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Unconstrained NEQ with full rank in GNSS networks

Crucial problem or trivial nuisance for TRF applications?

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Introduction

- The original (unconstrained) NEQ that are formed in daily, weekly or multi-epoch processing of GNSS networks are often **invertible**; with no datum defect.

- Applying "minimal constraints" to such full-rank systems creates distortions in the final solution!

  Particularly, it affects:

  - the estimated network geometry
  - well-estimable frame parameters
  - other inferences of scientific interest (assessment of loading effects in GNSS height time-series)

Problem description

Unconstrained NEQ (full-rank)

Frame information (origin, orientation, scale)

Inner-geometry information

Data noise and other observation-related errors

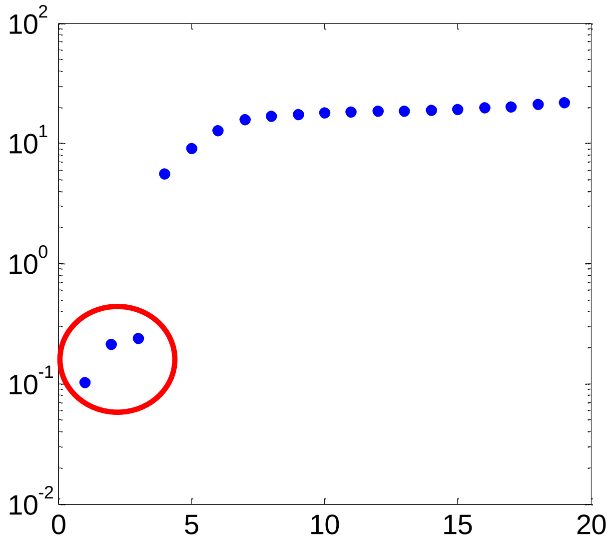
Additive "minimal" constraints (e.g. NNT)

→

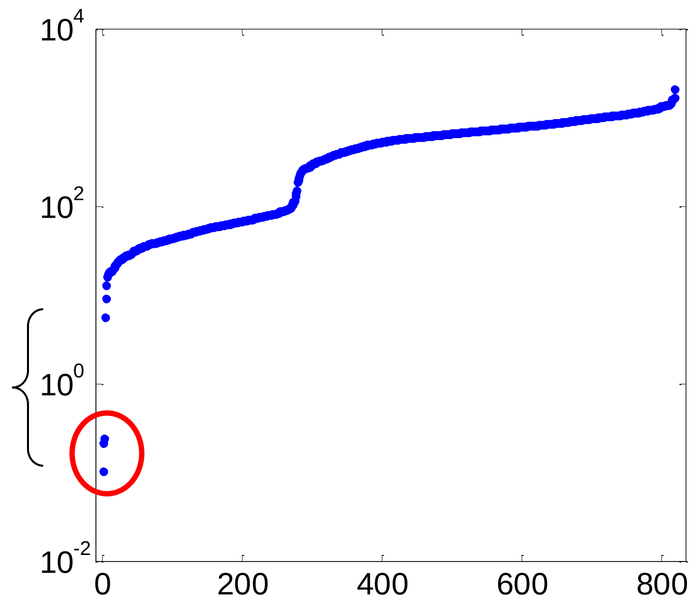
NEQ–MC "interference"

- This is not necessarily bad from an estimation viewpoint!
- Yet, it causes effects that may be unwanted in TRF applications.

Spectrum of weekly unconstrained normal matrix (EPN network)



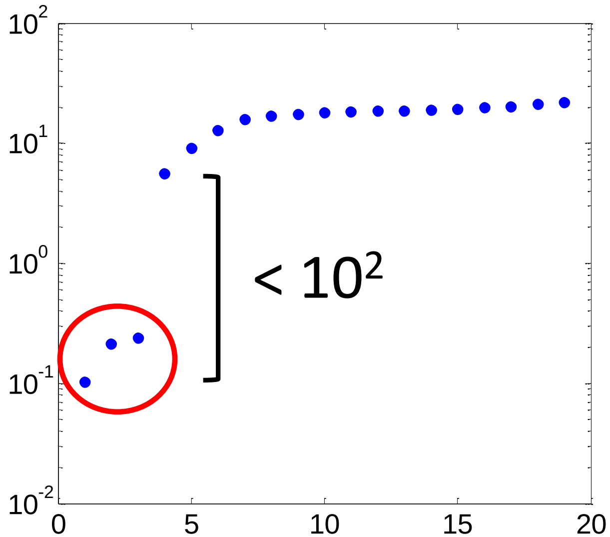
The first 20 eigenvalues



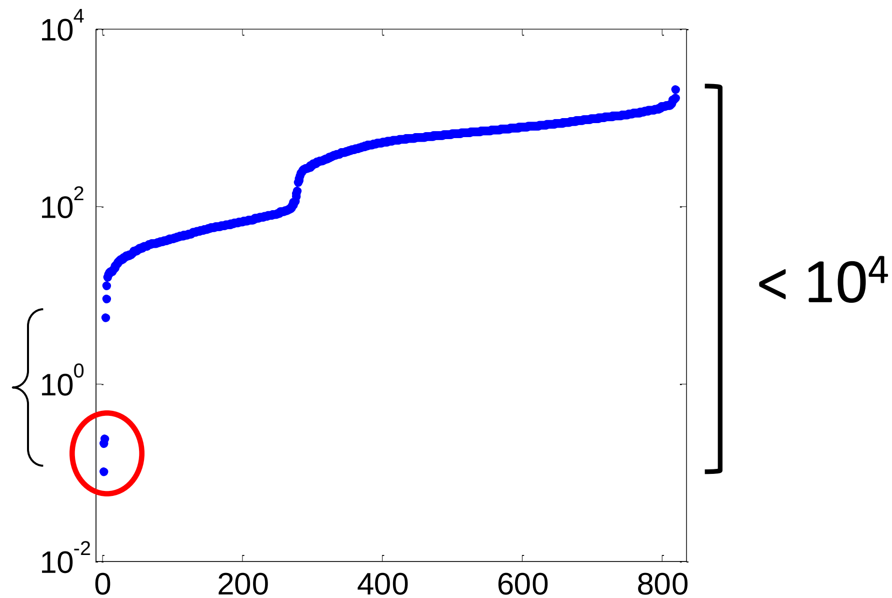
All eigenvalues

EUR18697.snx
(1-7/11/2015)

Spectrum of weekly unconstrained normal matrix (EPN network)



The first 20 eigenvalues



All eigenvalues

EUR18697.snx
(1-7/11/2015)

Well-conditioned invertible matrix !

Two key questions for the present study

- How significant is the distortion to "minimally constrained" EPN solutions due to absence of true rank defect in the original NEQ ?

- How can we convert unconstrained NEQ to truly singular systems with given rank defect for particular datum parameters, without altering their internal geometrical content ?

Detecting distortion due to absence of NEQ singularity

Compute and compare the following solutions from the original normal system $\mathbf{N}(\mathbf{X}-\mathbf{X}_o)=\mathbf{u}$

Free-net solution

$$\hat{\mathbf{X}} = \mathbf{X}_o + \mathbf{N}^{-1}\mathbf{u}$$

Constrained solution by a set of "minimal constraints" $\mathbf{H}(\mathbf{X}-\mathbf{X}_o)=\mathbf{c}$

$$\hat{\mathbf{X}} = \mathbf{X}_o + \left(\mathbf{N} + \mathbf{H}^T\mathbf{W}\mathbf{H}\right)^{-1}\left(\mathbf{u} + \mathbf{H}^T\mathbf{W}\mathbf{c}\right)$$

- The free-net solution always retains the full data information.
- Comparing the above solutions, after a Helmert transformation is applied between them, can reveal the distortion caused by the MCs.

Example

- Experiment with the original (unconstrained) NEQ from several EPN weekly combined solutions.

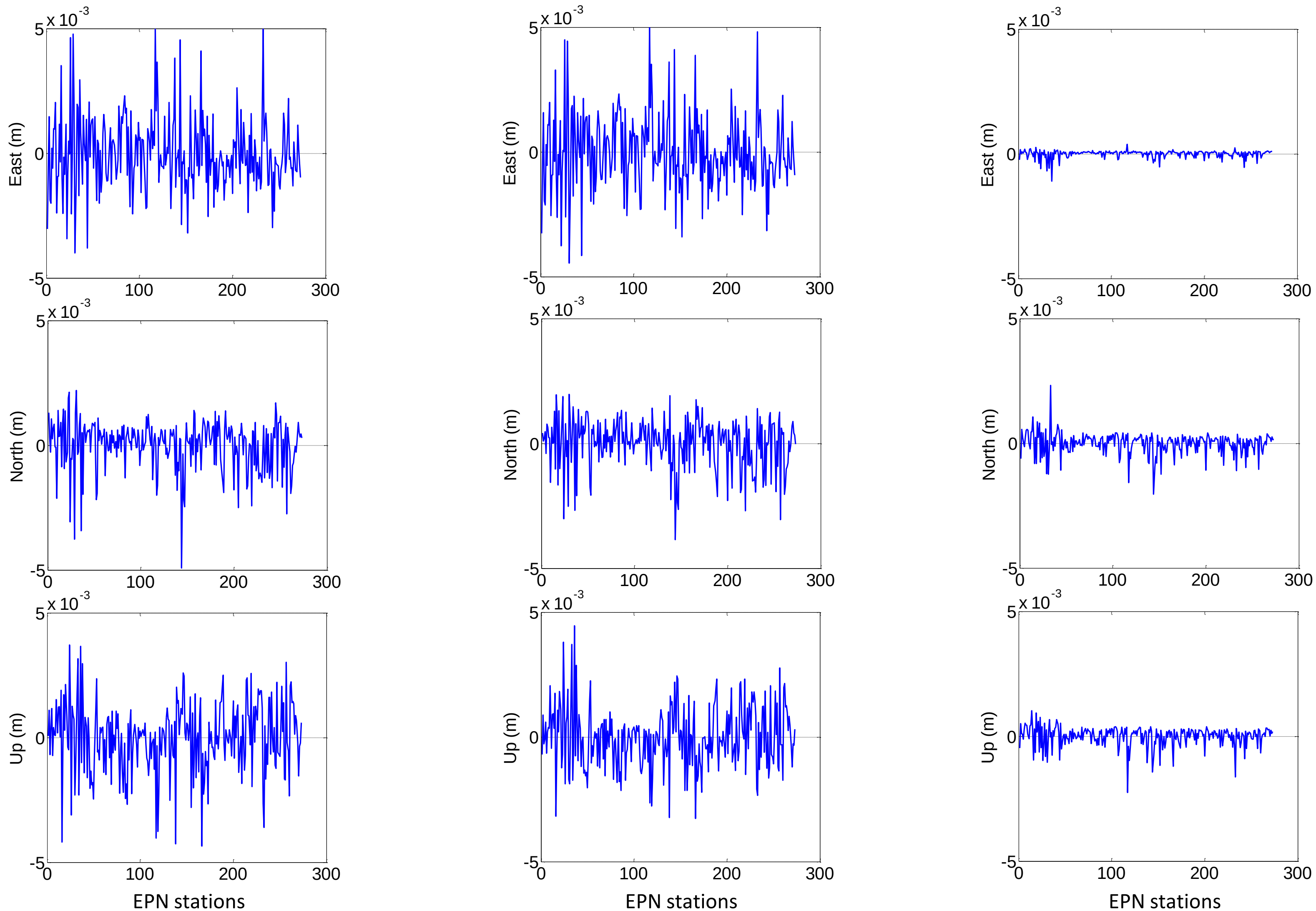
- For each week, determine the free-net solution and the NNT-constrained solution wrt. IGb08.
  - the weight matrix **W** is set to prior accuracy level of 10^{-5} m for frame origin fixation in each weekly solution

- Fit the above solutions by Helmert transformation, and assess their residuals and their transformation parameters for each week.

45 IGS reference stations for applying NNT wrt. IGb08

ANKR, BOR1, BRST, BUCU, GLSV, GRAS, GRAZ, HERS, HERT, HOFN, ISTA, JOZ2, JOZE, LROC, MAR6, MAS1, MATE, MDVJ, MEDI, METS, MORP, NICO, NOT1, NYA1, ONSA, PDEL, POLV, POTS, QAQ1, RABT, RAMO, RIGA, SCOR, SFER, SOFI, THU3, TLSE, TRO1, UZHL, VILL, WSRT, WTZR, YEBE, ZECK, ZIMM

Transformation residuals (week 1869)



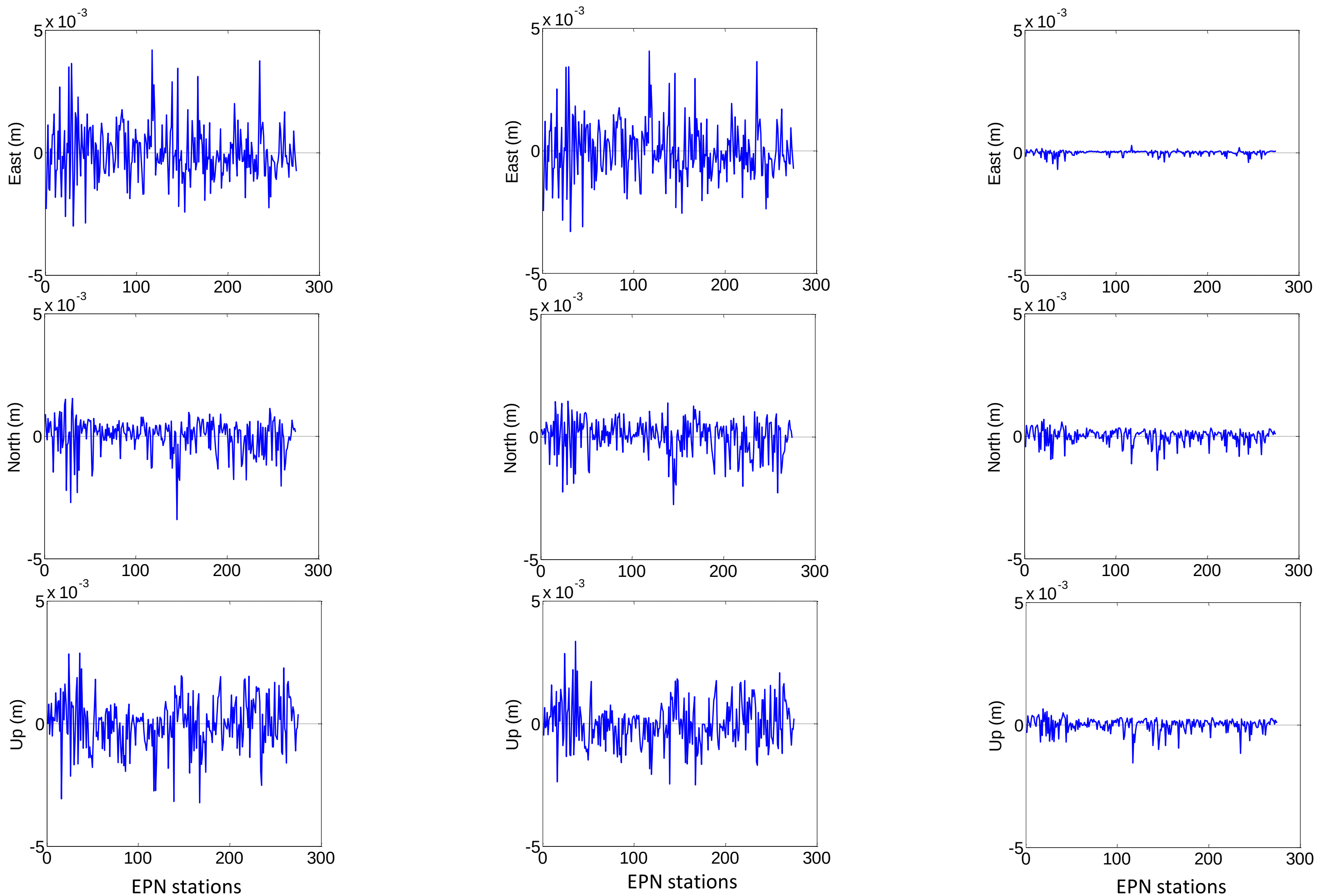
Shift-only model | Shift/roto model | Full-similarity model

Helmert transformation parameters btw free-net and NNT-constrained weekly EPN solutions (week 1869)

	Tx (cm)	Ty (cm)	Tz (cm)	Rx (mas)	Ry (mas)	Rz (mas)	Scale (ppb)
Shift-only	2.64	1.06	2.94				
Shift/roto	2.59	1.44	2.93	-0.10	-0.01	0.08	
Full-similarity	3.21	1.55	3.66	-0.10	-0.01	0.08	-1.55

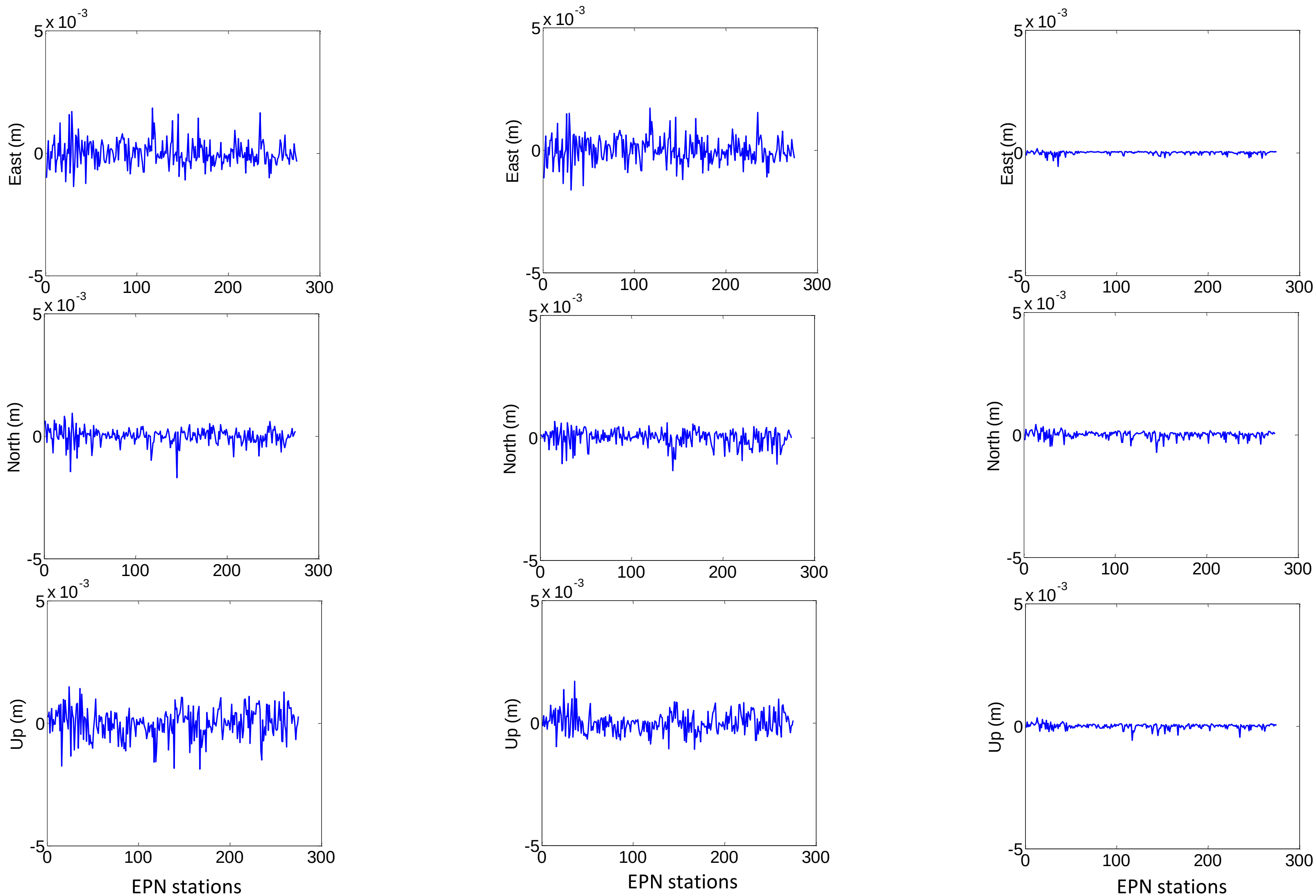
(*) It seems that the NNT constraints affect the scale and orientation of the weekly solution in IGb08!

Transformation residuals (week 1870)



Shift-only model   Shift/roto model   Full-similarity model

Transformation residuals (week 1871)

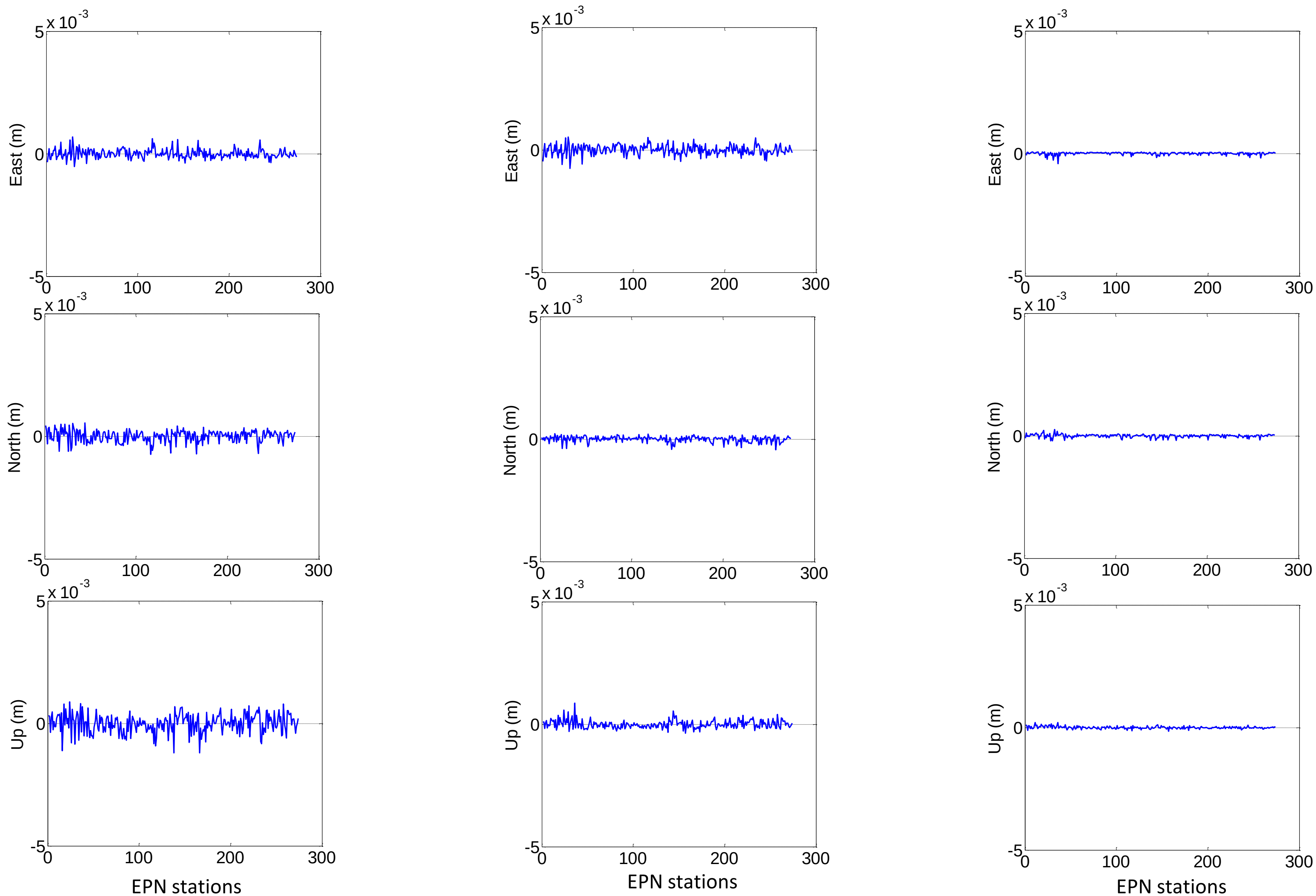


Shift-only model

Shift/roto model

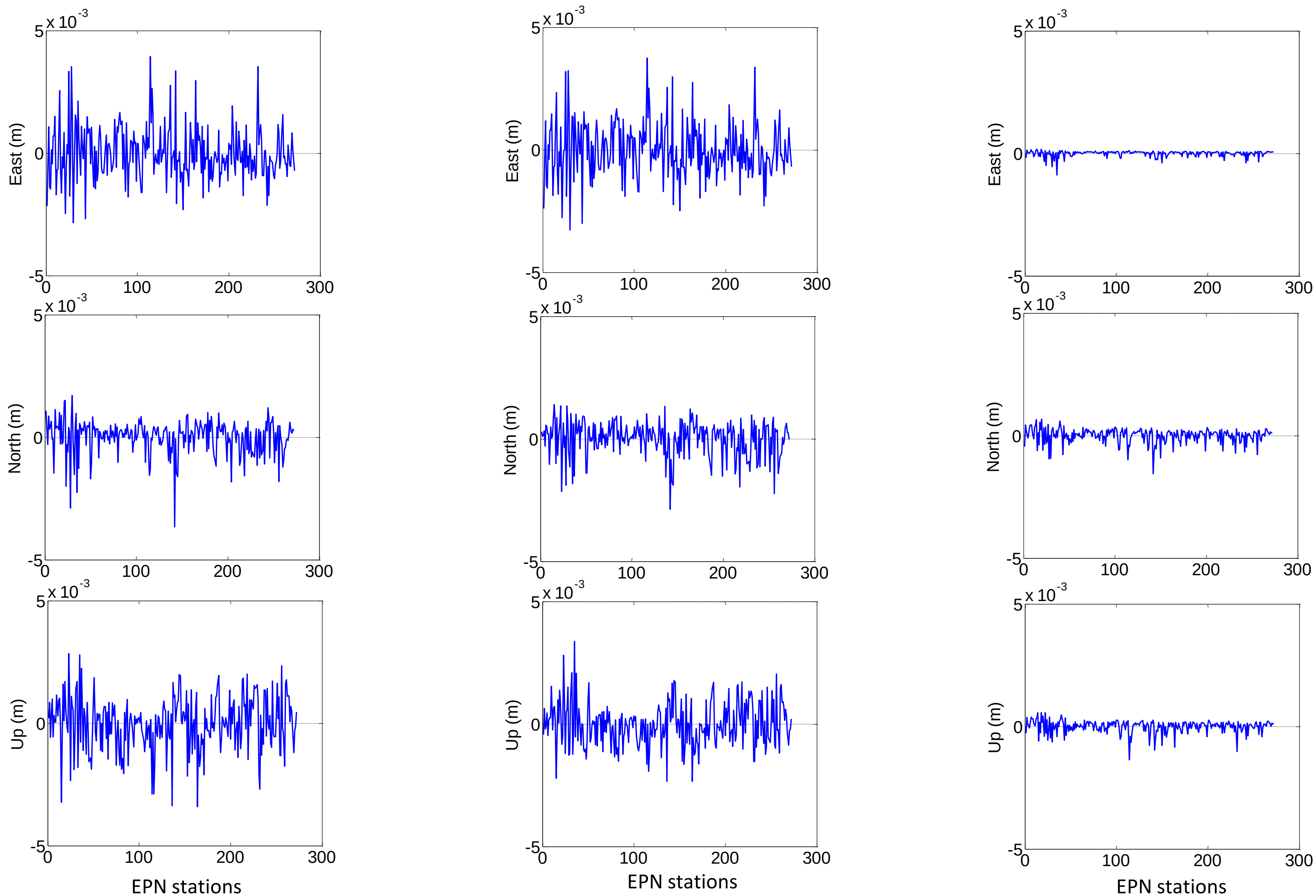
Full-similarity model

Transformation residuals (week 1872)



Shift-only model Shift/roto model Full-similarity model

Transformation residuals (week 1873)



Shift-only model

Shift/roto model

Full-similarity model

Helmert transformation parameters btw free-net and NNT-constrained weekly EPN solutions

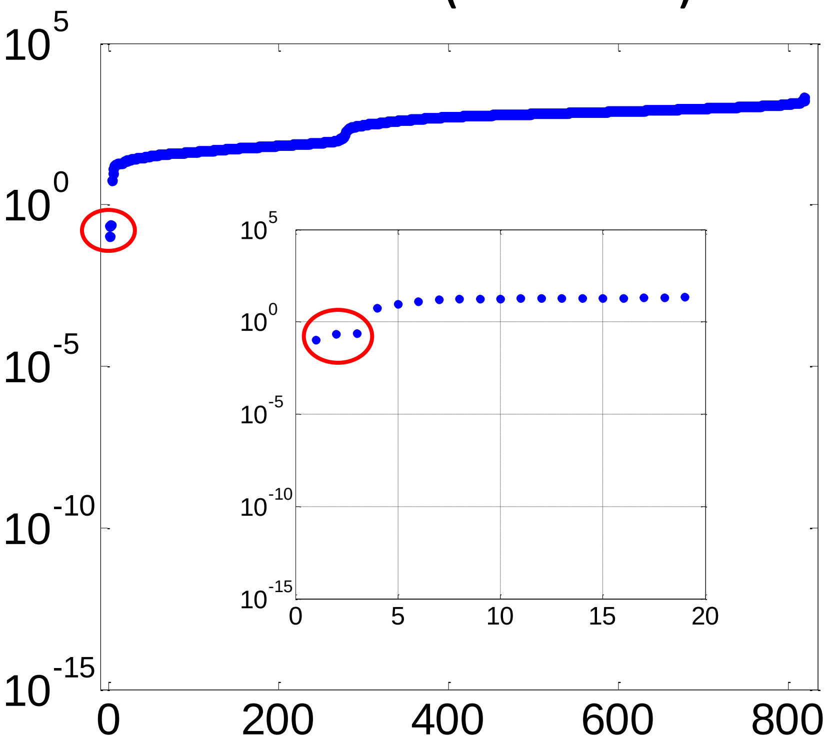
	Tx (cm)	Ty (cm)	Tz (cm)	Rx (mas)	Ry (mas)	Rz (mas)	Scale (ppb)
Week 1869	3.21	1.55	3.66	-0.10	-0.01	0.08	-1.55
Week 1870	2.32	0.98	2.87	-0.06	-0.01	0.05	-1.16
Week 1871	0.79	0.89	1.51	-0.06	-0.03	0.05	-0.53
Week 1872	-0.15	0.77	0.92	-0.05	-0.05	0.04	-0.20
Week 1873	2.07	1.33	2.89	-0.09	-0.03	0.07	-1.12

Removal of frame information from unconstrained NEQ

Unconstrained NEQ	$\mathbf{N}(\mathbf{X}-\mathbf{X}_o)=\mathbf{u}$ Original system without rank defect	$\mathbf{N}'(\mathbf{X}-\mathbf{X}_o)=\mathbf{u}'$ Modified system with "proper" rank defect
Helmert transformation matrix & handling of datum parameters	$\mathbf{G}=\begin{bmatrix}\mathbf{E}\\ \tilde{\mathbf{E}}\end{bmatrix}\begin{matrix}\}\,\boldsymbol{\theta}\\ \}\,\tilde{\boldsymbol{\theta}}\end{matrix}$	$\boldsymbol{\theta}$: ill-defined datum parameters to be fully removed from the NEQ $\tilde{\boldsymbol{\theta}}$: well-estimable datum parameters from the available data
Filtering of selected datum parameters (a.k.a. CDR)	$\mathbf{N}'=\left(\mathbf{I}-\mathbf{N}\mathbf{E}^T(\mathbf{E}\mathbf{N}\mathbf{E}^T)^{-1}\mathbf{E}\right)\mathbf{N}$ $\mathbf{u}'=\left(\mathbf{I}-\mathbf{N}\mathbf{E}^T(\mathbf{E}\mathbf{N}\mathbf{E}^T)^{-1}\mathbf{E}\right)\mathbf{u}$	
Preservation of original NEQ information	$\mathbf{N}'(\mathbf{N}^{-1}\mathbf{u})=\mathbf{u}'$ (*) the free-net solution of the original NEQ satisfies also the modified singular NEQ	

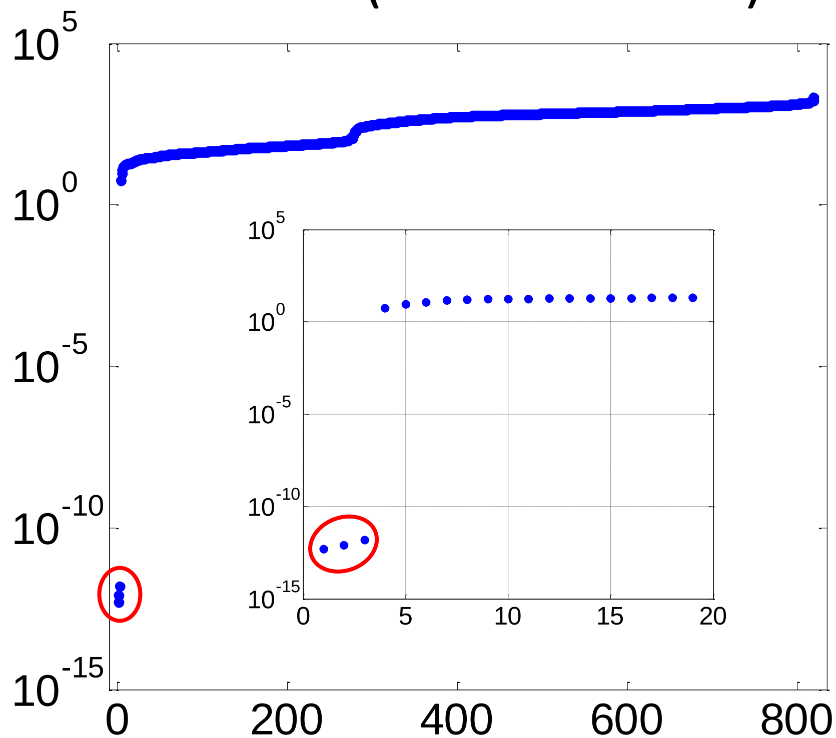
EPN weekly combined NEQ

BEFORE (full-rank)



Eigenvalues of
normal matrix **N**

AFTER (rank-deficient)



Eigenvalues of
normal matrix **N**′

EUR18697.snx

Testing on EPN weekly NEQ (2011-2015)

Week 1
Week 2
Weekly NEQ
(original, full-rank)

NNT alignment to IGb08
(*) pseudo MCs

Week 1
Week 2
Weekly estimated coordinates

Products:

- Station coordinate time series
- Velocity estimation and linear de-trending
- Reduced coordinate time series

Removal of frame origin information

Comparison

Week 1
Week 2
Weekly NEQ
(singular)

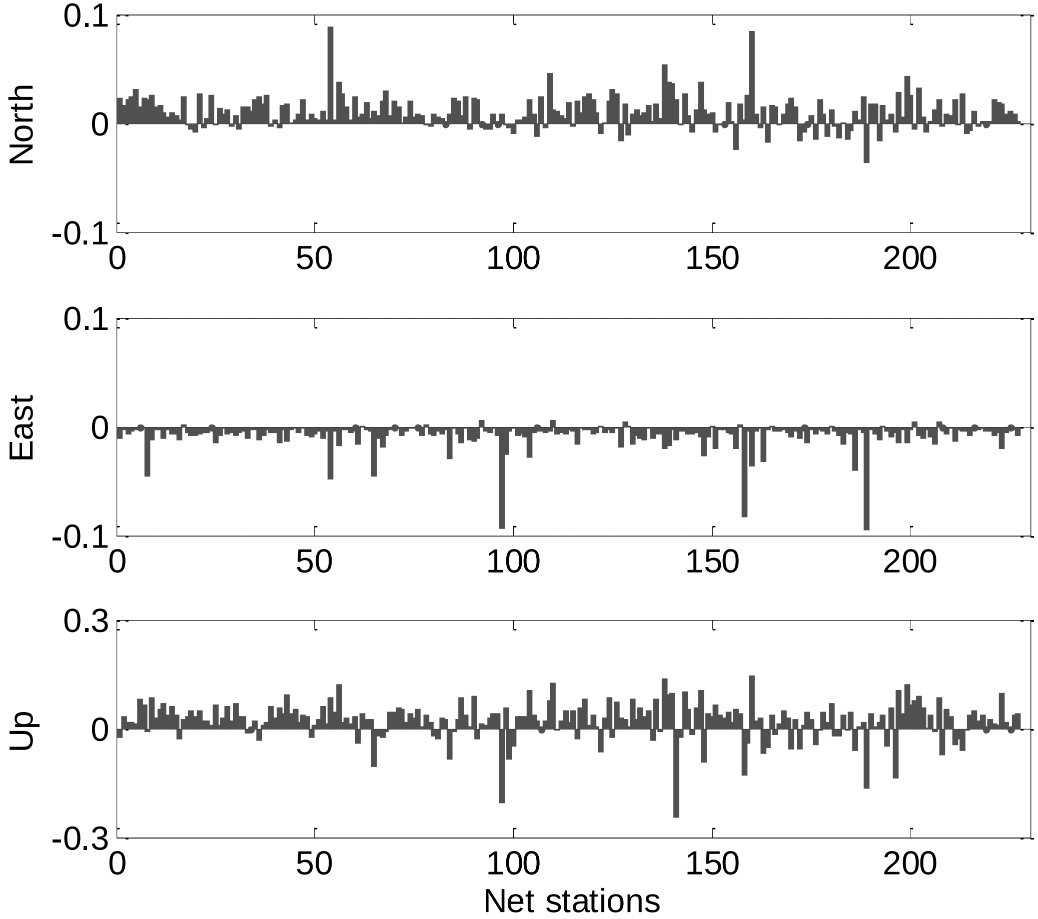
NNT alignment to IGb08
(*) true MCs

Week 1
Week 2
Weekly estimated coordinates

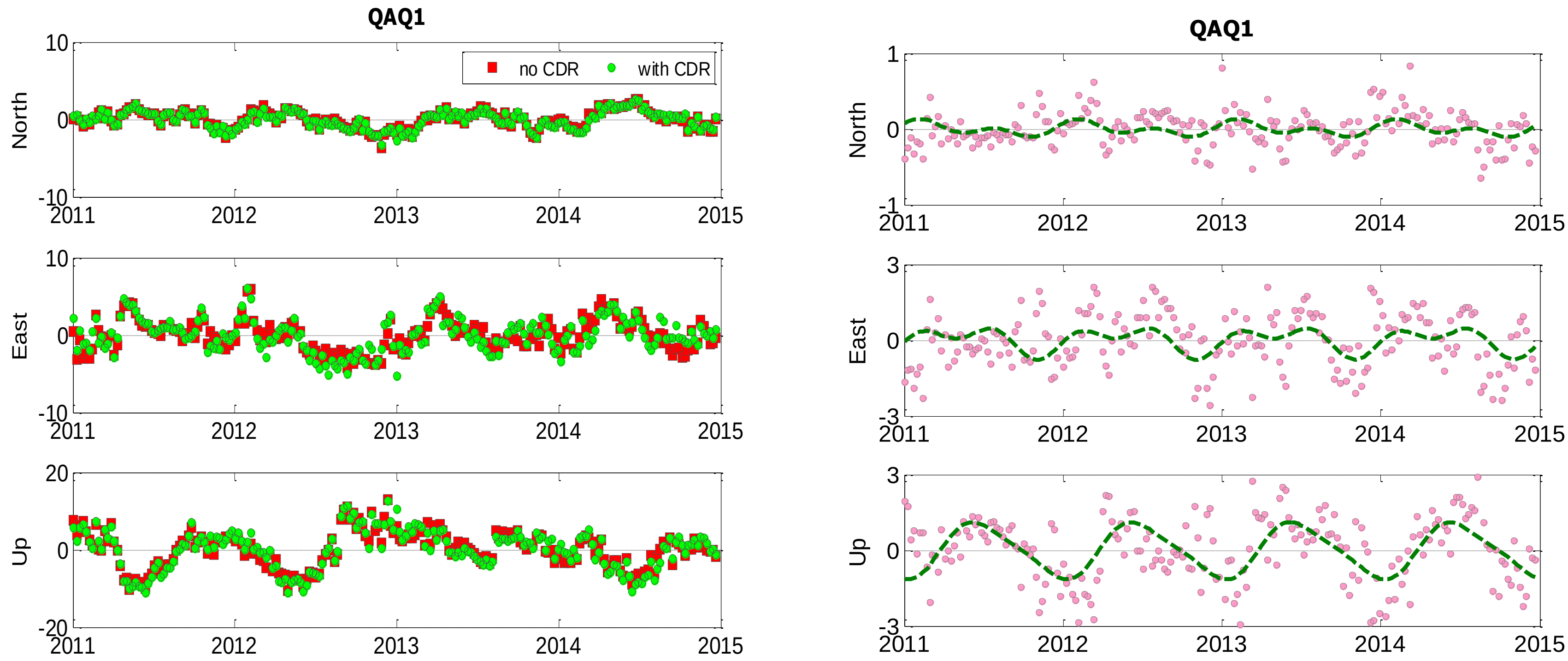
Products:

- Station coordinate time series
- Velocity estimation and linear de-trending
- Reduced coordinate time series

Differences (in mm/year) of the estimated EPN station velocities

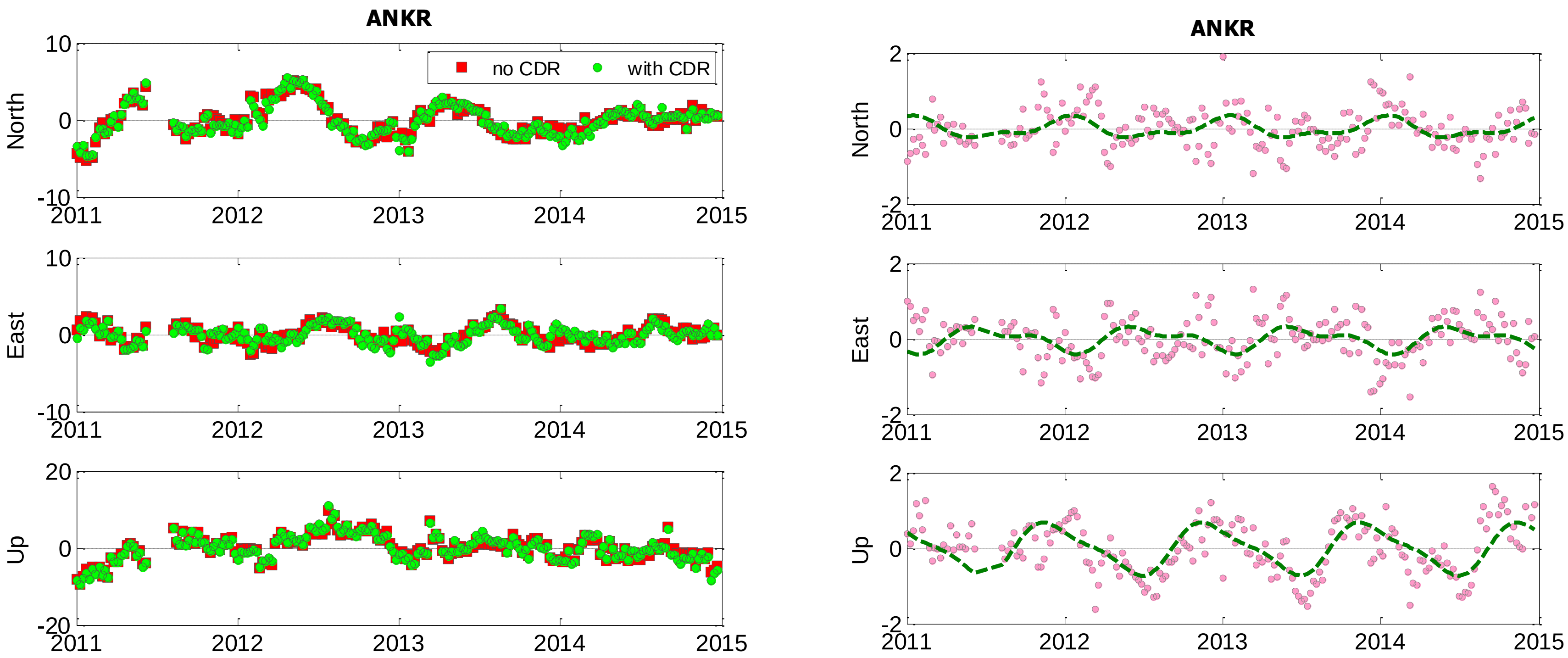


Reduced coordinate time series and their differences (in mm)



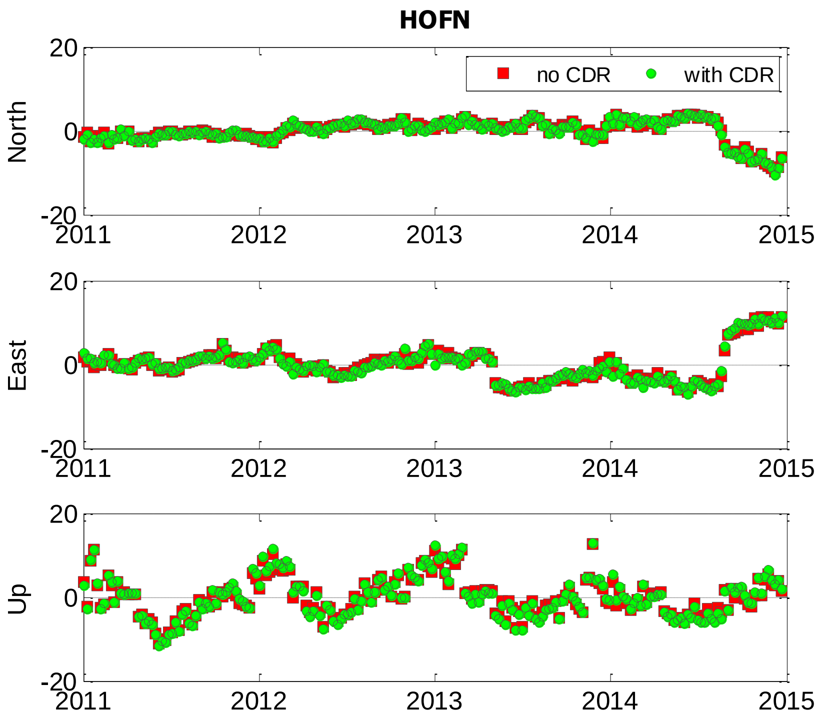
systematic effect on the annual and semi-annual variations (especially in the height time-series!)

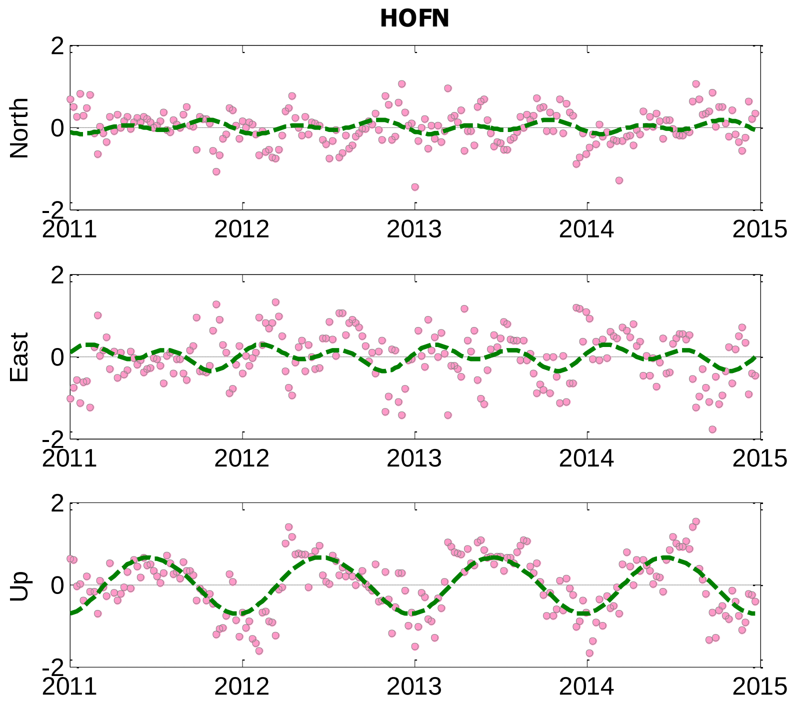
Reduced coordinate time series and their differences (in mm)



systematic effect on the annual and semi-annual variations (especially in the height time-series!)

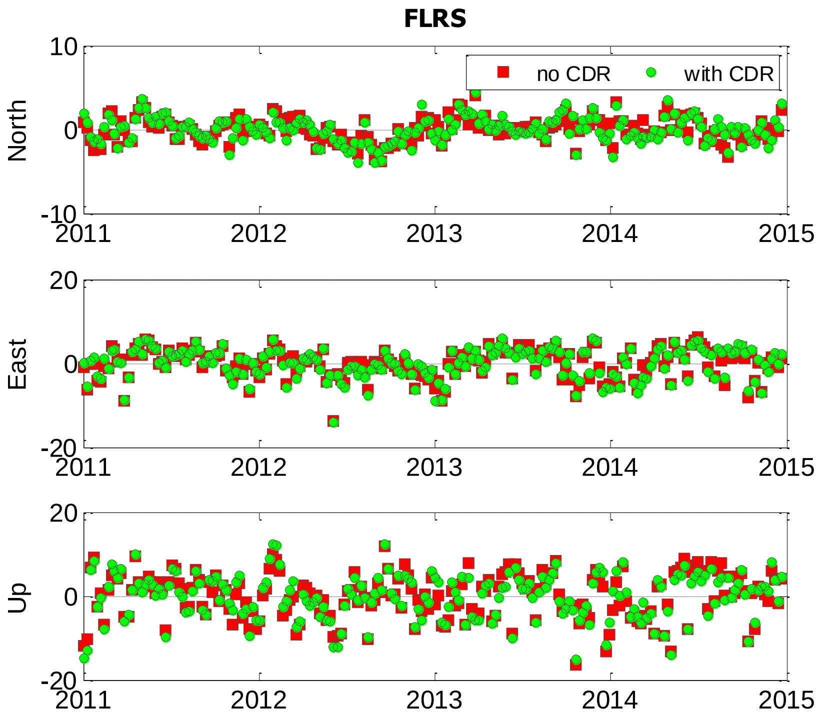
Reduced coordinate time series and their differences (in mm)

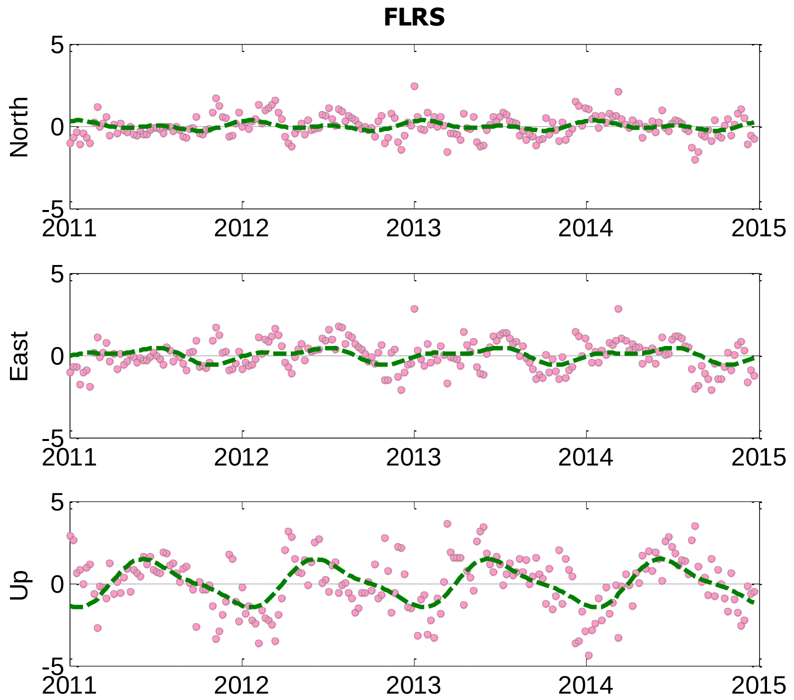




systematic effect on the annual and semi-annual variations
(especially in the height time-series!)

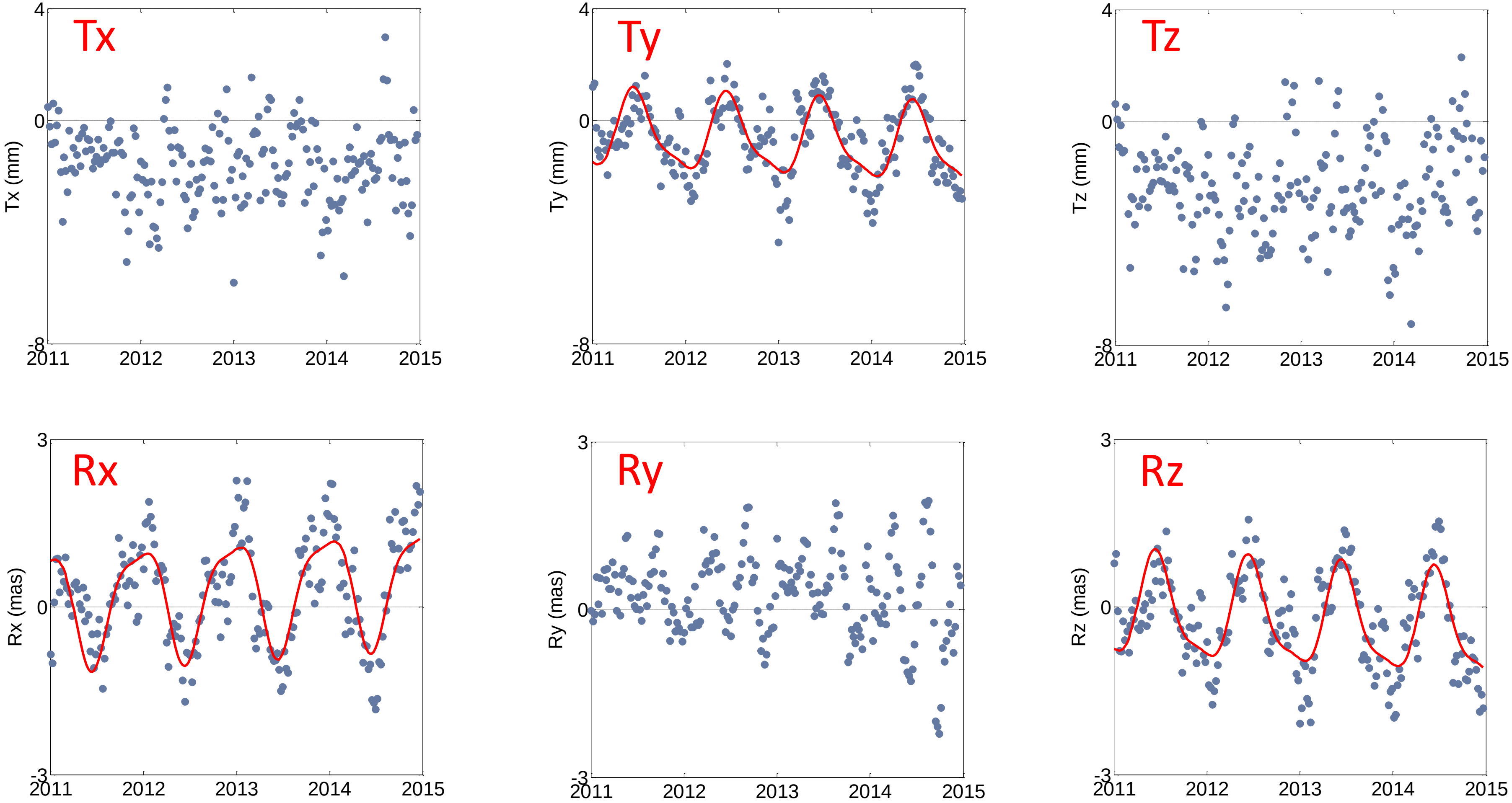
Reduced coordinate time series and their differences (in mm)



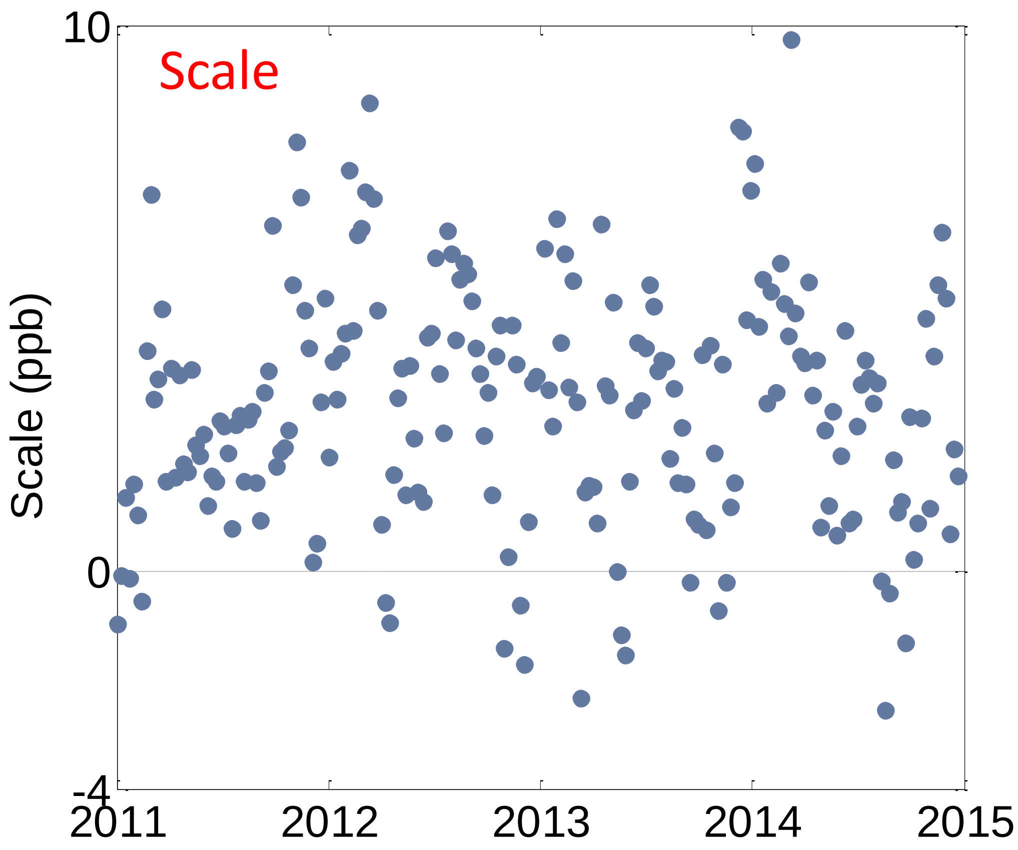


systematic effect on the annual and semi-annual variations
(especially in the height time-series!)

Helmert transformation parameters
(withCDR vs. noCDR weekly solutions)



Helmert transformation parameters
(withCDR vs. noCDR weekly solutions)



Conclusions

- The implementation of "minimal constraints" to unconstrained full-rank NEQ can create various types of distortion in the computed solutions.

- In the case of EPN's weekly combined NEQ:
  - geometrical distortions of **several mm** were identified in the NNT-constrained weekly solutions
  - a significant part of these distortions seems to be equivalent to a scale-like bias of several ppb
  - vertical velocities inferred from the corresponding time series showed a systematic effect of more than **0.1-0.2 mm/yr**
  - additional systematic effects occur in the annual/semi-annual periodic variations in the reduced time series

Conclusions

- The aforementioned distortions can be prevented by a simple "pre-filtering" of selected frame parameters from the original normal system.

  - the removed information may refer only to ill-defined frame parameters (as in the present study) or it could be related to a complete de-stripping of the original NEQ's datum content!

  - this is useful not only in GNSS network analysis, but also in multi-technique combination strategies for TRF estimation at the NEQ level

Backup slides

Bernese test network
(13 stations, DOY 205, 2011)

```
TRANSFORMATION IN EQUATORIAL SYSTEM (X, Y, Z):
RESIDUALS IN LOCAL SYSTEM (NORTH, EAST, UP)
```

NUM	NAME	FLG	RESIDUALS IN MILLIMETERS			
71	GANP 11515M001	A A	-0.38	1.79	0.58	
89	HERT 13212M010	W A	0.17	-2.27	0.40	
106	JOZ2 12204M002	W A	0.62	1.83	0.90	
123	LAMA 12209M001	A A	1.17	1.74	1.55	
136	MATE 12734M008	W A	-3.39	1.18	0.16	
174	ONSA 10402M004	W A	2.51	0.10	1.72	
193	PTBB 14234M001	A A	0.90	-0.25	0.36	
236	TLSE 10003M009	W A	-2.35	-2.11	-0.60	
258	WSRT 13506M005	W A	1.13	-1.01	0.00	
259	WTZR 14201M010	W A	-0.16	0.31	-0.63	
260	WTZZ 14201M014	A A	-0.16	0.30	-0.62	
270	ZIM2 14001M008	A A	-1.00	-0.83	-0.92	
272	ZIMM 14001M004	W A	-1.13	-0.90	-0.12	
	RMS / COMPONENT		1.57	1.39	0.86	
	MEAN		-0.16	-0.01	0.21	
	MIN		-3.39	-2.27	-0.92	
	MAX		2.51	1.83	1.72	

```
NUMBER OF PARAMETERS  :     3
NUMBER OF COORDINATES :    39
RMS OF TRANSFORMATION :    1.31 MM
```

```
PARAMETERS:

TRANSLATION IN  X     :        -70.47    +-  0.36    MM
TRANSLATION IN  Y     :         -4.89    +-  0.36    MM
TRANSLATION IN  Z     :        -44.46    +-  0.36    MM

NUMBER OF ITERATIONS  :     2

------------------------------------------------------------------
>>> CPU/Real time for pgm "HELMR1": 0:00:00.010 / 0:00:00.020
>>> Program finished successfully
```

Bernese test network

(13 stations, DOY 205, 2011)

```
TRANSFORMATION IN EQUATORIAL SYSTEM (X, Y, Z):
RESIDUALS IN LOCAL SYSTEM (NORTH, EAST, UP)



 ---------------------------------------------------------------------
 | NUM |        NAME          | FLG |     RESIDUALS IN MILLIMETERS    |   |
 ---------------------------------------------------------------------
 |     |                      |     |                                 |   |
 |  71 | GANP 11515M001       | A A |      -0.39      1.83      0.23  |   |
 |  89 | HERT 13212M010       | W A |       0.16     -2.24      0.69  |   |
 | 106 | JOZ2 12204M002       | W A |       0.63      1.91      0.17  |   |
 | 123 | LAMA 12209M001       | A A |       1.21      1.83      0.63  |   |
 | 136 | MATE 12734M008       | W A |      -3.33      1.08      0.98  |   |
 | 174 | ONSA 10402M004       | W A |       2.61      0.11      0.78  |   |
 | 193 | PTBB 14234M001       | A A |       0.94     -0.23      0.10  |   |
 | 236 | TLSE 10003M009       | W A |      -2.25     -2.01      0.59  |   |
 | 258 | WSRT 13506M005       | W A |       1.16     -1.00     -0.19  |   |
 | 259 | WTZR 14201M010       | W A |      -0.12      0.33     -0.62  |   |
 | 260 | WTZZ 14201M014       | A A |      -0.13      0.33     -0.61  |   |
 | 270 | ZIM2 14001M008       | A A |      -0.94     -0.80     -0.39  |   |
 | 272 | ZIMM 14001M004       | W A |      -1.07     -0.87      0.42  |   |
 |     |                      |     |                                 |   |
 ---------------------------------------------------------------------
 |     | RMS / COMPONENT      |     |       1.56      1.39      0.58  |   |
 |     | MEAN                 |     |      -0.12      0.02      0.21  |   |
 |     | MIN                  |     |      -3.33     -2.24     -0.62  |   |
 |     | MAX                  |     |       2.61      1.91      0.98  |   |
 ---------------------------------------------------------------------


NUMBER OF PARAMETERS  :     6
NUMBER OF COORDINATES :    39
RMS OF TRANSFORMATION :    1.31 MM
```

```
PARAMETERS:

TRANSLATION IN  X     :          -76.45      +-   3.67     MM
TRANSLATION IN  Y     :           -2.07      +-   4.95     MM
TRANSLATION IN  Z     :          -39.95      +-   3.21     MM
ROTATION AROUND X-AXIS: -   0   0   0.000054 +-   0.000145 "
ROTATION AROUND Y-AXIS: -   0   0   0.000243 +-   0.000156 "
ROTATION AROUND Z-AXIS:     0   0   0.000080 +-   0.000134 "


NUMBER OF ITERATIONS  :     2
```

0.5/2.4/0.8 mas

```
-------------------------------------------------------------------------
>>> CPU/Real time for pgm "HELMR1": 0:00:00.010 / 0:00:00.0
>>> Program finished successfully
```

Bernese test network
(13 stations, DOY 205, 2011)

```
TRANSFORMATION IN EQUATORIAL SYSTEM (X, Y, Z):
RESIDUALS IN LOCAL SYSTEM (NORTH, EAST, UP)
```

NUM	NAME	FLG	RESIDUALS IN MILLIMETERS			
71	GANP 11515M001	A A	-0.03	-0.09	0.03	
89	HERT 13212M010	W A	-0.02	0.16	0.43	
106	JOZ2 12204M002	W A	-0.01	-0.15	-0.06	
123	LAMA 12209M001	A A	-0.04	-0.16	0.38	
136	MATE 12734M008	W A	-0.25	-0.06	0.59	
174	ONSA 10402M004	W A	0.07	0.01	0.50	
193	PTBB 14234M001	A A	0.10	-0.02	-0.03	
236	TLSE 10003M009	W A	-0.04	0.14	0.24	
258	WSRT 13506M005	W A	0.15	0.04	-0.35	
259	WTZR 14201M010	W A	0.10	0.02	-0.73	
260	WTZZ 14201M014	A A	0.09	0.02	-0.72	
270	ZIM2 14001M008	A A	0.05	0.07	-0.54	
272	ZIMM 14001M004	W A	-0.08	-0.01	0.26	
	RMS / COMPONENT		0.11	0.10	0.46	
	MEAN		0.01	-0.00	0.00	
	MIN		-0.25	-0.16	-0.73	
	MAX		0.15	0.16	0.59	

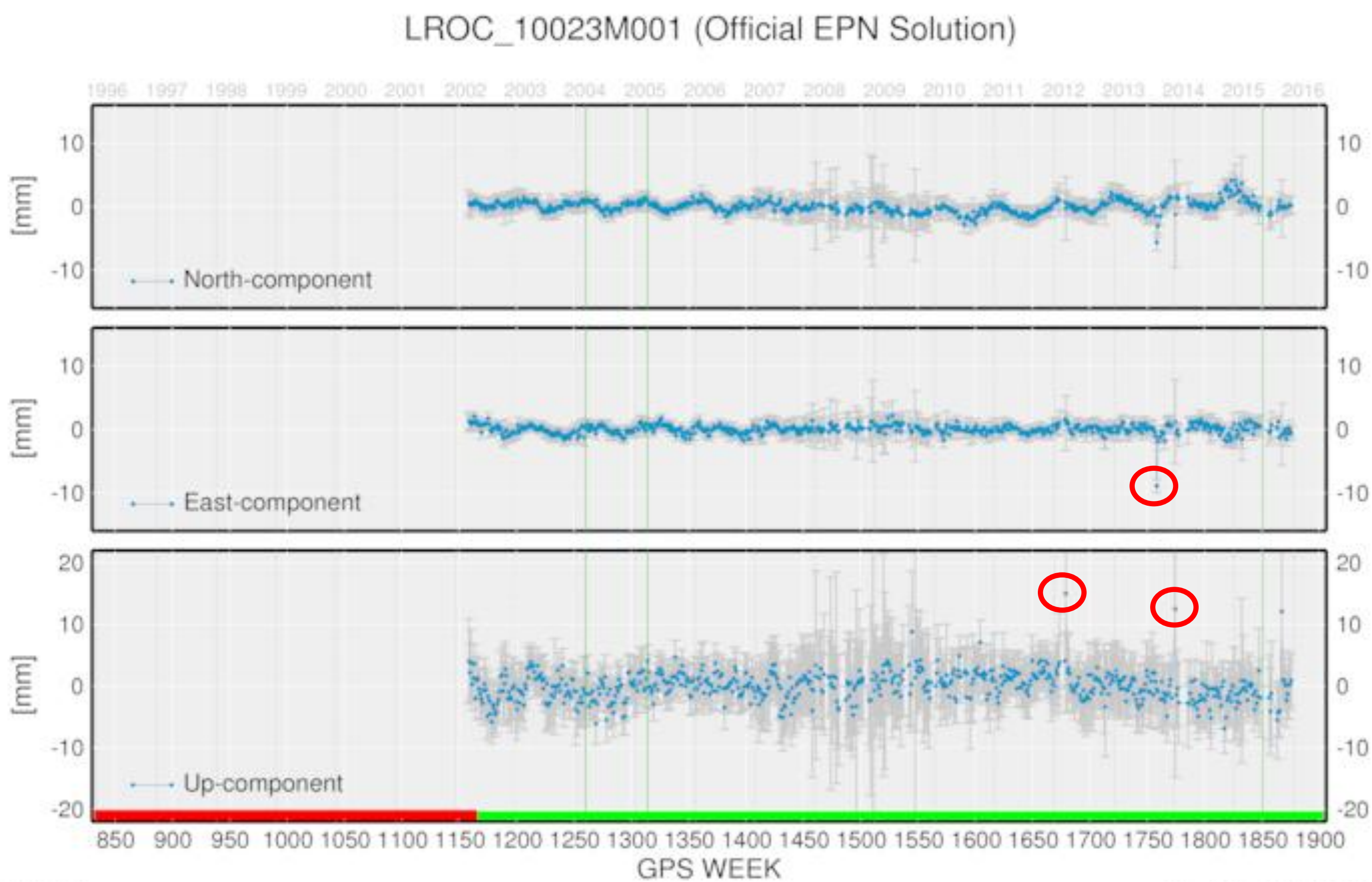
```
NUMBER OF PARAMETERS  :     7
NUMBER OF COORDINATES :    39
RMS OF TRANSFORMATION :    0.29 MM
```

```
PARAMETERS:

TRANSLATION IN  X      :          -88.62      +-  0.96      MM
TRANSLATION IN  Y      :           -4.53      +-  1.12      MM
TRANSLATION IN  Z      :          -54.53      +-  0.93      MM
ROTATION AROUND X-AXIS:   -  0  0  0.000054 +-  0.000033 "
ROTATION AROUND Y-AXIS:   -  0  0  0.000243 +-  0.000035 "
ROTATION AROUND Z-AXIS:      0  0  0.000080 +-  0.000030 "
SCALE FACTOR           :            0.00302  +-  0.00012 MM/KM

NUMBER OF ITERATIONS   :     2
```

3 ppb

```
>>> CPU/Real time for pgm "HELMR1": 0:00:00.010 / 0:00:00.009
>>> Program finished successfully
```

The case of LROC



The case of LROC