Unification of height systems using GNSS-leveling data and global satellite gravity models

R. Barzaghi*, A. Gatti, D. Carrion*, M. Gilardoni*, M. Reguzzoni*, G. Venuti*
*DICA, Politecnico di Milano, P.zza Leonardo da Vinci 32, Milano, Italy

The context

- The realization of the International Height Reference System has been recognized as one important topic for IAG (a resolution on that will be presented at the next IAG/IUGG in Prague)

- In GGOS we have Theme 1 – Unified Height System

- We have new detailed global geopotential models based on satellite observations only (e.g. GOCE)

- GNSS techniques can provide highly accurate estimate of ellipsoidal heights

Merging different height systems at regional/global scale

The normal height

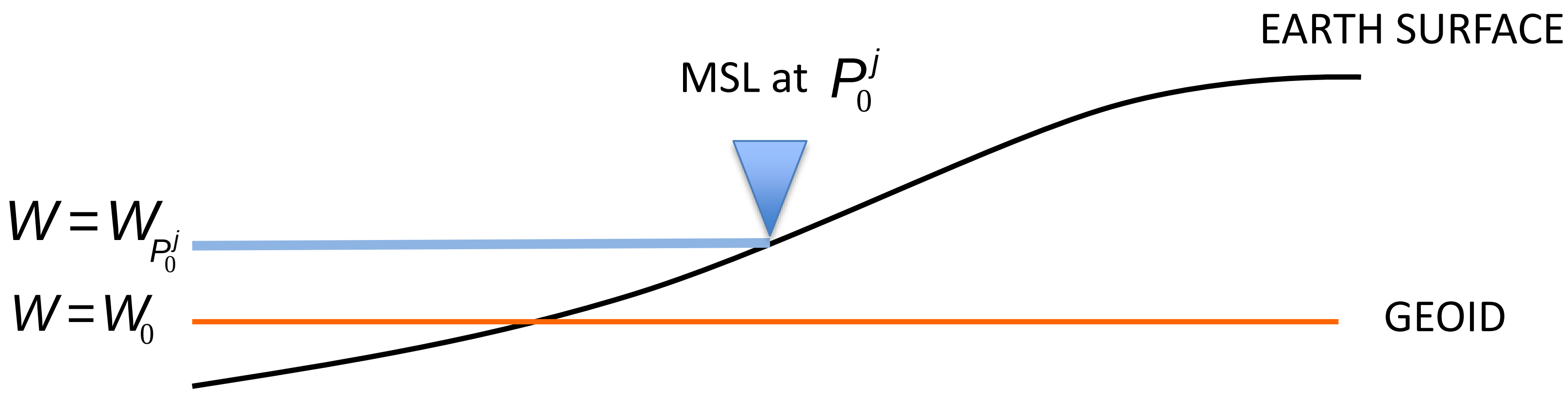
$$\int_0^Q g\,dh = \int_0^Q -dU = U_0 - U_Q = W_0 - W_P$$

$$h^* = \frac{1}{\bar{g}_Q}\int_0^Q g\,dh = \frac{W_0 - W_P}{\bar{g}_Q}$$

$$\bar{g}_Q = \frac{1}{h^*}\int_0^Q g\,dh$$

The normal height at a tide gauge point

$$h^*_{P_0^j} = \frac{W_0 - W_{P_0^j}}{\bar{g}_{P_0^j}} = \frac{W_0 - \left(W_0 + dW_j\right)}{\bar{g}_{P_0^j}} = -\frac{dW_j}{\bar{g}_{P_0^j}} = b_{P_0^j}$$



P_0^j *= tide gauge point for the jth domain (patch)*

The biased normal height

In the jth patch normal heights are biased and one can write the equation

$$h - \tilde{h}^{*j} = h - h^{*j} + b^j = z + b^j = \frac{T}{g} + b^j$$

h *= ellipsoidal height (GNSS)*

$\tilde{h}^{*j}$ *= biased normal height in the jth patch (spirit leveling)*

$$z = \frac{T}{g}$$

b^j *= bias of the jth patch*

The observation equation for estimating b^j

$$\tilde{z} = h - \tilde{h}^{*j} = \frac{T}{g} + b^j$$

$$\boxed{b^j = \tilde{z}(P_k^j) - \frac{T(P_k^j)}{g(P_k^j)}}$$

P_k^j *= observation point k in the jth patch*

The observation equation: the anomalous potential problem

T(P) cannot be estimated using ground gravity data (they are biased too)

EGM2008 cannot be used for the same reason

Global geopotential models based on satellite data are not affected by the biases and can be used in the estimation equation (GOCE models)

However, to reduce the omission error, EGM2008 can be used at high degree
$T_{EGM2008}$ is biased but e.g. in the range $201<n<2160$
it can be proved that the induced error in terms of geoid is $\varepsilon(N)<0.5$ cm

$$\boxed{T(P_k^j) = T^L(P_k^j) + T^H(P_k^j)}$$

$T^L(P_k^j)$ = GOCE global model to d/o ≅ 200

$T^H(P_k^j)$ = EGM2008 global model from d/o ≅ 201 to higher degree

The final observation equation for estimating b^j in the jth patch

$$b^j = \tilde{z}_{GNSS/lev}(P_k^j) - \frac{T^L(P_k^j)}{g} - \frac{T^H(P_k^j)}{g} \quad k = 1,\ldots,N$$

In the l.s. solving equation the proper covariance structure must be considered

$$C = C_{\tilde{z}} + C_{T^L} + C_{T^H}$$

$$C_{\tilde{z}} = s_{\tilde{z}}^2 I$$

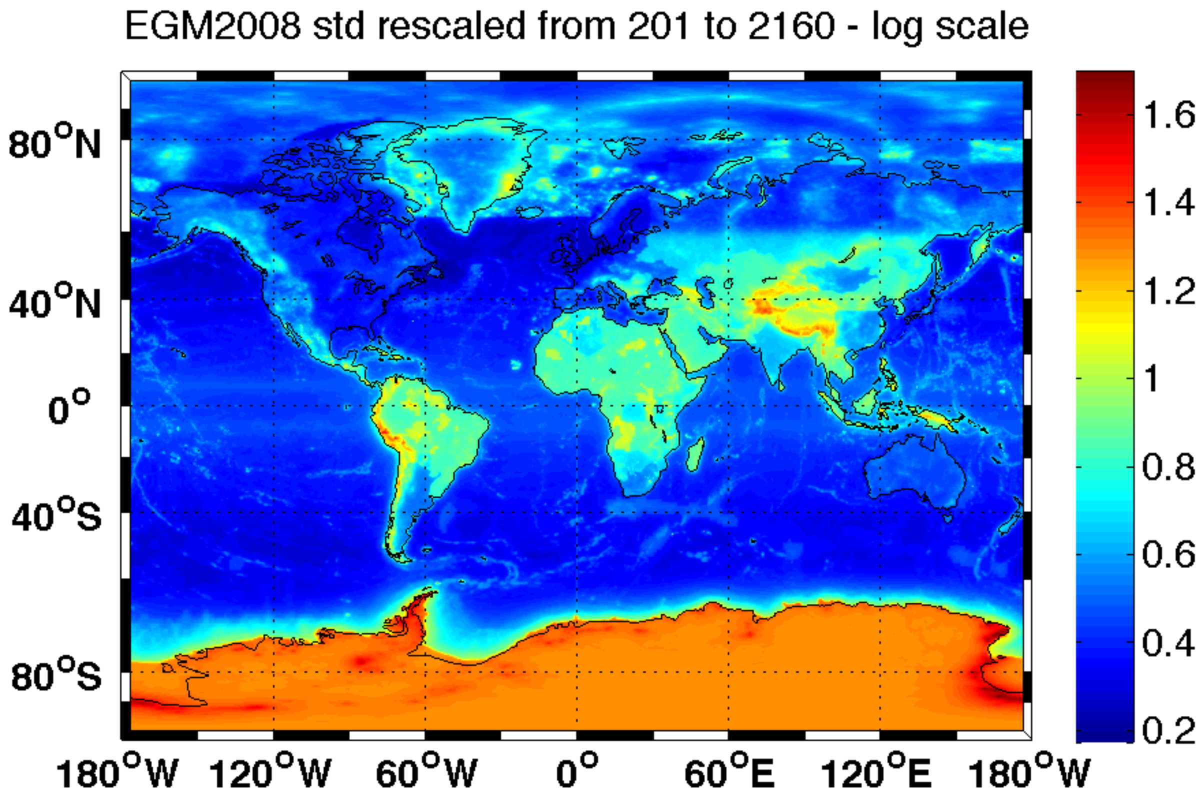
C_{T^L} and C_{T^H} from global model covariance information

The covariance structure of the T^L and T^H

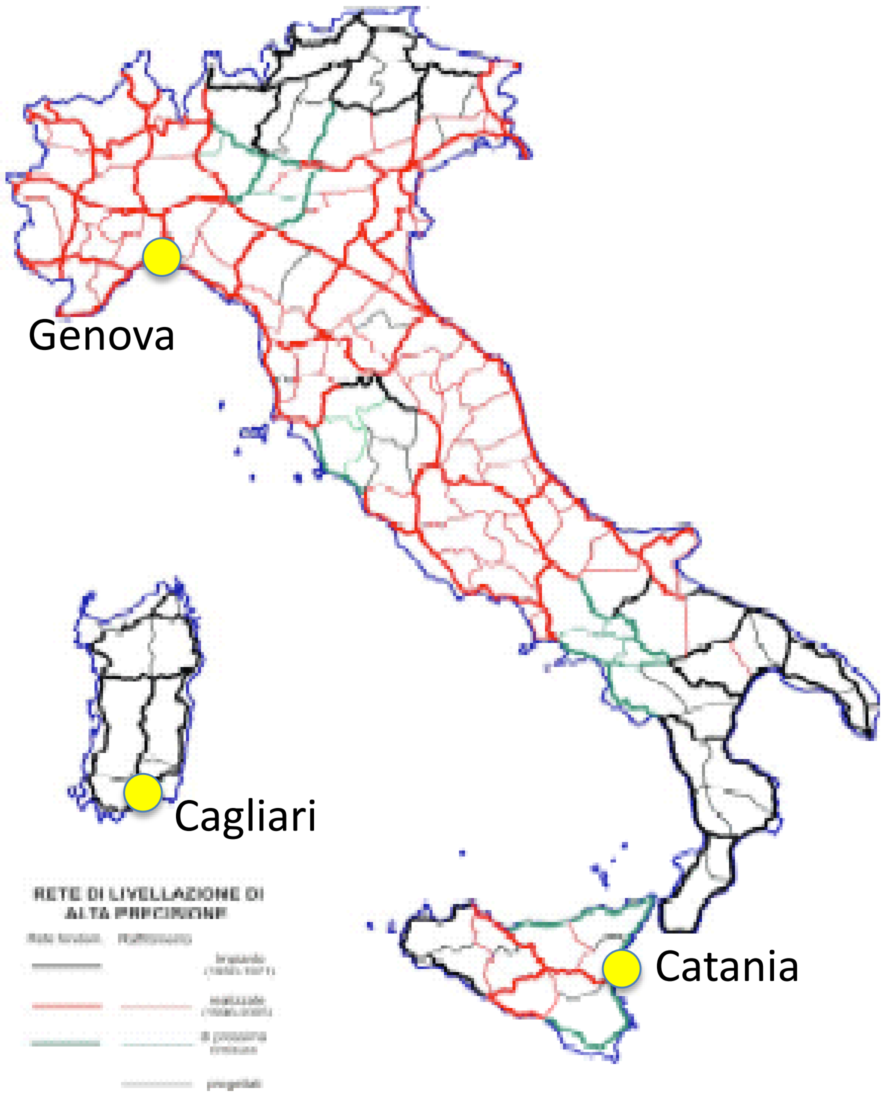
- Coviariance matrix of GOCE model (**T^L**) is computed following the GOCO model by propagating the block diagonal structure (Pail and Gerlach)

- The covariance matrix of **T^H** is computed using EGM2008 errors geographically rescaled

log (m)



A test computation for Italy



- Three main tide gauges

1) Genova (Peninsular Italy)

1) Catania (Sicily Island)

1) Cagliari (Sardinia Island)

3 biases to be estimated

The estimation problem setting

$$b^j = \tilde{z}_{GNSS/lev}(P_k^j) - \frac{T^L(P_k^j)}{g} - \frac{T^H(P_k^j)}{g} \quad k = 1,\dots,N \quad j = 1,2,3$$

- GPS/lev observations supplied by IGM: 977 in the mainland; 43 in Sicily Island; 48 in Sardinia Island

- $\boldsymbol{T^L}$ component estimated using the GOCO-03 based on ITG-Bonn GRACE and GOCE (TIM-R3) solutions

- $\boldsymbol{T^H}$ component estimated EGM2008 in the domain (L+1; 900) (error budget has been performed for different L values)

The bias estimation (L=250)

Computation has been done by setting to zero the mainland bias

$s_n(z)$ = 1 cm	Bias estimate [cm]	Bias st.dev. [cm]
Mainland	0	0.59
Sicily	9.48	2.75
Sardinia	-12.81	4.92

H_o: σ_0^2=1 rejected

$s_n(z)$ = 12 cm	Bias estimate [cm]	Bias st.dev. [cm]
Mainland	0	0.52
Sicily	9.82	2.57
Sardinia	-20.68	2.72

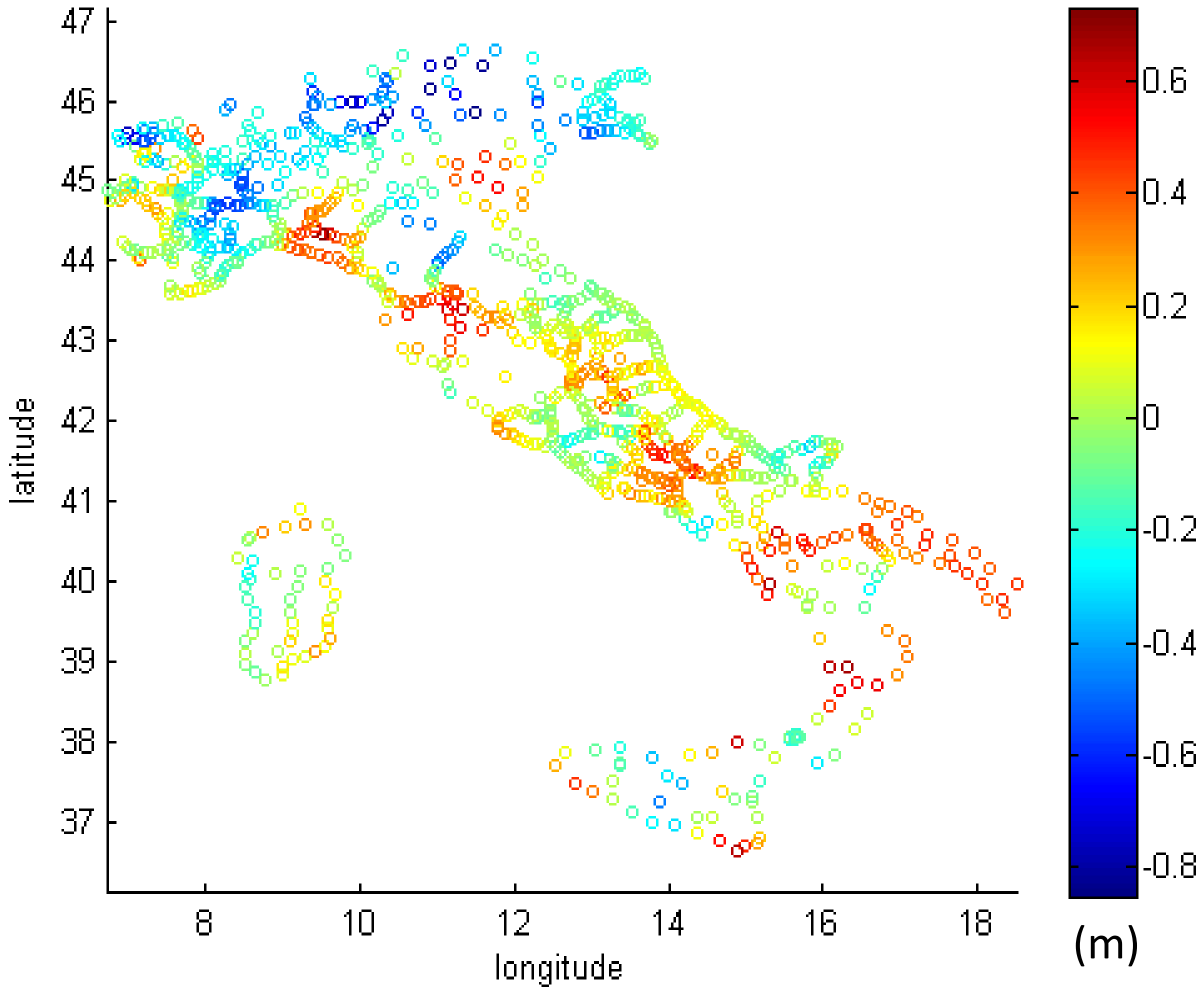
H_o: σ_0^2=1 accepted

t_{emp} = 4.59, $t_{0.975,1065}$=1.96 → H_o: b_{sicily} =0 rejected

t_{emp} = -4.73, $t_{0.975,1065}$=1.96 → H_o: $b_{sardinia}$=0 rejected

Remark: the bias Mainland-Sicily has been directly estimated equal to 14.1 cm by IGM

Least-squares residuals



Remarks and conclusions

- The devised method can give proper results based on the error propagation for global geopotential model that has been set up

- The estimated biases are in a reasonable agreement with the estimated DOT in the Mediterranean Sea and with the direct estimate of the bias Mainland-Sicily by IGM

- Further investigations are needed for explaining the existing tilt (?) that is present in the least squares residuals (improvements in the levelling data)