in Greece

S. Bitharis(1), D. Ampatzidis(2), C. Pikridas(1), A. Fotiou(1) and D. Rossikopoulos(1)

(1)Aristotle University of Thessaloniki, School of Rural & Surveying Eng. Lab. of Geodetic Methods and Satellite Applications, Greece

(2)German Center for Geosciences (GFZ), Department 1.2 Geomonitoring and Gravity Field, Wessling, Germany

Abstract

In the present study, we describe the mathematical models for the realization of an Optimal Reference Frame (ORF) in Greece. The Hellenic area is one of the most deforming areas in Europe, with inhomogeneous horizontal velocity field as verified by numerous studies. The main idea based on the minimization of a long-term reference frame's total kinetic energy, in order to provide a more stable reference frame. The proposed strategy ensures that the geodetic velocities are reduced more than 60 percent in two different approaches for the International Terrestrial Reference Frame (ITRF) and the European Terrestrial Reference System 1989 (ETRS89), as resulted from our analysis in a well-distributed GNSS Network. In particular, the geodetic velocities are derived by seven years data analysis in a network of 151 continuous Global Positioning System (cGPS). The advantages of this coherent strategy is that could be applied both in two and three-dimensions, also provide a directly transformation between global/regional Terrestrial Reference Frames (TRFs) and ORF and vice versa. Subsequently, the minimum velocity leading to a more stable TRF which recommended for national cartographic and geodetic purposes.

The mathematical proof of ORF

The general idea behind the ORF (Ampatzidis 2011; Bitharis et al. 2016), based on the Helmert type velocity similarity transformation between two different reference frames ***A*, *B*** as descripted in the following equation (Eq. 1)

$$\mathbf{v}_i^B = \mathbf{v}_i^A + \dot{\mathbf{T}} + \dot{D}\mathbf{x}_i + \dot{\mathbf{R}}\mathbf{x}_i \quad (1)$$

Where, $\mathbf{v}_i$ is the 3D velocity vector and the $\dot{\mathbf{T}} = \begin{bmatrix} \dot{t}_x & \dot{t}_y & \dot{t}_z \end{bmatrix}^T$ includes the translation rates, $\dot{D}$ is the scale rate and $\dot{\mathbf{R}}$ is the 3x3 anti-symmetric matrix that contains the orientation rates.

Alternative, the equation 1, could the expressed in more compact form:

$$\mathbf{v}_i^{ORF} = \mathbf{v}_i^{TRF} + \mathbf{E}_i\dot{\theta} \quad (2)$$

Where, $\dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{t}_x & \dot{t}_y & \dot{t}_z & \dot{D} & \dot{r}_x & \dot{r}_y & \dot{r}_z \end{bmatrix}^T$ contains the transformation rate parameters and $\mathbf{E}$ is the design matrix.

According to weighted least square method, the kinetic energy of the RF minimized by the following optimal criterion:

$$\phi = \mathbf{v}^{ORF\ T} \mathbf{P}\ \mathbf{v}^{ORF} = \min \quad (3)$$

Where ϕ is the minimal quantity and $\mathbf{P}$ the weight matrix.

The transformation rate parameters are estimated through the relation:

$$\frac{\partial\phi}{\partial\dot{\theta}} = 0 \Rightarrow \dot{\theta} = -\ \mathbf{E}^T\mathbf{P}\mathbf{E}^{\ -1}\ \mathbf{E}^T\mathbf{P}\mathbf{v}^{ORF} \quad (4)$$

Respectively, the $\mathbf{v}^{ORF}$ velocities of the ORF will be expressed as:

$$\mathbf{v}^{ORF} = \left(\mathbf{I} - \mathbf{E}\ \mathbf{E}^T\mathbf{P}\mathbf{E}^{\ -1}\ \mathbf{E}^T\mathbf{P}\right)\mathbf{v}^{TRF} \quad (5)$$

Remarks of the ORF

- In local-scale analysis should be estimate only the three rotation parameters, due to the high correlation with the shift-rate parameters. Hence, the **physical meaning** described by the estimation of a single set of Euler Pole Parameters (EPPs):

$$\dot{r}_x = \omega_{EP}\cos\varphi_{EP}\cos\lambda_{EP},\ \dot{r}_y = \omega_{EP}\cos\varphi_{EP}\sin\lambda_{EP},\ \dot{r}_z = \omega_{EP}\sin\varphi_{EP}$$

- The spatial connection between existing TRFs and the new optimal LRF carried out under the following condition: $\mathbf{x}_i^{ORF}\ t_0 \equiv \mathbf{x}_i^{TRF}\ t_0$ The 3-D position vector to any epoch t in the ORF is computed through the following expression:

$$\mathbf{x}_i^{ORF}(t) = \mathbf{x}_i^{TRF}(t_0) + (t - t_0)\mathbf{v}_i^{ORF}$$

$$\mathbf{x}_i^{ORF}(t) = \mathbf{x}_i^{ORF}(t) + (t - t_0)\mathbf{v}_i^{ORF}$$

$$= \underbrace{\mathbf{x}_i^{ORF}(t_0) + (t - t_0)\mathbf{v}_i^{TRF}}_{\mathbf{x}_i^{TRF}(t)} + (t - t_0)\mathbf{E}_i\dot{\boldsymbol{\theta}} \quad \text{(Take into account the Eq. 2)}$$

$$= \mathbf{x}_i^{TRF}(t) + (t - t_0)\mathbf{E}_i\dot{\boldsymbol{\theta}}$$

Case study/Numerical Results

The GPS data analysis was carried out with the GAMIT/GLOBK s/w (Herring et al. 2010) and the station coordinates and velocities estimated by a Kalman filtering sequential approach. The data span is seven years (2008 to 2014) of continuously 30-sec daily rate observations (Bitharis et al. 2015).

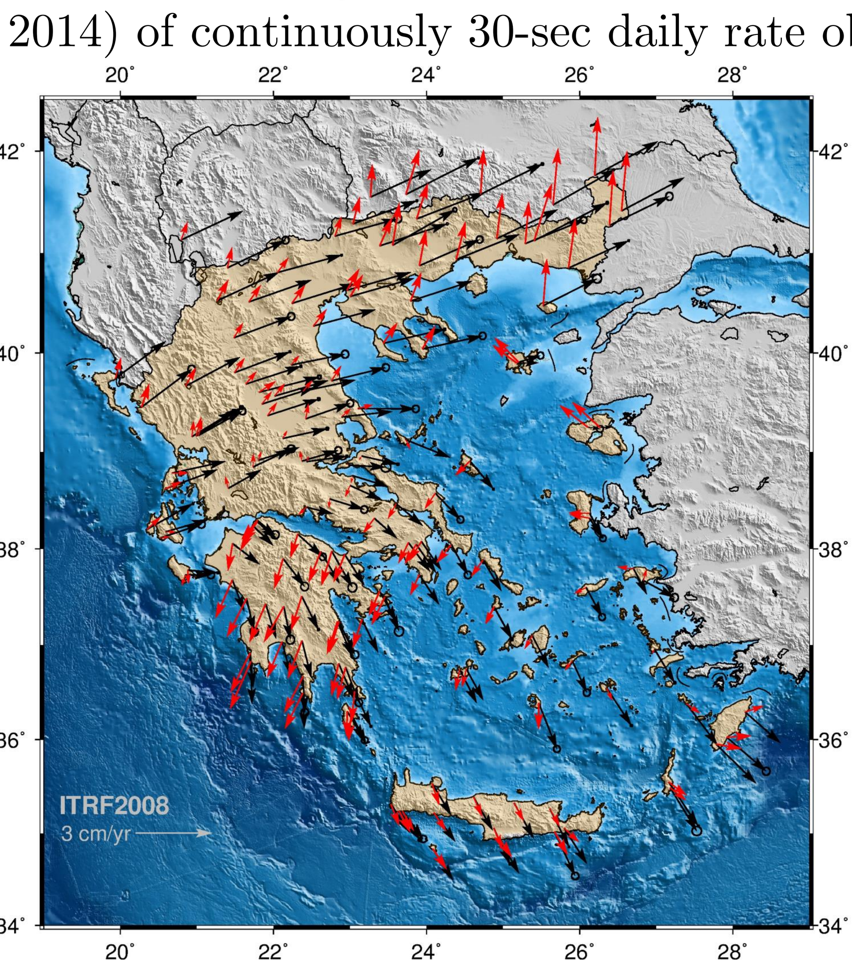

Fig. 1 Horizontal velocities at Greek area in the ITRF2008 (black) and the optimal RF (red)

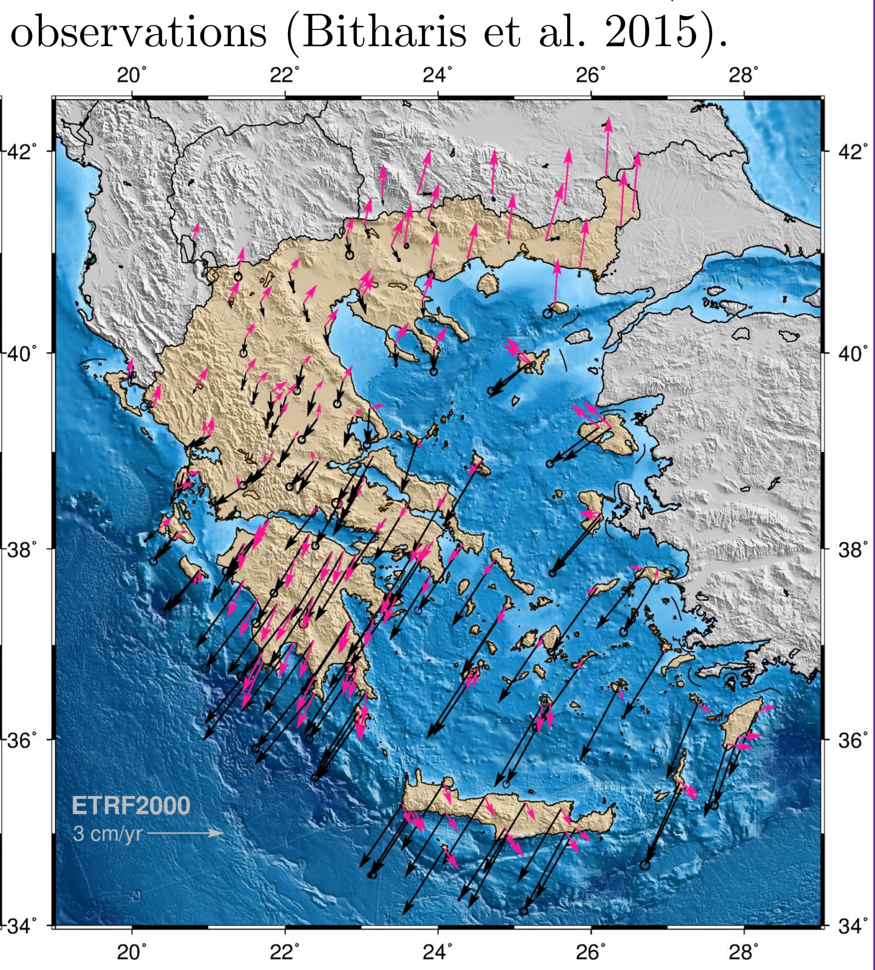

Fig. 2 Horizontal velocities at Greek area in the ETRF2000 (black) and the optimal RF (pink)

Reference frame	ITRF	ITRF (MKEC)	ETRF	ETRF (MKEC) ,*(units: mm/yr)*
Min	4.8	0.5	0.9	0.6
Max	32.2	23.6	38.9	23.8
Std	5.7	4.7	12	4.7
Bias	18	9.7	20.7	9.7
RMS	18.9	10.8	23.9	10.8
Median	15.5	8.8	26	8.8
Kinetic energy	53811.2	17681.7	86414.5	17493.3

We choose to use only the 2D horizontal geodetic velocities. According to the statistics we shown that the ETRF2000 implementation does not provide any significant advantage in compare with the ITRF2008 (Altamimi et al. 2011).

The average magnitude of the new geodetic velocities are better than 1 cm/yr in both scenarios. Hence, the proposed strategy led to a more stable RF in order to generate the cartographic materials in national/local coverage. In both approaches the Minimization of the Kinetic Energy Criterion (MKEC) gives reduction more than 60 percent as depicted in Fig 3.

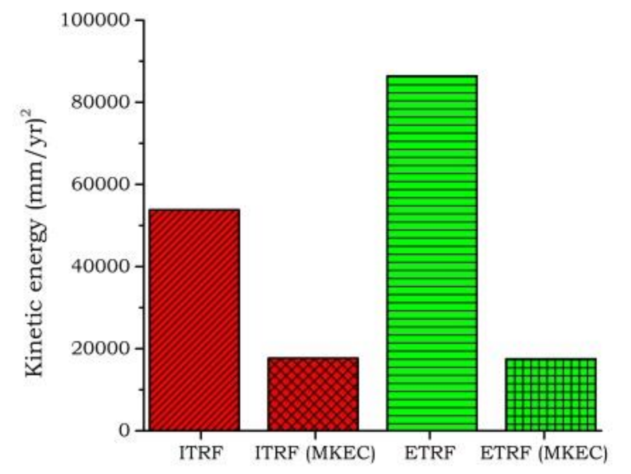

Fig. 3 Kinetic energy between initial and optimal reference frames (ITRF-ETRF)

Strategies	Advantages	Disadvantages
$ITRF_{YY}$	Represents the nature of the crust displacements, globally. The most models and products (orbits) are generated in ITRF. Is a multi-technique combination of high accurate geodetic techniques (GNSS, SLR, VLBI, DORIS).	Not recommended in areas with inhomogeneous velocities. A dynamic TRF which is not easily understand in order to generate the cartographic materials and intended for expert users.
ETRF2000	Directly transformation with $ITRF_{YY}$. Homogenous velocity field in central Europe. Recommended datum for regional mapping and surveying applications in Europe. Useful for geophysical purposes	Strong and inhomogeneous velocities field in south Greece. Always need an updated velocity field (e.g Greece) due to transform the measurements at each current epoch back to previous reference epoch (2005).
Local Euler Pole	In small areas gives good results and represents the local characteristics of the inhomogeneous velocity field.	The results are high correlated with microplates boundaries. The accuracy of the estimated EPPs is low in small areas.
Minimization of Kinetic Energy Criterion	The velocity is minimal and led to a more stable TRF. Directly transformation between global/regional TRFs and ORF. Could be applied both in 2D and 3D. Recommended for national cartographic and geodetic purposes.	The velocity field in the most cases remain inhomogeneous. Do not reflects the geodynamic behavior of the area and not recommended for geophysical studies.

References

Altamimi Z, Collilieux X, Métivier L (2011) ITRF2008: an improved solution of the international terrestrial reference frame. J Geod 85:457–473. doi: 10.1007/s00190-011-0444-4

Ampatzidis D (2011) Study for an optimal Geodetic Reference Frame realization in the Hellenic area. PhD Dissertation, Department of Geodesy and Surveying, Aristotle University of Thessaloniki (in Greek).

Bitharis S, Ampatzidis D, Pikridas C (2016) An optimal geodetic dynamic reference frame realization for Greece: Methodology and application. *Arab J Geosci (under review)*

Bitharis S, Fotiou A, Pikridas C, Rossikopoulos D (2015) A new crustal velocity field of Greece based on seven years (2008-2014) continuously operating GPS station data. In: IUGG 26th General Assembly. Prague, Czech Republic.

Herring T a, King RW, McClusky SC (2010) Documentation of the GAMIT GPS Analysis Software release 10.4. Dep Earth Planet Sci Massachusetts Inst Technol Cambridge, Massachusetts 1–171

Acknowledgments

We are grateful to the National Cadastre and Mapping Agency S.A of Greece, METRICA S.A., the National Observatory of Athens and the Dionysos Satellite Observatory of NTUA for providing GPS data used in this study

EUREF 2016 Symposium

Contact Person:
Stylianos I. Bitharis
e-mail: stylbith@gmail.com

San Sebastián – Spain, May 2016
SESSION 1 - SYSTEMS: ETRS89, EVRS