RECONSTRUCTION OF VERTICAL MOVEMENTS OF THE EARTH'S CRUST, ACCORDING TO TIDE GAUGE OBSERVATIONS

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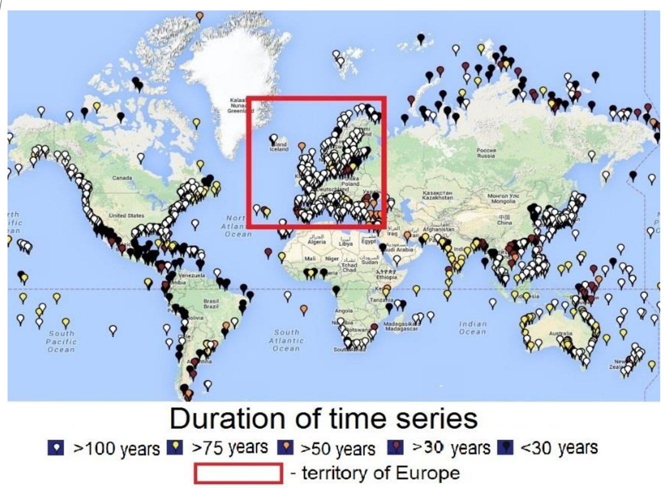
Introduction

Study of modern crustal movements - one of the actual problem of Earth sciences, which has significant theoretical and practical value.

The aim of this work is to study the precision of vertical crustal movements using tide gauge observations and to reconstruct vertical crustal movements at the given average epoch of observations (for example Avalonya territory).

Study of precision of vertical crustal movements using tide gauge observations

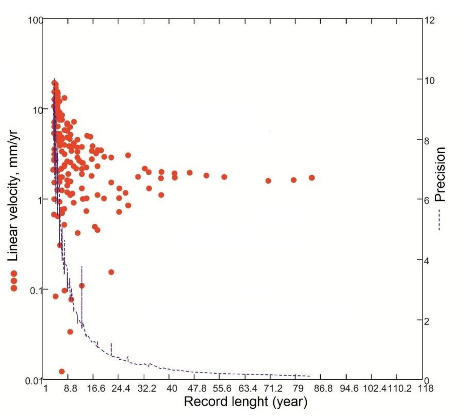


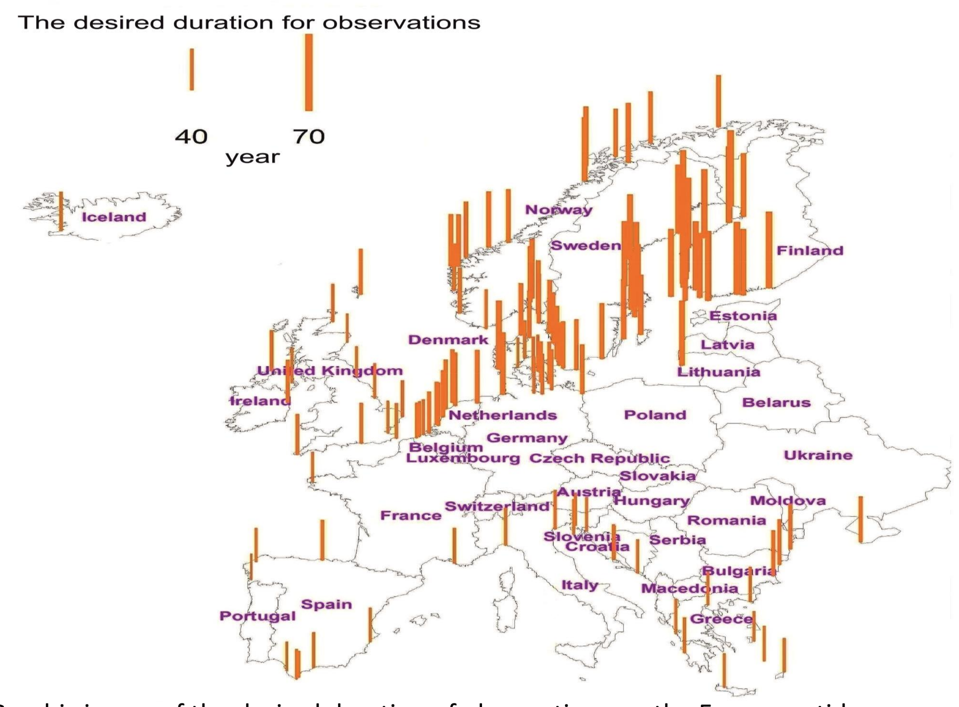
Data for study of time series and selection of tide gauges was conducted using data from the following web-site http://www.psmsl.org

Fragment of calculation results V_{TG} and $m_{V_{TG}}$ depending on the duration of observations on the example of Travemunde tide gauge (Germany)

V_{TG} mm/yr	$m_{V_{TG}}$ mm/yr	Duration of observation (yr)	Start of observation	Finish of observation
1,7	0,4	37,5	1928,0	1965,5
1,1	0,4	37,5	1965,5	2003,0
2,0	0,4	37,6	1974,4	2012,0
1,9	0,3	41,6	1928,0	1969,6
1,7	0,3	41,7	1969,7	2011,4
2,0	0,3	46,3	1928,0	1974,3
1,8	0,2	51,5	1928,0	1979,5

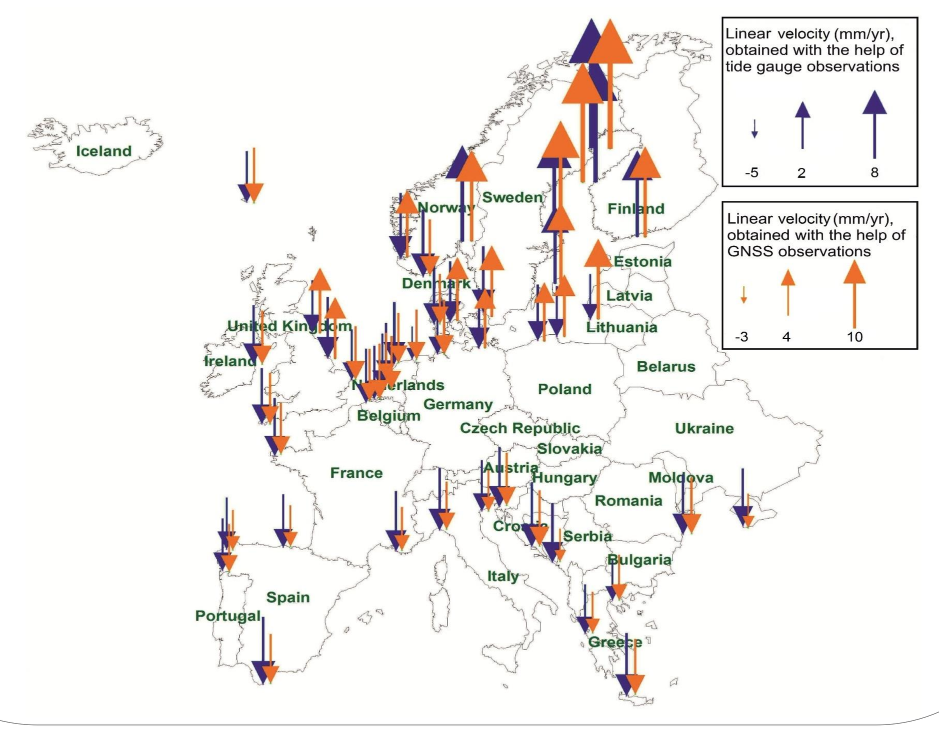
Dependence V_{TG} and $m_{V_{TG}}$ for different periods of observation, depending on their duration on the example of Travemunde tide gauge (Germany)





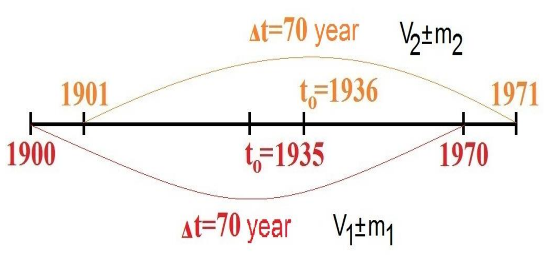
Graphic image of the desired duration of observations on the European tide gauges to achieve a given precision of vertical crust movements determination $m_{V_{TG}}$=0.3 mm/yr

Diagram of velocity (mm/yr) of vertical movements of the Earth's crust, based on the results of tide gauge and GNSS observations

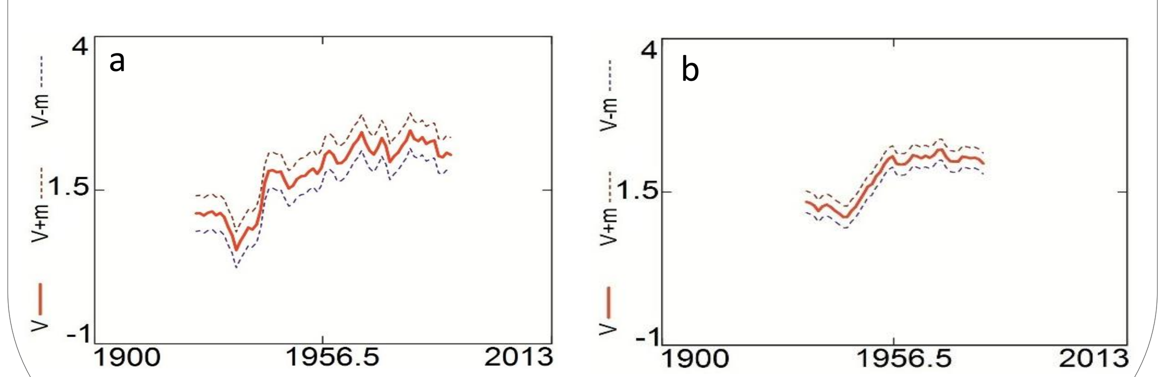


Reconstruction of vertical movements of the Earth's crust

Location of selected for the study tide gauges

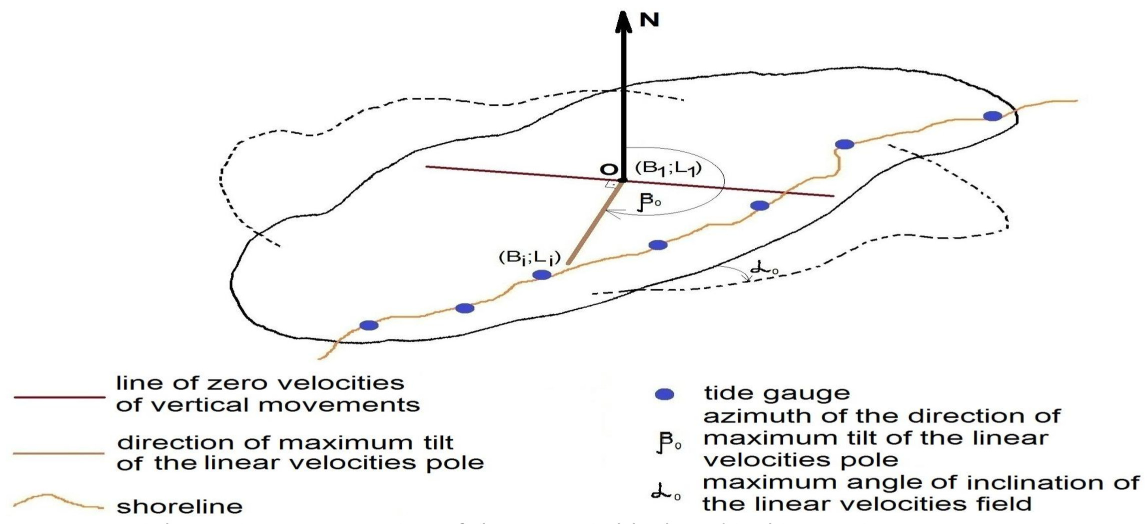


Computation of linear velocities V_{TG} at a given average epoch t0 depending on averaging period Δt



Change of averaged velocities, derived from the results of observation at tide gauge № 9 for Δt =50 years (a) and Δt =70 (b) years

Methodology of kinematic parameters determination of vertical velocity field of the tectonic block



Schematic representation of the tectonic block and its kinematic parameters

Zero approximation

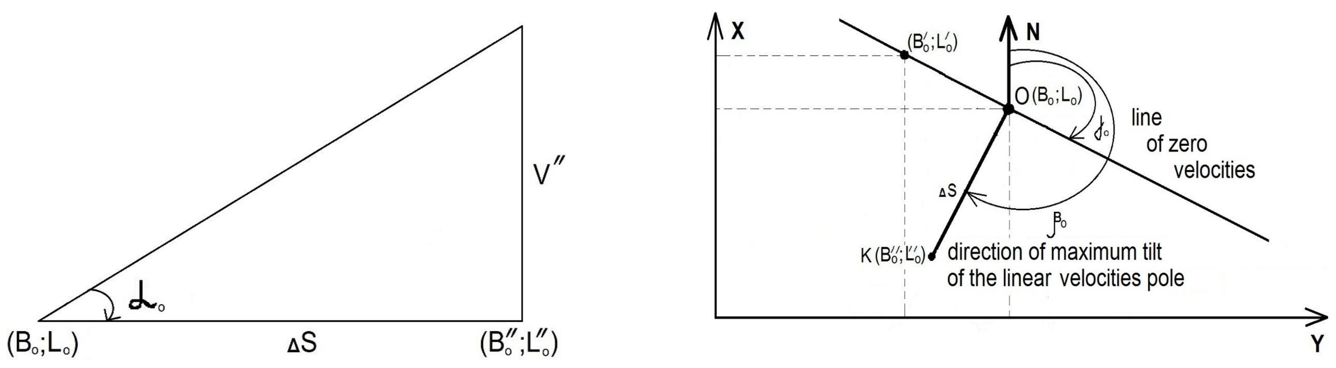
$$m \cdot B + n \cdot L + p = V_{TG}\ , \tag{1}$$

where B and L – coordinates of the tide gauge, V_{TG} – averaged velocity of the tide gauge; m, n, p – unknown coefficients

$$L_0 = \frac{-m \cdot B_0 - p}{n}\ , \tag{2}$$

where (B0; L0) – coordinates of the point, through which the line of zero velocities is passed

$$L_0'' = L_0 + \Delta L \cdot \sin\beta_0\ , \quad (3) \qquad B_0'' = B_0 + \Delta S \cdot \cos\beta_0, \quad (4)$$



Parameters of kinematics of the tectonic block

First approximation

$$V = \alpha \cdot d \cdot \cos\beta\ , \tag{5}$$

$$d = R \cdot \sqrt{(B_2 - B_1)^2 + q^2 \cdot (L_2 - L_1)^2}, \tag{6}$$

where R – radius of the Earth, equal 6371 km

$$q = \frac{B_2 - B_1}{\Delta\varphi} \quad (6) \qquad \Delta\varphi = \ln\left(\frac{\tan\left(\frac{\pi}{4} + \frac{B_2}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{B_1}{2}\right)}\right) \quad (7) \qquad \beta = \arctan\left(\frac{L_2 - L_1}{\Delta\varphi}\right) \quad (8)$$

where α – maximum angle of inclination of the linear velocities pole (''/year), β – azimuth of the direction of maximum tilt of linear velocities pole, d – distance between points

$$V = \alpha \cdot \left(R \cdot \sqrt{\left(B_i - B_1'\right)^2 + \frac{\left(B_i - B_1'\right)^2}{\ln\left(\frac{\tan\left(\frac{\pi}{4} + \frac{B_i}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{B_1'}{2}\right)}\right)^2} \cdot \left(L_i - L_1'\right)^2} \right) \cdot \cos\left(\arctan\left(\frac{L_i - L_1'}{\ln\left(\frac{\tan\left(\frac{\pi}{4} + \frac{B_i}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{B_1'}{2}\right)}\right)} \right) - \beta + \begin{cases} 0, (B_2 > B_1) \\ \pi, (B_1 > B_2) \end{cases} \right) \tag{9}$$

Second approximation

$$\frac{\partial V}{\partial B_1} \cdot \delta B_1 + \frac{\partial V}{\partial L_1} \cdot \delta L_1 + \frac{\partial V}{\partial \alpha} \cdot \delta\alpha + \frac{\partial V}{\partial \beta} \cdot \delta\beta + (V_{measure} - V_{approx}) = 0 \tag{10}$$

1) If $B_i - B_{opt} > 0$, then

$$\frac{\partial V}{\partial B_1} = \frac{R \cdot \alpha \cdot \left(\tan\left(\frac{\pi}{4} + \frac{B_{opt}}{2}\right) + 1\right) \cdot \ln(p)^2 \cdot \left(B_{opt} - B_i\right)^2 \cdot \left(\sin(k) \cdot \left(L_{opt} - L_i\right) \cdot \ln(p)^3 + \cos(k) \cdot \left(L_{opt} - L_i\right)^2\right)}{2 \cdot \tan\left(\frac{\pi}{4} + \frac{B_{opt}}{2}\right) \cdot \ln(p)^2 \cdot \ln(p)^3 \cdot \left(B_{opt} - B_i\right)^2 \cdot \left(\ln(p)^2 + \left(L_{opt} - L_i\right)^2\right)} - \frac{2 \cdot R \cdot \alpha \cdot \cos(k) \cdot \left(B_i - B_{opt}\right) \cdot \ln(p)^3 \cdot \tan\left(\frac{\pi}{4} + \frac{B_{opt}}{2}\right) \cdot \left(\ln(p)^2 + \left(L_{opt} - L_i\right)^2\right)}{2 \cdot \tan\left(\frac{\pi}{4} + \frac{B_{opt}}{2}\right) \cdot \ln(p)^2 \cdot \ln(p)^3 \cdot \left(B_{opt} - B_i\right)^2 \cdot \left(\ln(p)^2 + \left(L_{opt} - L_i\right)^2\right)} \tag{11}$$

$$\frac{\partial V}{\partial L_1} = \frac{R \cdot \alpha \cdot \left(\sin(k) \cdot \ln(p)^2 - \cos(k) \cdot \ln(p) \cdot \left(L_i - L_{opt}\right)\right)}{\ln(p) \cdot \left(\ln(p)^2 + \left(L_{opt} - L_i\right)^2\right)} \tag{12}$$

$$\frac{\partial V}{\partial \alpha} = R \cdot \cos(k)\ , \quad (13) \qquad \frac{\partial V}{\partial \beta} = R \cdot \alpha \cdot \sin(k)\ , \quad (14)$$

where

$$k = \left(-\beta - a\tan\left(\frac{L_{opt} - L_i}{\ln p}\right)\right) \cdot \sqrt{\left(B_{opt} - B_i\right)^2 + \frac{\left(B_{opt} - B_i\right)^2 \cdot \left(L_{opt} - L_i\right)^2}{\ln p^2}} \tag{15}$$

$$p = \frac{\tan\left(\frac{\pi}{4} + \frac{B_i}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{B_{opt}}{2}\right)} \tag{16}$$

2) If $B_i - B_{opt} < 0$, then in this case the derivative by relevant values can be calculated by similar formulas (11) - (14), just putting before them the sign "-"

3) If $B_i = B_{opt}$, and $L_i - L_{opt} > 0$, then

$$\frac{\partial V}{\partial B_i} = \frac{R \cdot \alpha \cdot \sin\beta \cdot \left(\cos\left(B_{opt}\right) \cdot \sin\left(B_{opt}\right) \cdot \left(L_{opt} - L_i\right)^2 - B_{opt} + B_i\right)}{\sqrt{\left(B_{opt} - B_i\right)^2 + \cos\left(B_{opt}\right)^2 \cdot \left(L_{opt} - L_i\right)^2}} \tag{17}$$

$$\frac{\partial V}{\partial L_i} = \frac{R \cdot \alpha \cdot \sin\beta \cdot \cos\left(B_{opt}\right)^2 \cdot \left(L_i - L_{opt}\right)}{\sqrt{\left(B_{opt} - B_i\right)^2 + \cos\left(B_{opt}\right)^2 \cdot \left(L_{opt} - L_i\right)^2}} \tag{18}$$

$$\frac{\partial V}{\partial \alpha} = R \cdot \sin\beta \cdot \sqrt{\left(B_{opt} - B_i\right)^2 + \cos\left(B_{opt}\right)^2 \cdot \left(L_{opt} - L_i\right)^2} \tag{19}$$

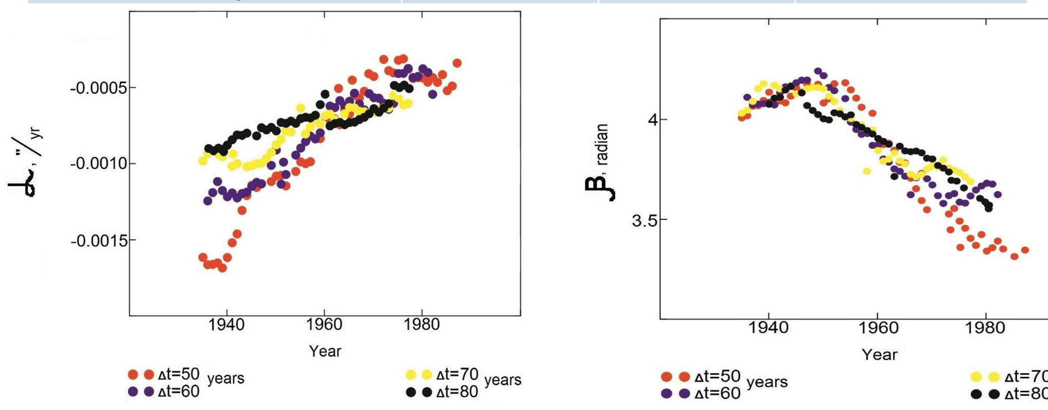
$$\frac{\partial V}{\partial \beta} = R \cdot \alpha \cdot \cos\beta \cdot \sqrt{\left(B_{opt} - B_i\right)^2 + \cos\left(B_{opt}\right)^2 \cdot \left(L_{opt} - L_i\right)^2} \tag{20}$$

4) If $B_i = B_{opt}$, and $L_i - L_{opt} < 0$, then in this case the derivative by relevant values can be calculated by similar formulas (15) - (18), just putting before them the sign "-". Determination of values B, L, α, β is conducted by using the least squares method.

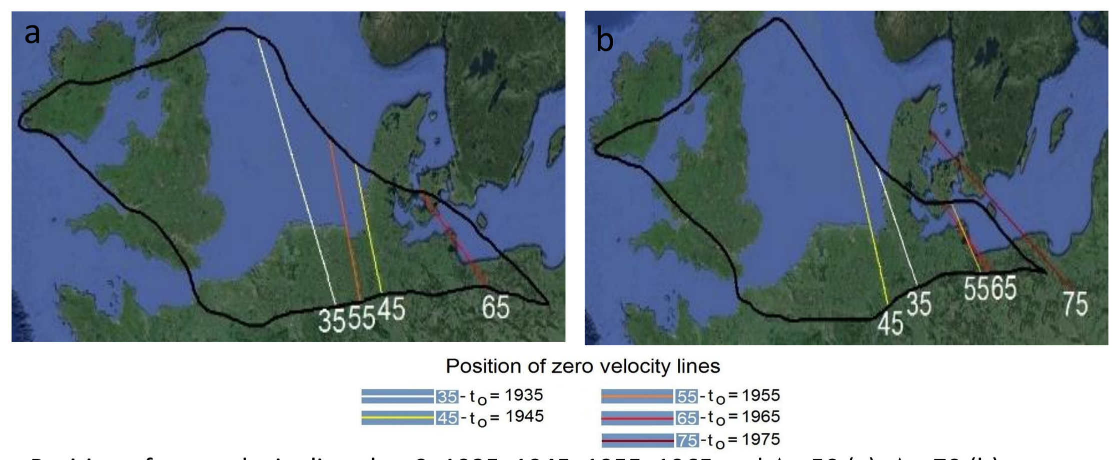
Results

Results of kinematic parameters calculating of the tectonic block on the results of tide gauge observations by approximation method on average epoch t0=1957 and Δt=70 years

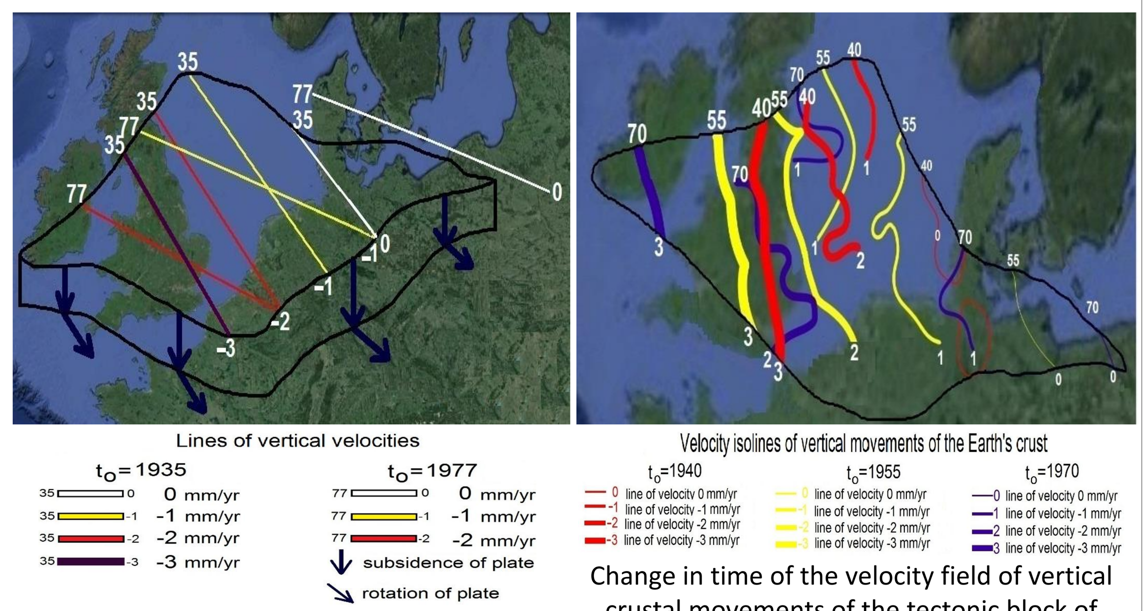
	Zero approximation	First approximation	Second approximation
B, radian	0,9301	0,9301	0,9292±0,0005
L, radian	0,8477	0,6994	0,7006±0,0008
α, ''/yr	-0,0002	-0,0002	-0,0002±0,000138
β, radian	3,4903	3,9870	3,9869±0,00004
$m_{V_{TG}}$**, mm/yr**			0,39



Change of parameters α and β by Δt=70 years for the tectonic block of Avalonia

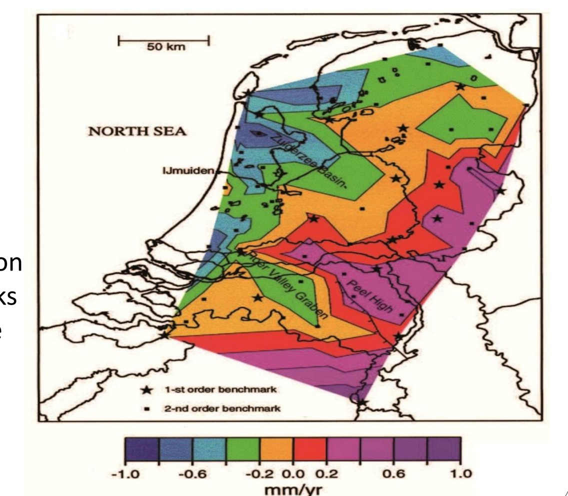


Position of zero velocity lines by t0=1935, 1945, 1955, 1965 and Δt=50 (a), Δt=70 (b) years



Change of the linear velocity field of vertical crustal movements of the tectonic block of Avalonia on average epochs t0=1935 and t0=1977 at Δt=70 years

Change in time of the velocity field of vertical crustal movements of the tectonic block of Avalonia on average epochs t0=1940, 1955 and 1970 at Δt=70 years



Results of vertical movements determination in the Netherlands obtained by benchmarks levelling and the tectonic structure of the region [Kooi H., 1998]

Conclusion and recommendation

- The methodology of precision estimation of vertical crustal movements according to long-period data of tide gauge observations is developed.
- Research of velocity change of secular vertical crustal movements depending on averaging period of tide gauge observation results is performed. It was determined, that with increasing period of averaging the amplitude of averaged velocities is decreasing, indicating that regional geodynamic factors are removed from the general trend of velocity.
- The methodology of determination of kinematic parameters of the tectonic block's velocity field according to long-period data of tide gauge observations is developed.
- According to the results of observations, performed on the tectonic structure of Avalonia, change of kinematic parameters of velocity field in time is determined and kinematic model of the vertical movement of the plate is developed.
- Obtained methodology coincides with data of GNSS observations on the investigated area and has a high degree of convergence with the results of precise levelling, the results of tide gauge observations processing by other researchers and models of velocities of vertical crustal movements.
- Proposed methodology can be used for similar studies in other regions of the world coast.

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