RESTRICTED MAXIMUM LIKELIHOOD ESTIMATION OF SPATIAL COVARIANCE PARAMETERS OF GEODETIC DATA

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Least-squares collocation (prediction, interpolation…?) – one kind of data

LSC (interpolation)

$$\hat{\mathbf{z}}_r = \mathbf{C}_P^{\mathrm{T}} \left(\mathbf{C} + \mathbf{N} \right)^{-1} \mathbf{z}_r$$

Gauss-Markov
3rd order model.
Spherical distance ψ used

$$GM3(C_0, CL, \psi) = C_0 \left(1 + \frac{\psi}{CL} + \frac{\psi^2}{3 \cdot CL^2} \right) \cdot \exp\left(\frac{-\psi}{CL} \right)$$

Homogeneous noise
(except last example)

$$\mathbf{D} = \delta n^2 \cdot \mathbf{I}_n$$

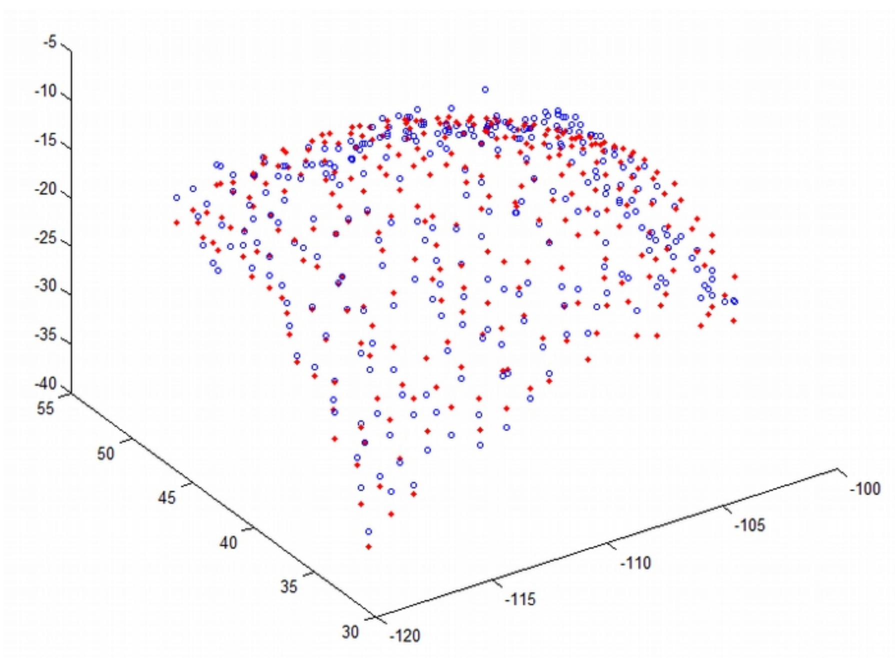
Lower order trend
- Projection matrix

$$\mathbf{P} = \mathbf{I}_n - \mathbf{X} (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}}$$

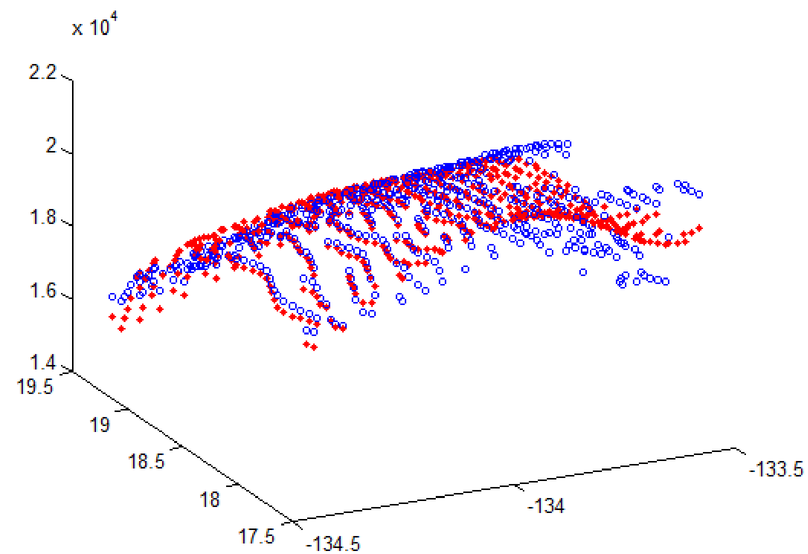
$$\mathbf{X} = \begin{bmatrix} 1 & \varphi_1 & \lambda_1 & \varphi_1{}^2 & \lambda_1{}^2 & \varphi_1 \lambda_1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \varphi_n & \lambda_n & \varphi_n{}^2 & \lambda_n{}^2 & \varphi_n \lambda_n \end{bmatrix}$$

Detrending of the data

Second order

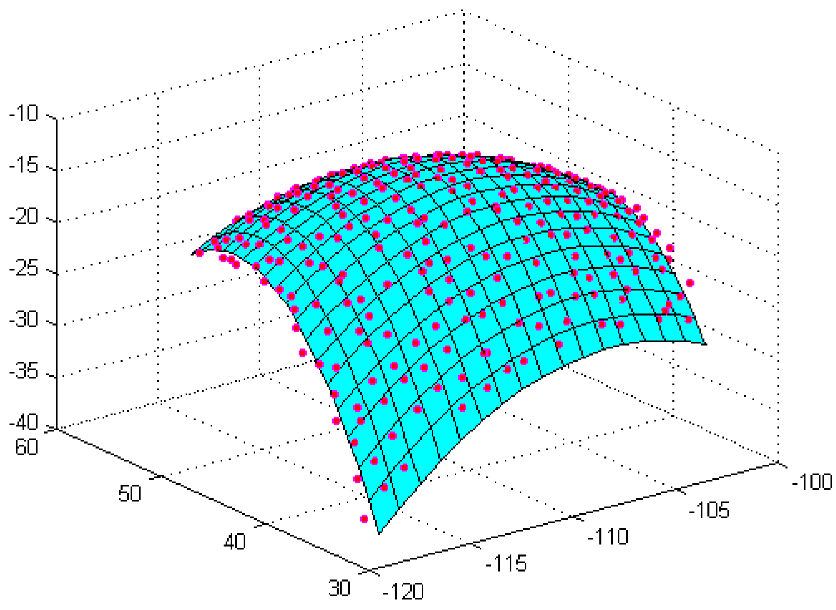


Third order

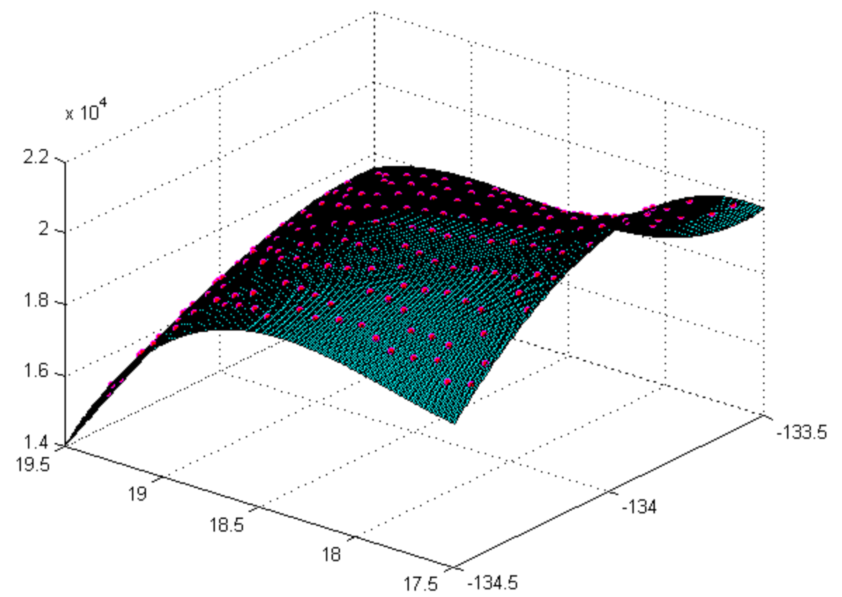


Analytical trend from points, computed e.g. in grid nodes

Second order



Third order



Restricted Maximum Likelihood (REML)

Distribution function

$$p(\mathbf{z},\boldsymbol{\theta}) = \left|\mathbf{C}(\boldsymbol{\theta})\right|^{-1/2} \left|\mathbf{X}^{\mathrm{T}}\mathbf{C}(\boldsymbol{\theta})^{-1}\mathbf{X}\right|^{-1/2} \exp\left(-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\mathbf{R}(\boldsymbol{\theta})\mathbf{z}\right)$$

Negative log-likelihood

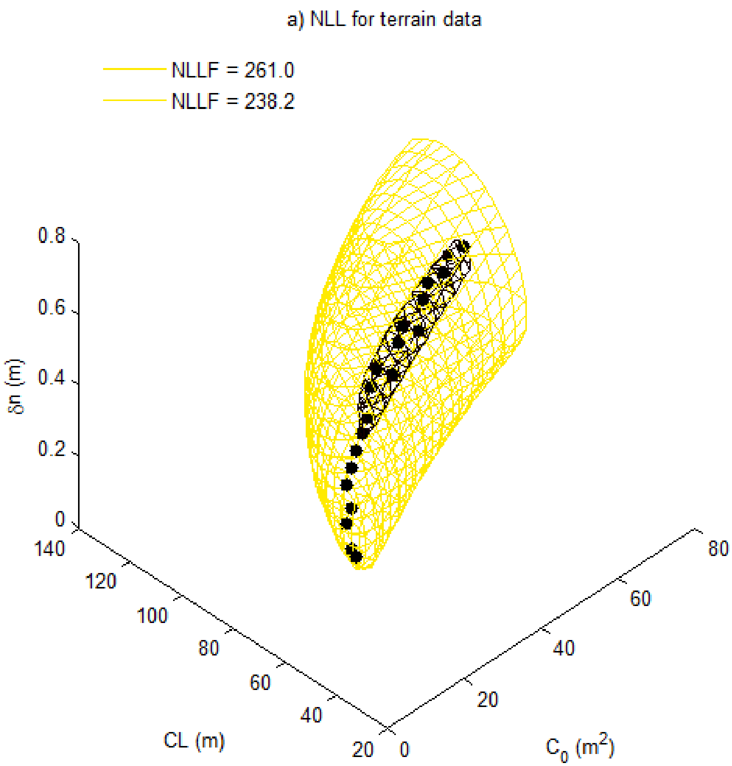
$$NLLF(\mathbf{z},\boldsymbol{\theta}) = \frac{1}{2}\ln\left|\mathbf{C}(\boldsymbol{\theta})\right| + \frac{1}{2}\ln\left|\mathbf{X}^{\mathrm{T}}\mathbf{C}(\boldsymbol{\theta})^{-1}\mathbf{X}\right| + \frac{1}{2}\left(\mathbf{z}^{\mathrm{T}}\mathbf{R}(\boldsymbol{\theta})\mathbf{z}\right)$$

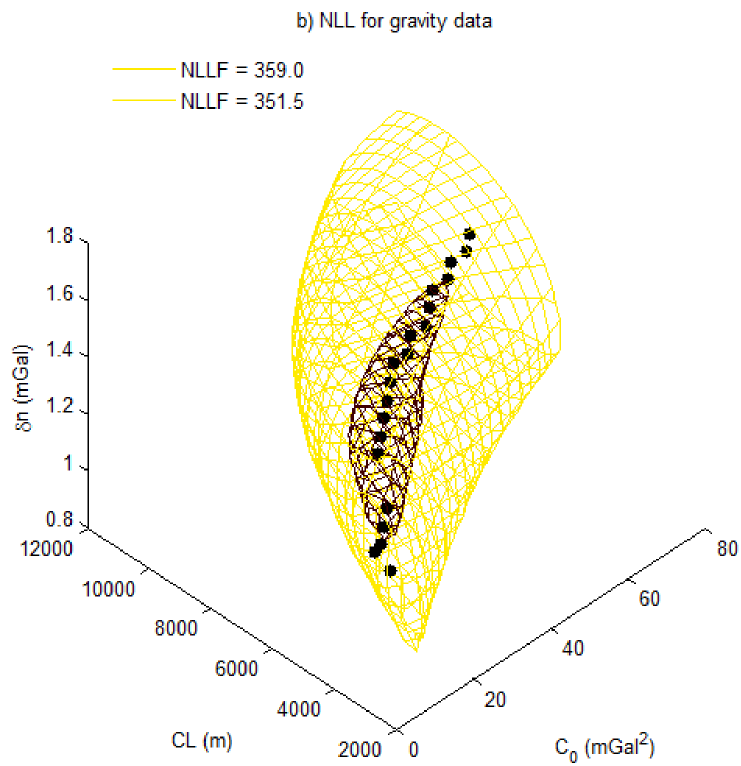
R matrix (including projection)

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{C}(\boldsymbol{\theta})^{-1}\mathbf{P} = \mathbf{C}(\boldsymbol{\theta})^{-1}\left[\mathbf{I} - \mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{C}(\boldsymbol{\theta})^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{C}(\boldsymbol{\theta})^{-1}\right]$$

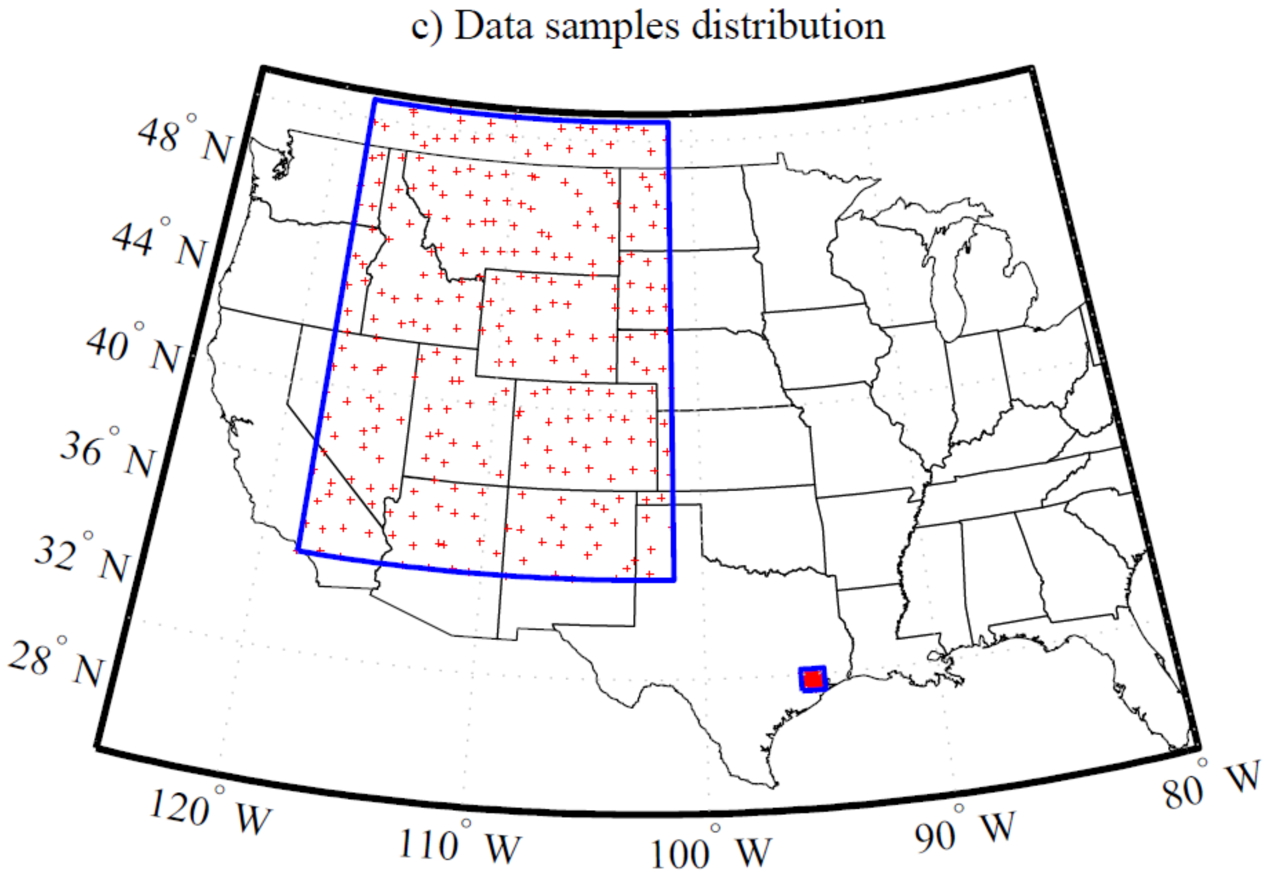
Additionally crosss-validation.
Here - leave-one-out (LOO) validation.
Std dev. of differences between the estimates and point data.

REML in space of three parameters: Signal variance (C_0), noise variance (δn), correlation length of the model (CL)





Example of GNSS/leveling geoid data



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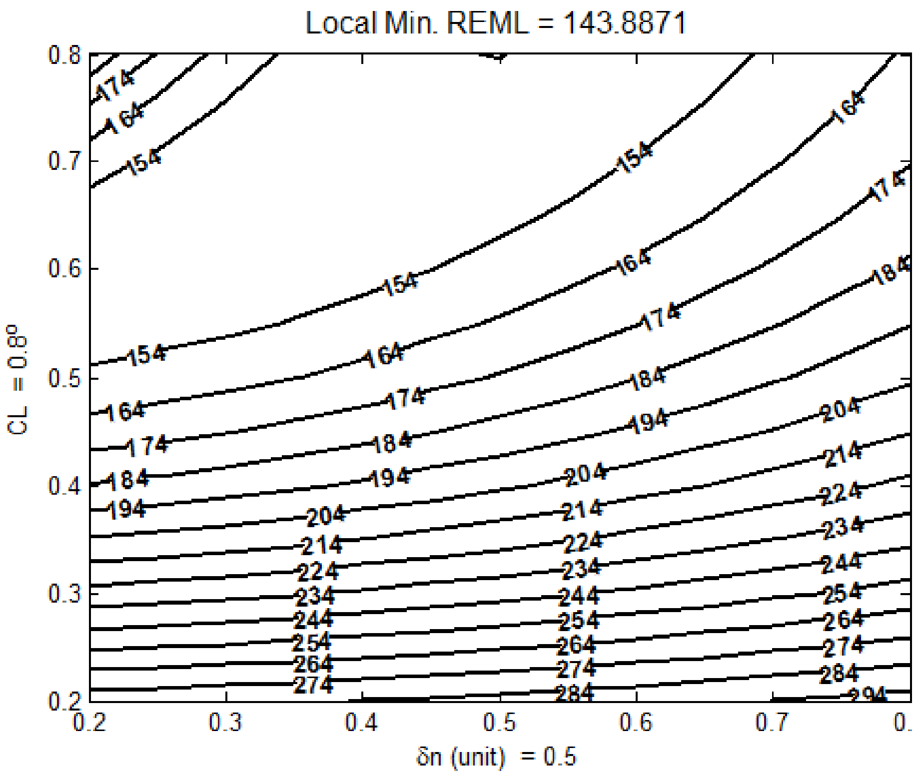
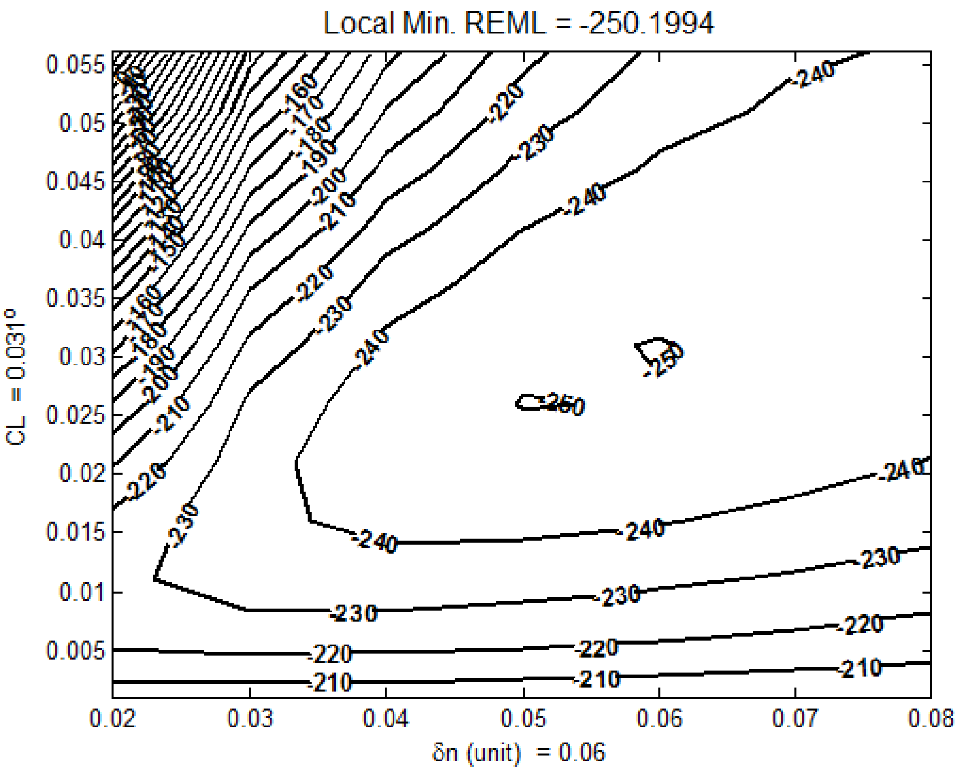
REML. NLLF minima for GNSS/lev. geoid

Western USA



Houston



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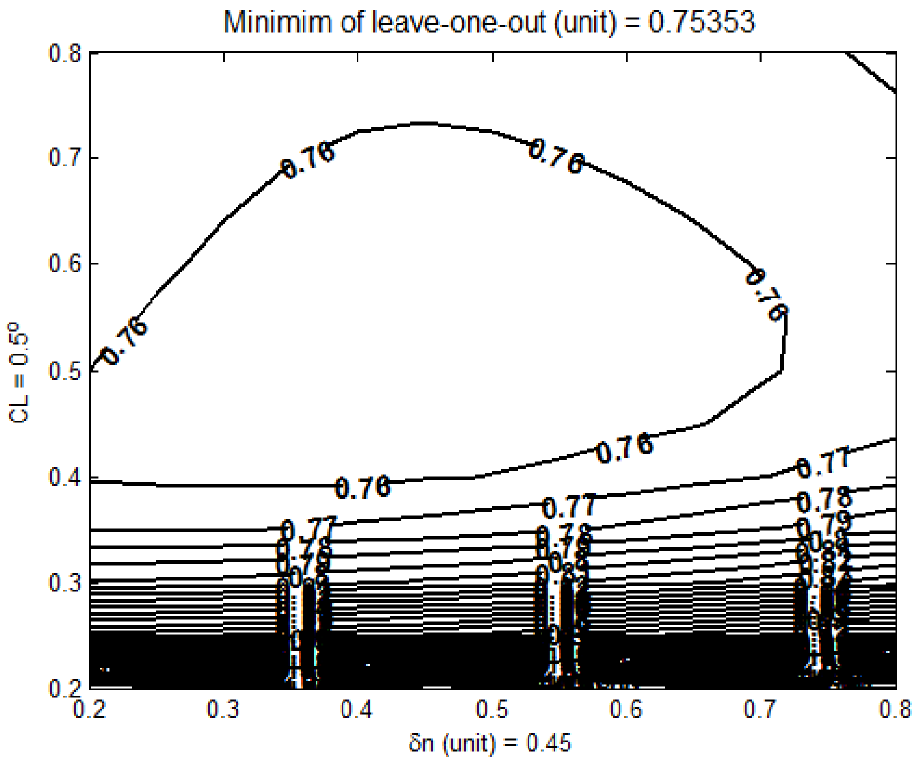
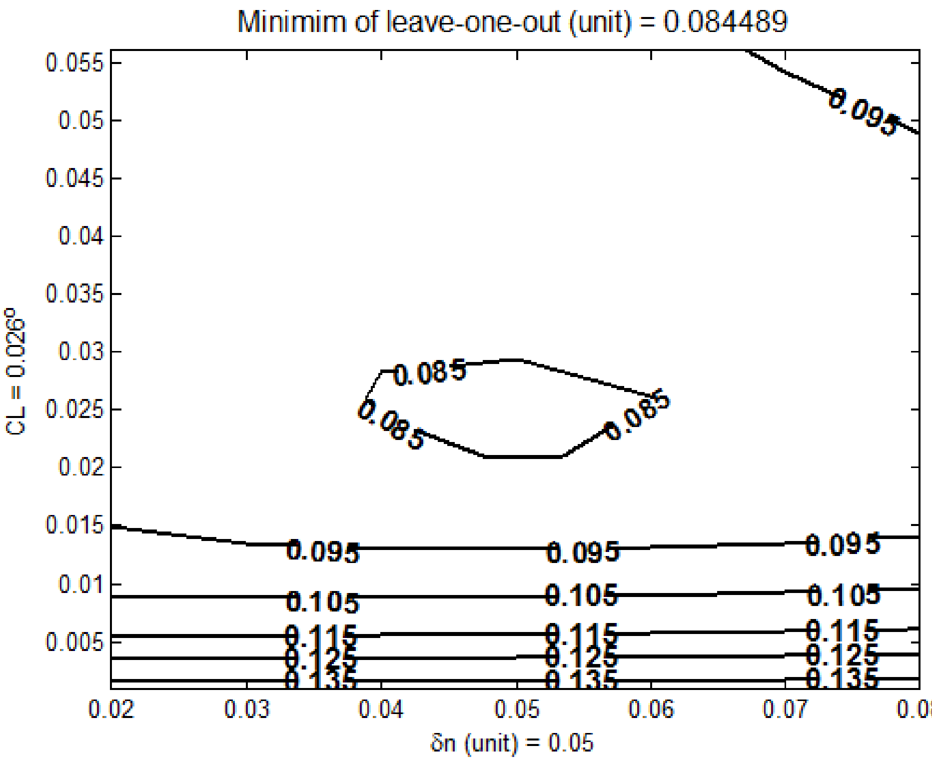
LOO. Std dev. of differences for GNSS/lev. geoid

Western USA



Houston



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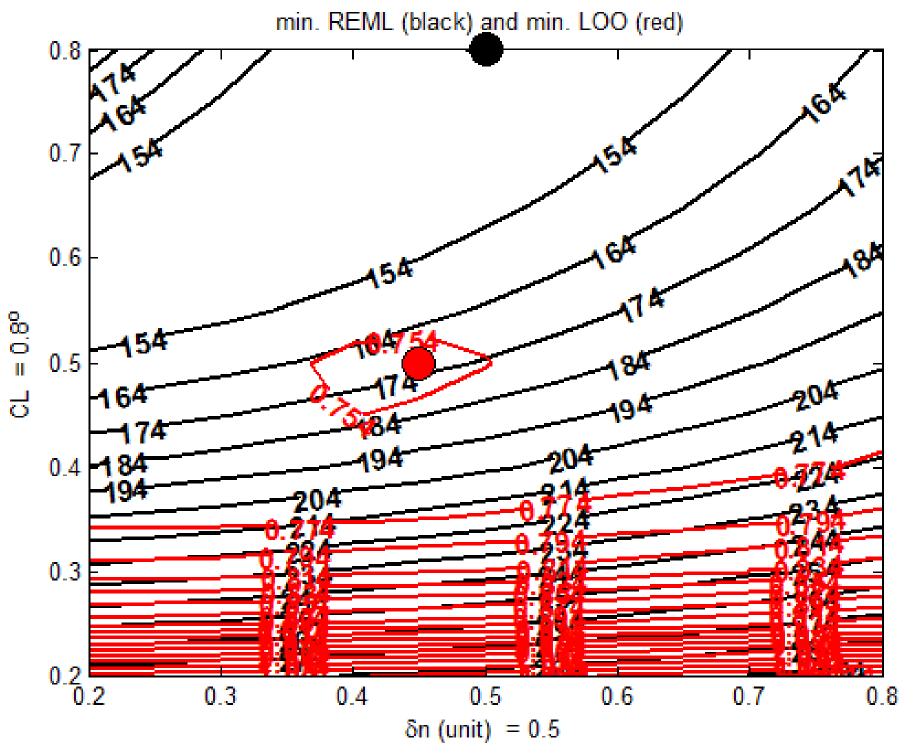
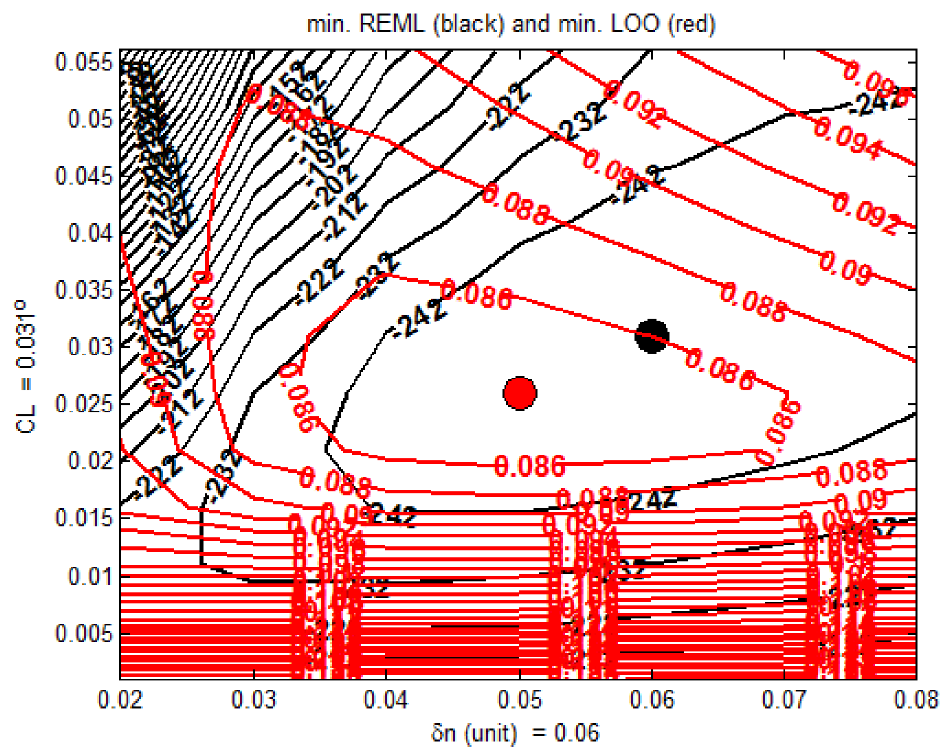
REML compared to LOO (GNSS/lev.)

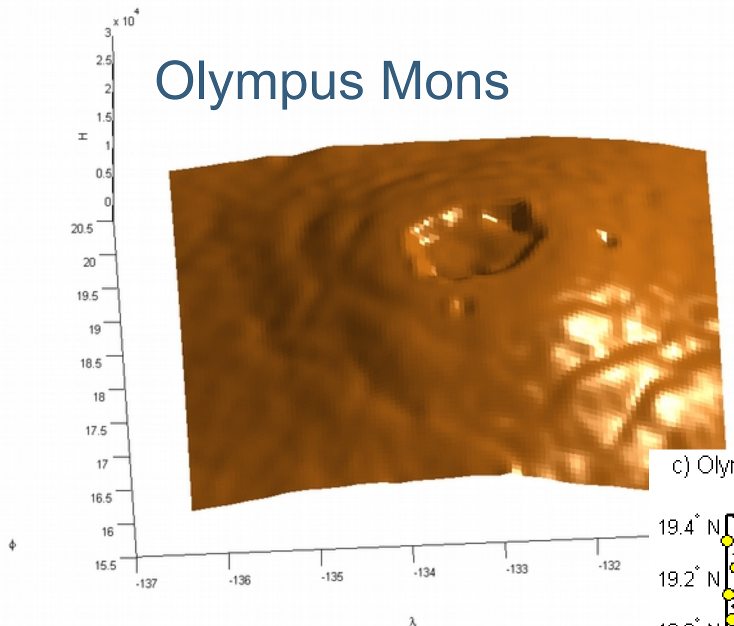
Western USA

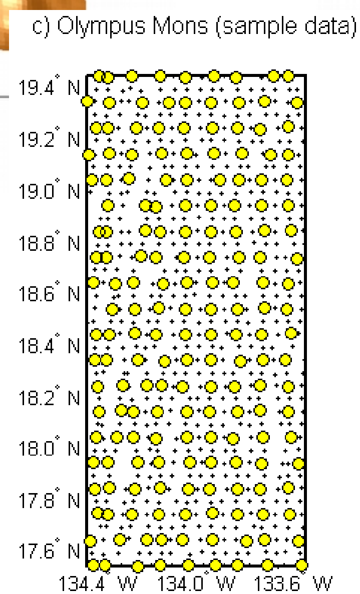


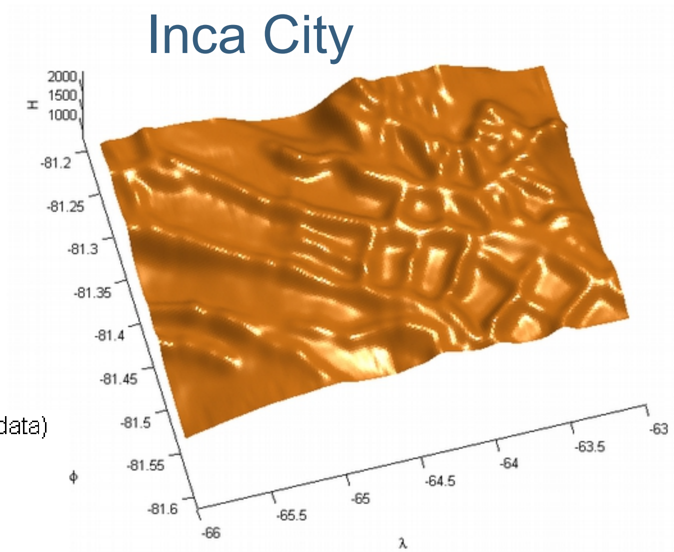
Houston

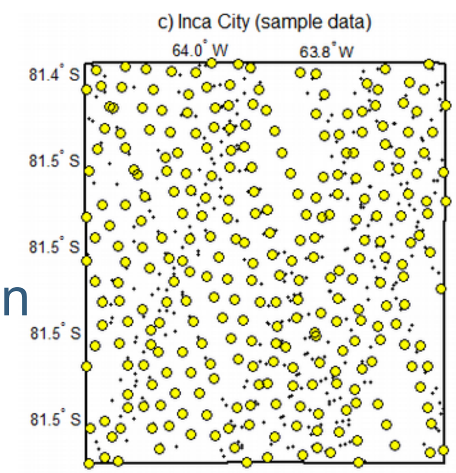


MOLA topography example.
(Mars Orbiter Lunar Altimeter)







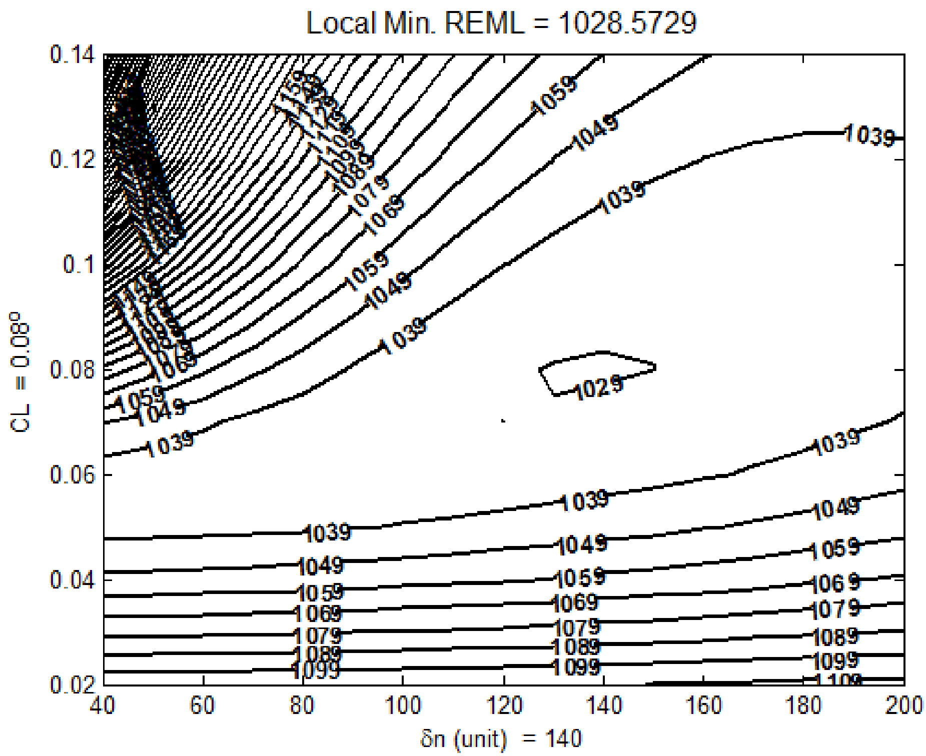


Sample is used
(appx. Resolution 0.1°)

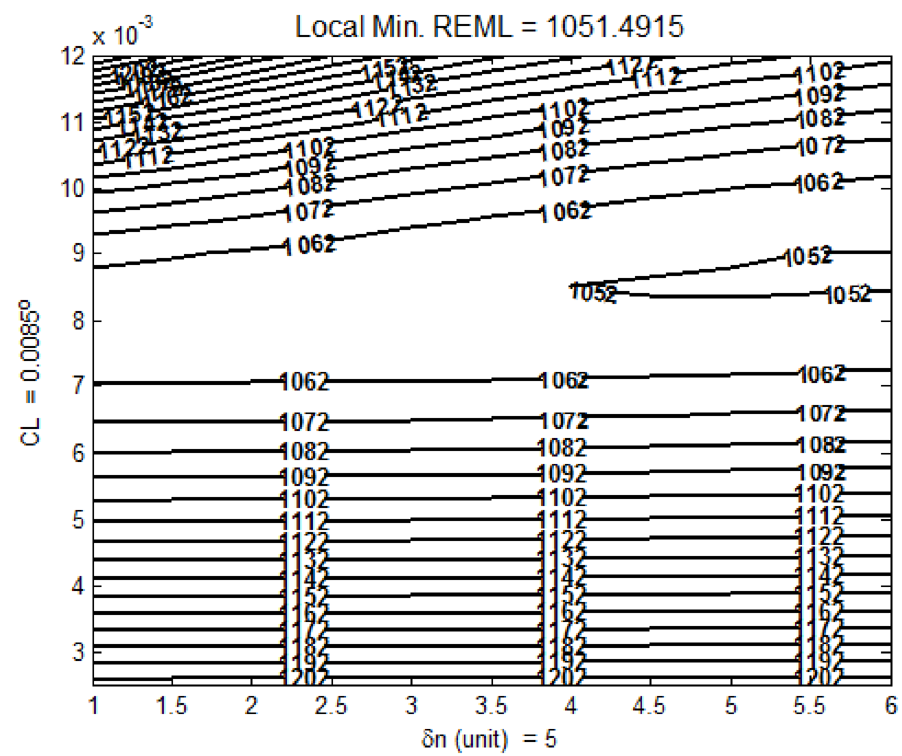
Sample is used
(appx. Resolution
0.005°)

REML. NLLF minima for MOLA altimetry

Olympus Mons



Inca City



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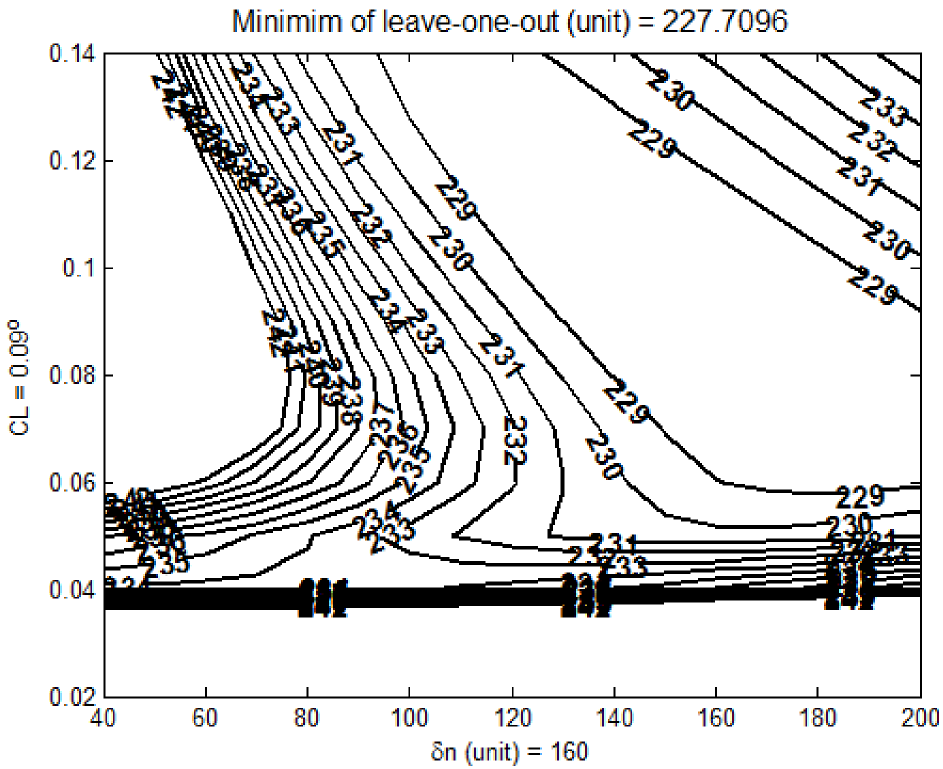
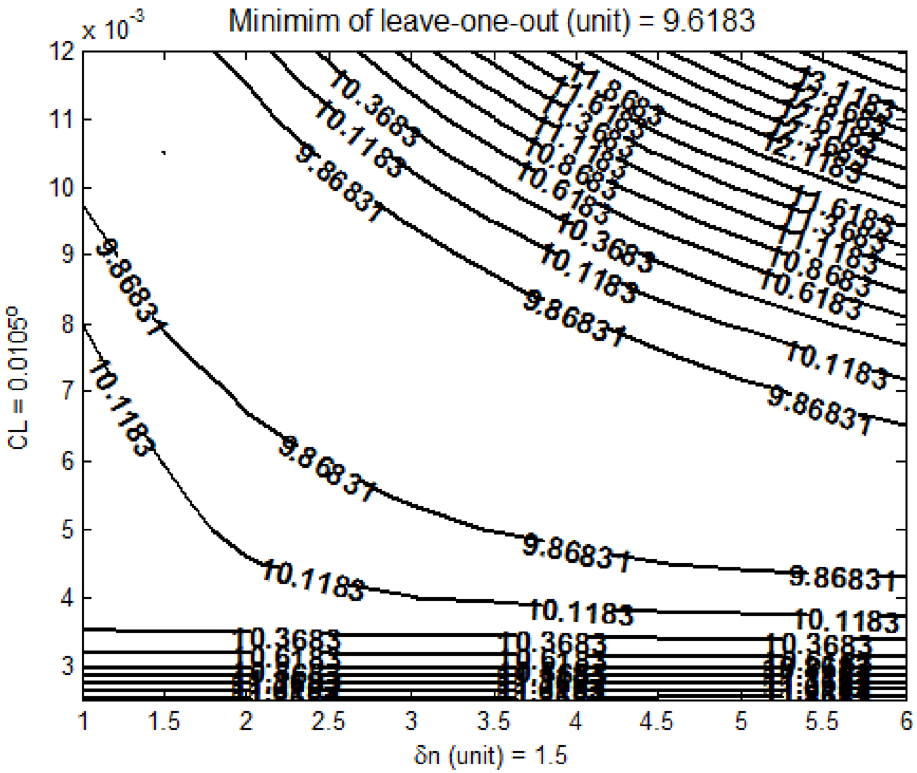
LOO. Std dev. of differences for MOLA altimetry

Olympus Mons

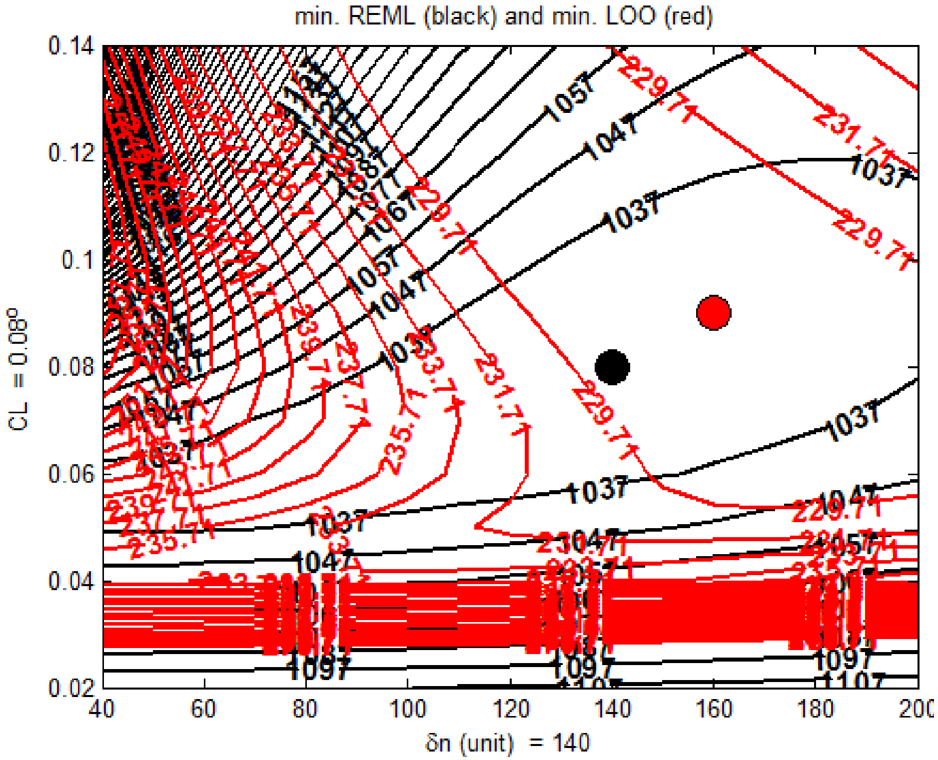


Inca City

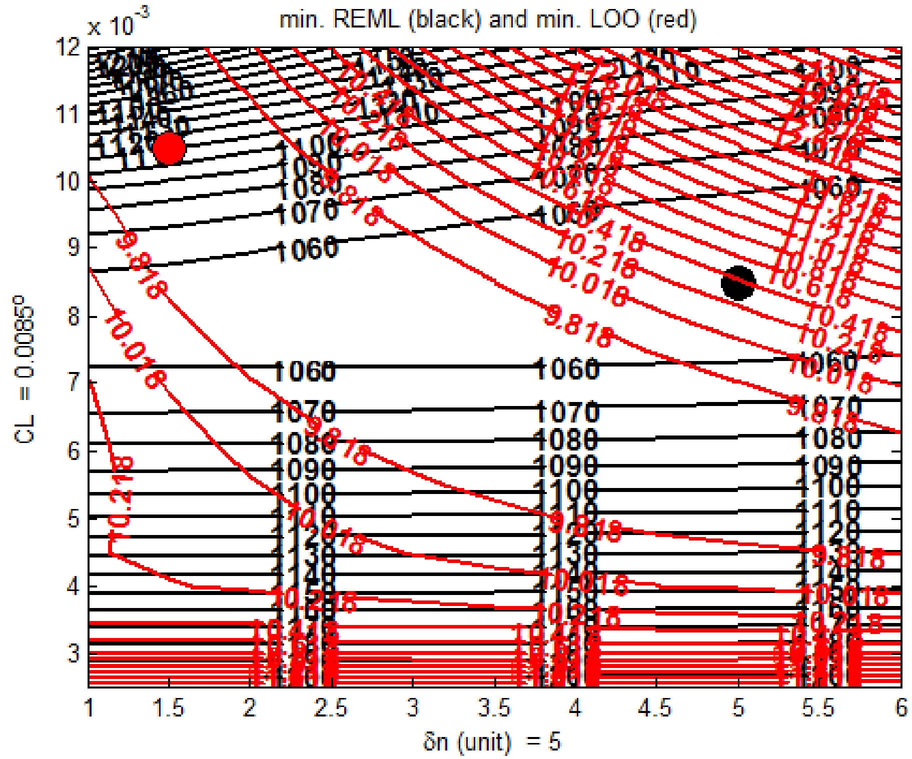


REML compared to LOO (MOLA)

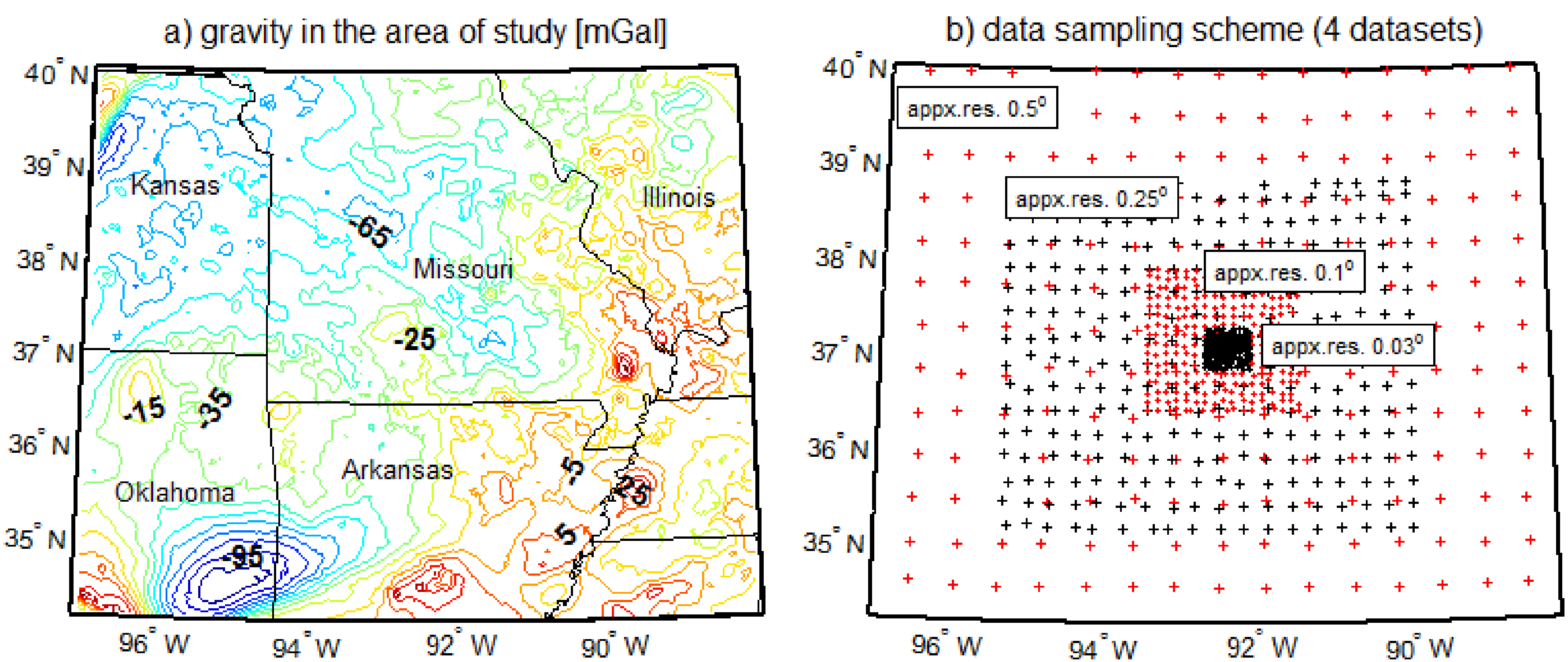
Olympus Mons



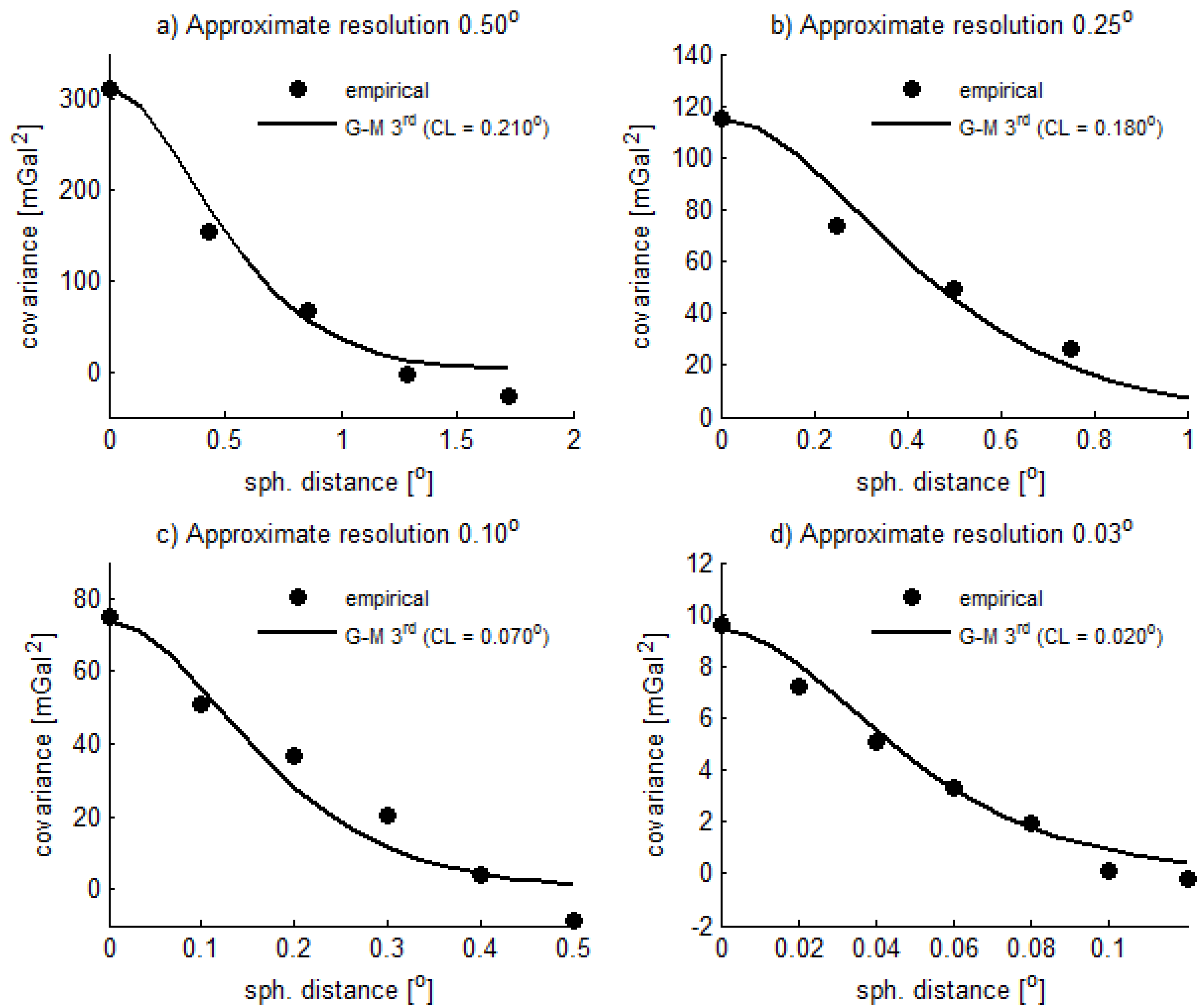
Inca City



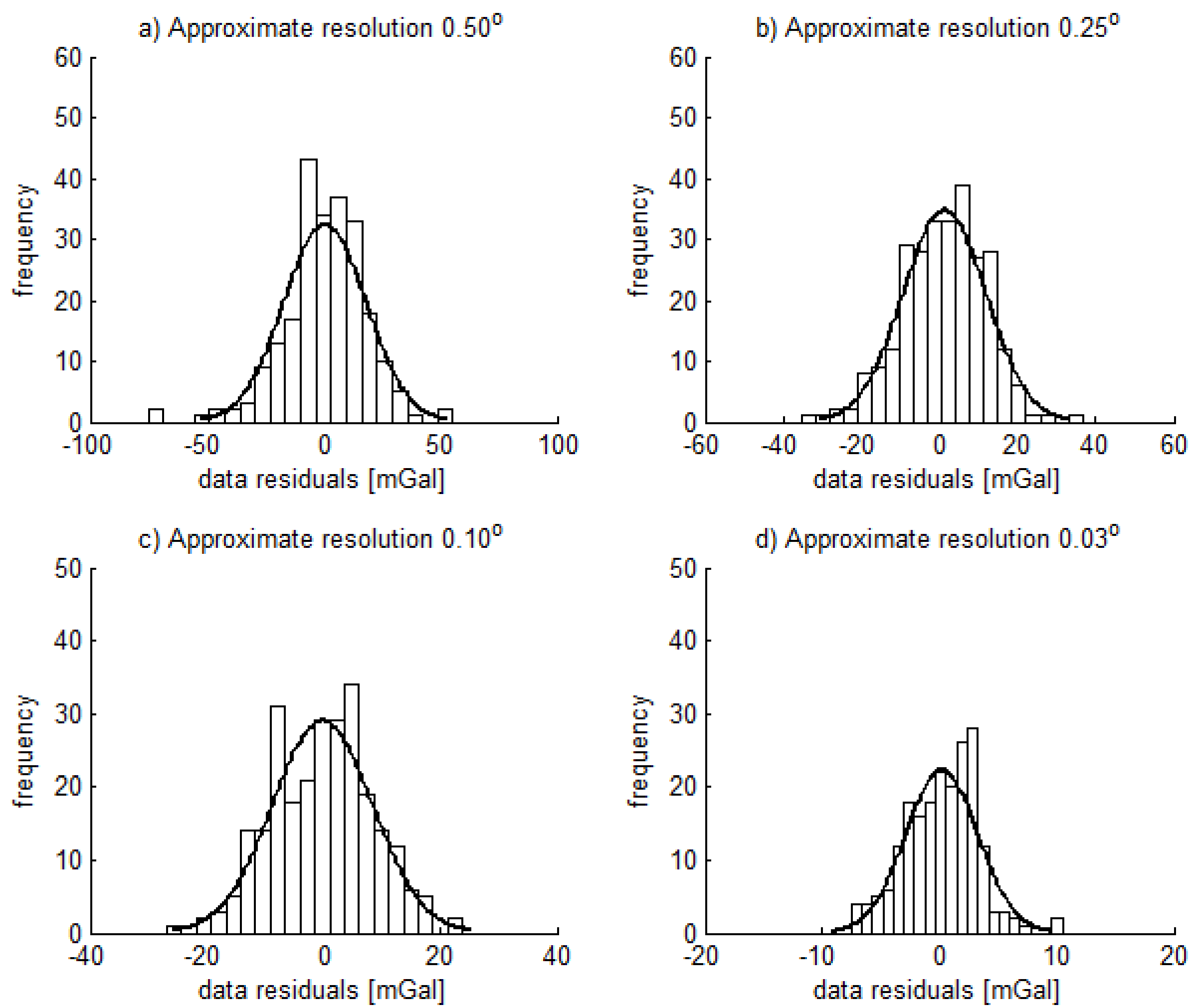
Bouguer gravity sampled with the use of different resolutions



Fit of the covariance function

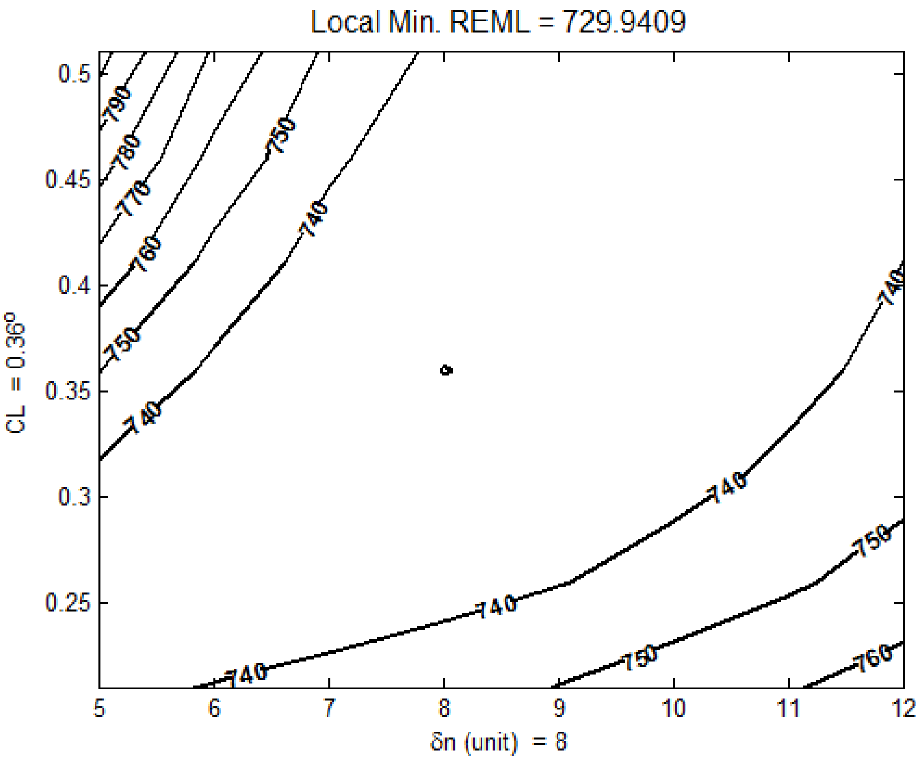


Histograms

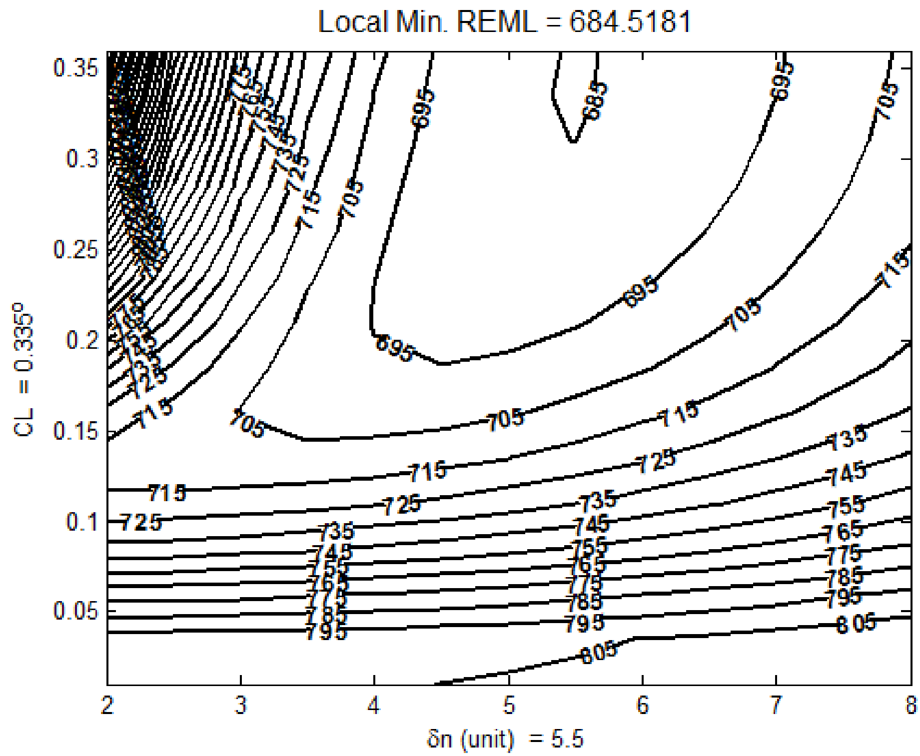

a) Approximate resolution 0.50°
b) Approximate resolution 0.25°
c) Approximate resolution 0.10°
d) Approximate resolution 0.03°
frequency
data residuals [mGal]
60
50
40
30
20
10
0
-100
-50
0
50
100
-60
-40
-20
20
40
60
-40
-10
10

REML. NLLF minima for gravity anomalies

Δg_B 0.5˚

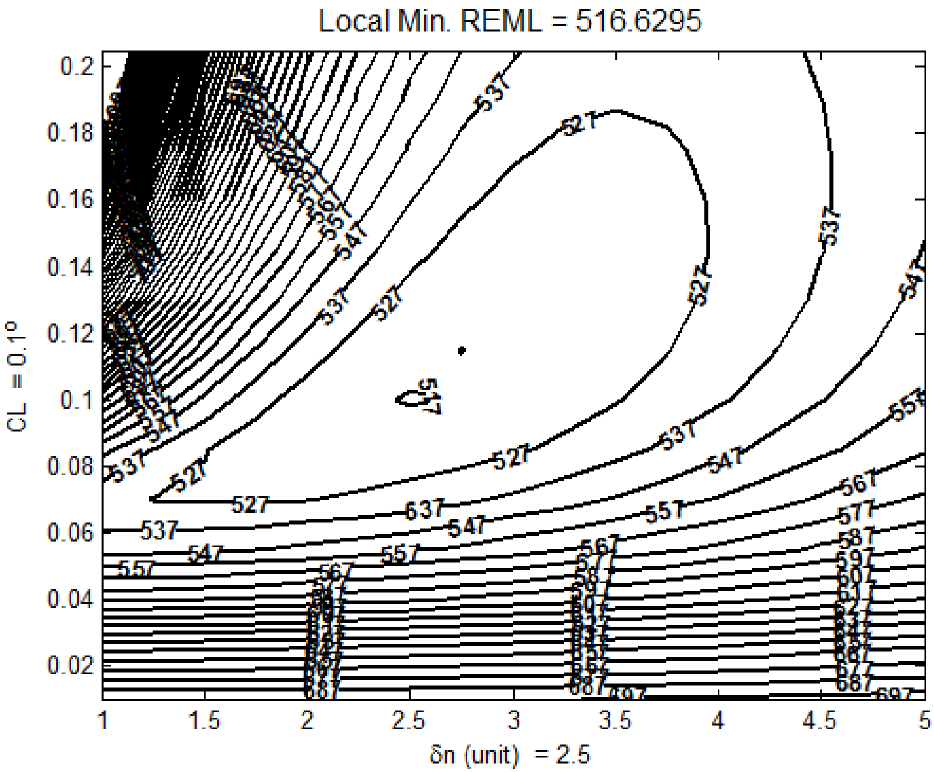


Δg_B 0.25˚

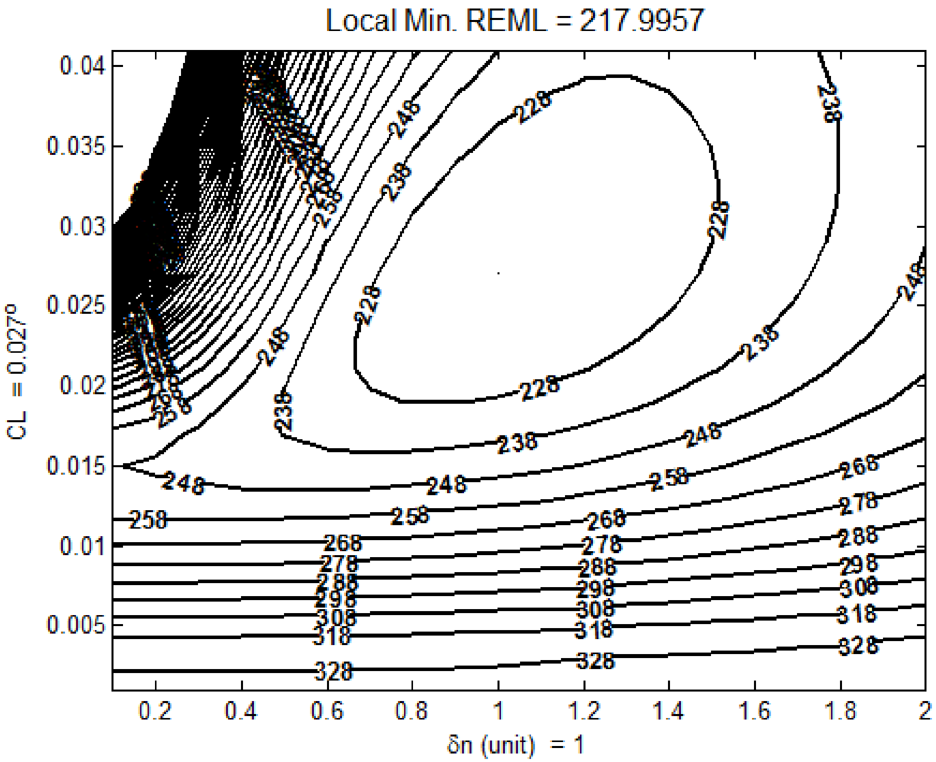


REML. NLLF minima for gravity anomalies

Δg_B 0.1˚

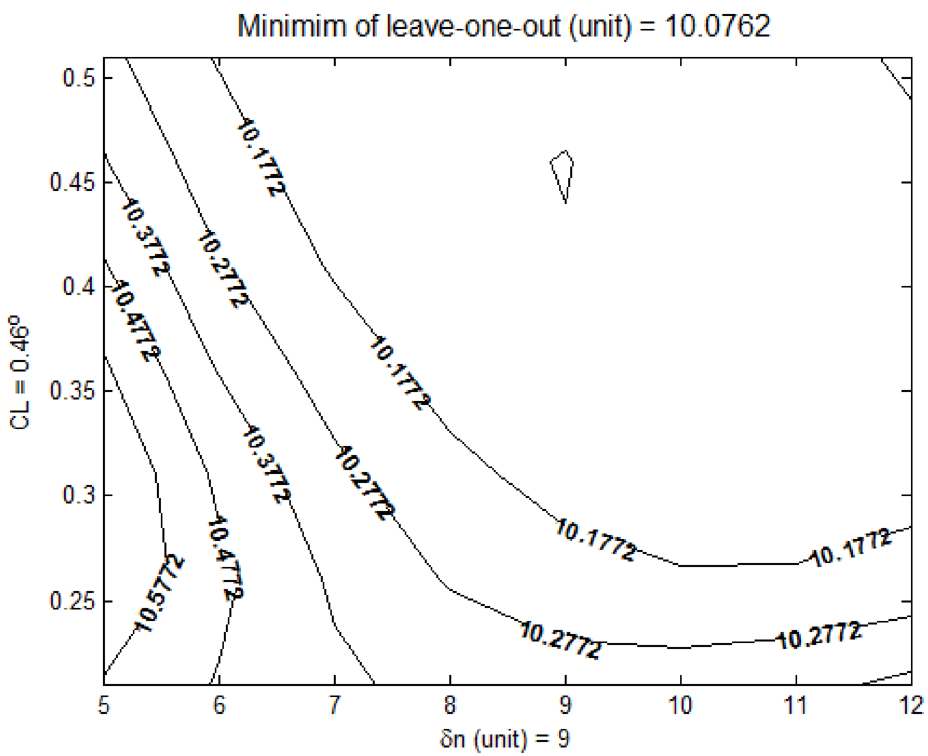


Δg_B 0.03˚

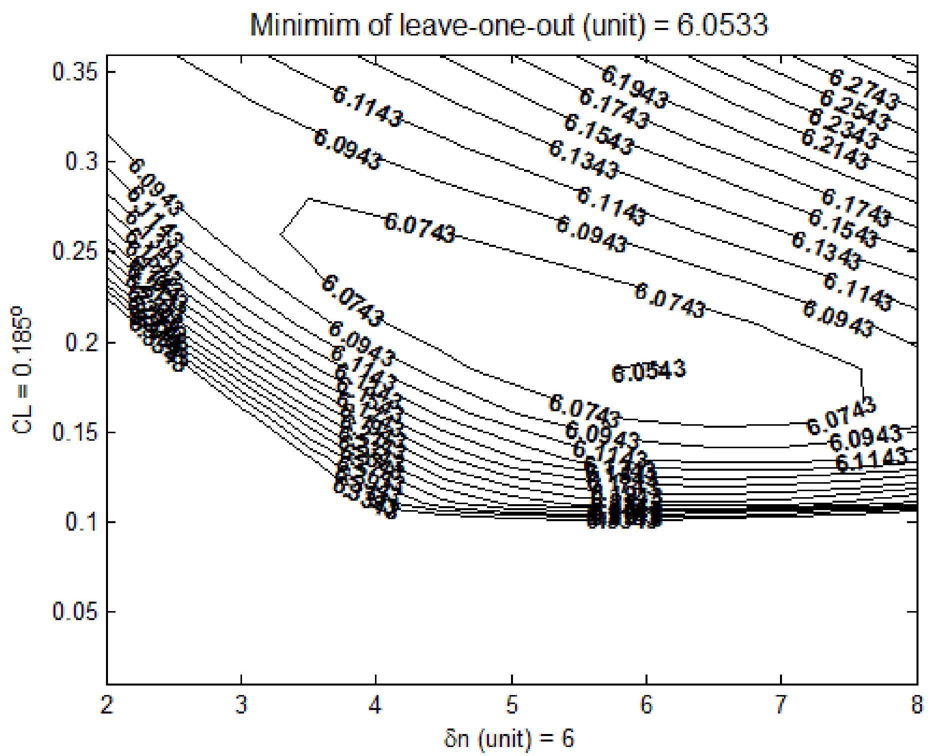


LOO. Std dev. of differences for gravity anomalies

Δg_B 0.5°



Δg_B 0.25°



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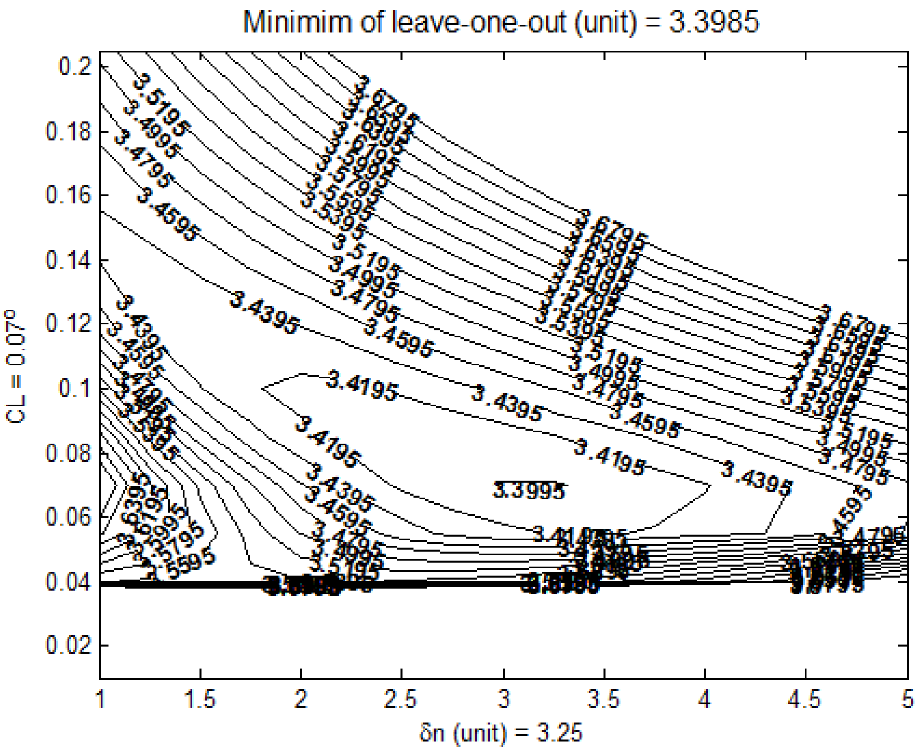
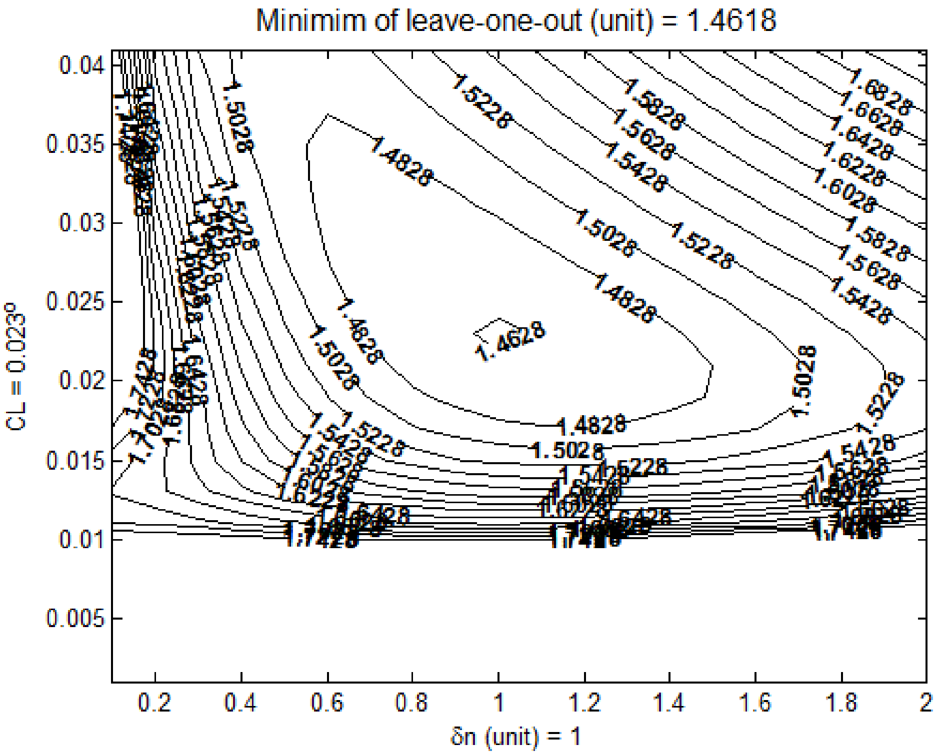
LOO. Std dev. of differences for gravity anomalies

Δg_B 0.1˚

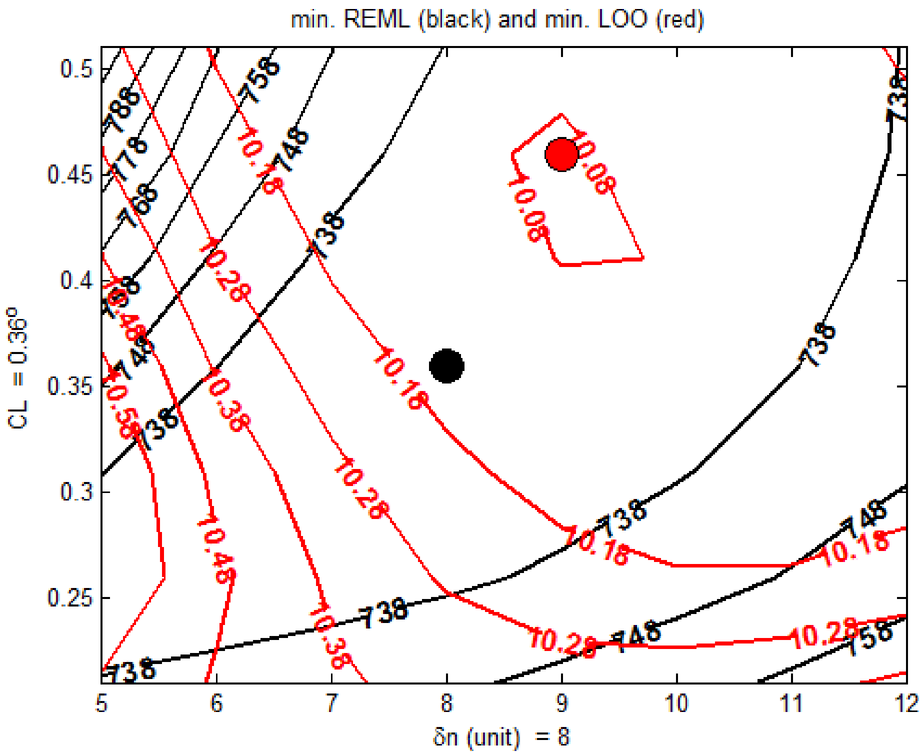


Δg_B 0.03˚

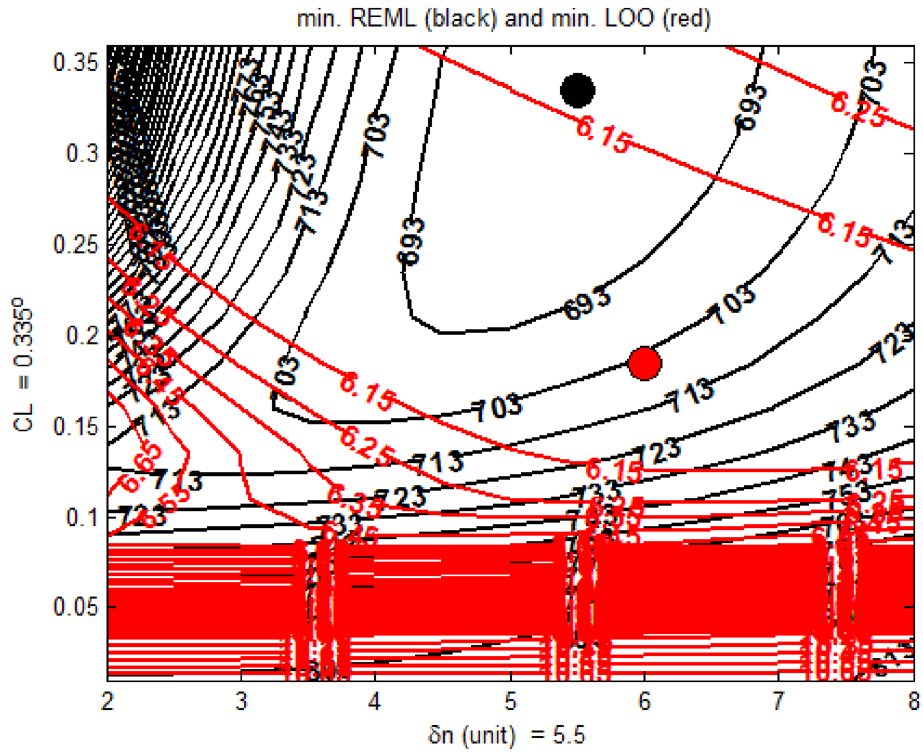


REML compared to LOO (gravity anomalies)

Δg_B 0.5˚

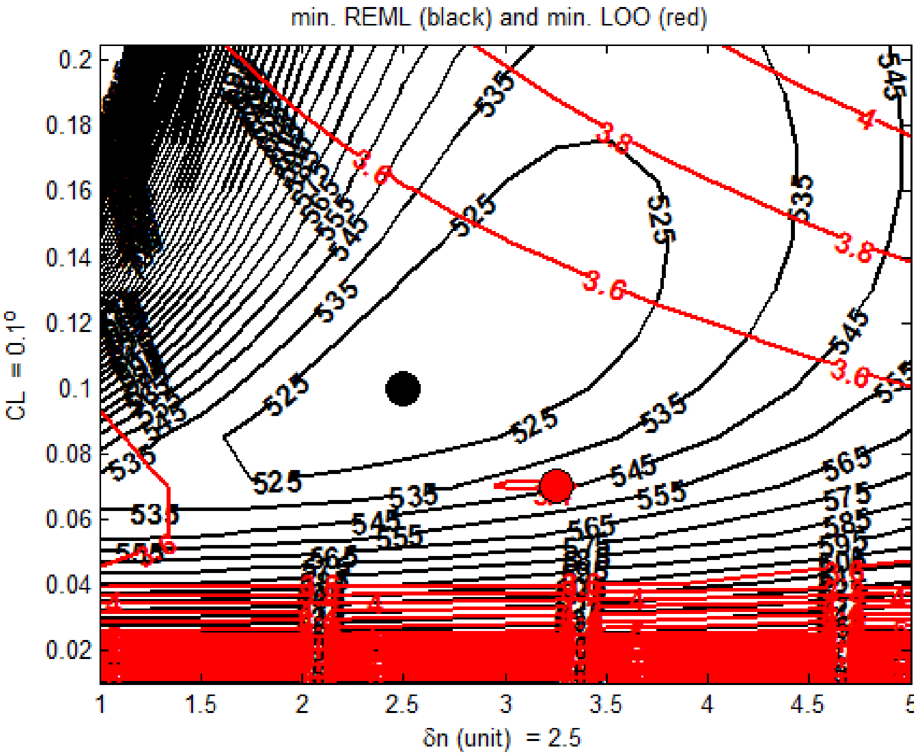


Δg_B 0.25˚

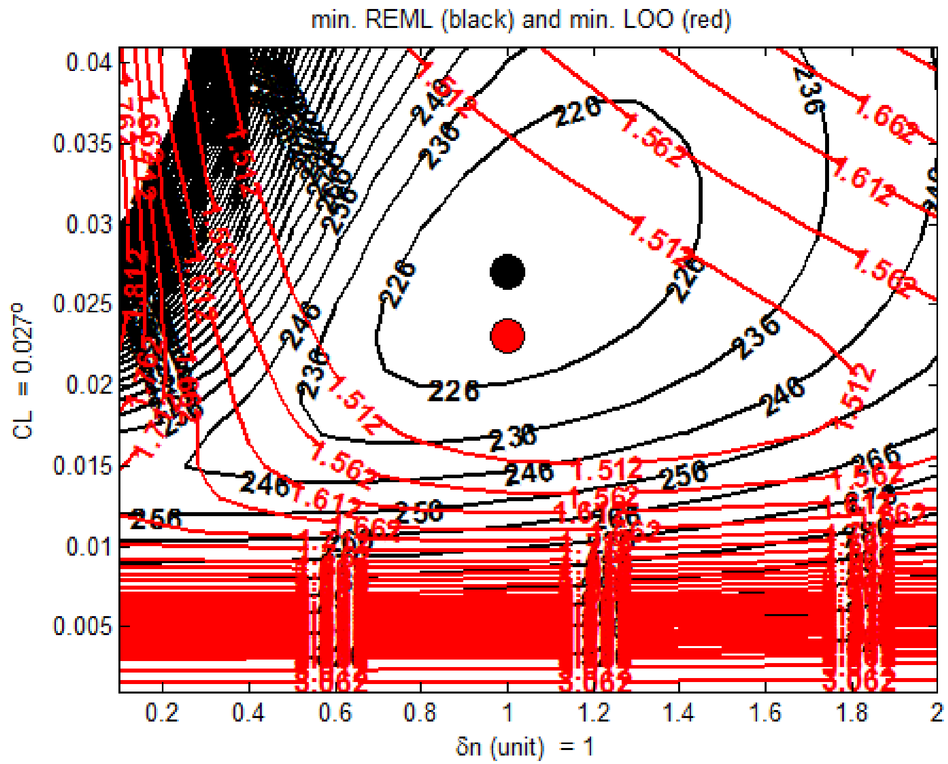


REML compared to LOO (gravity anomalies)

Δg_B 0.1˚



Δg_B 0.03˚



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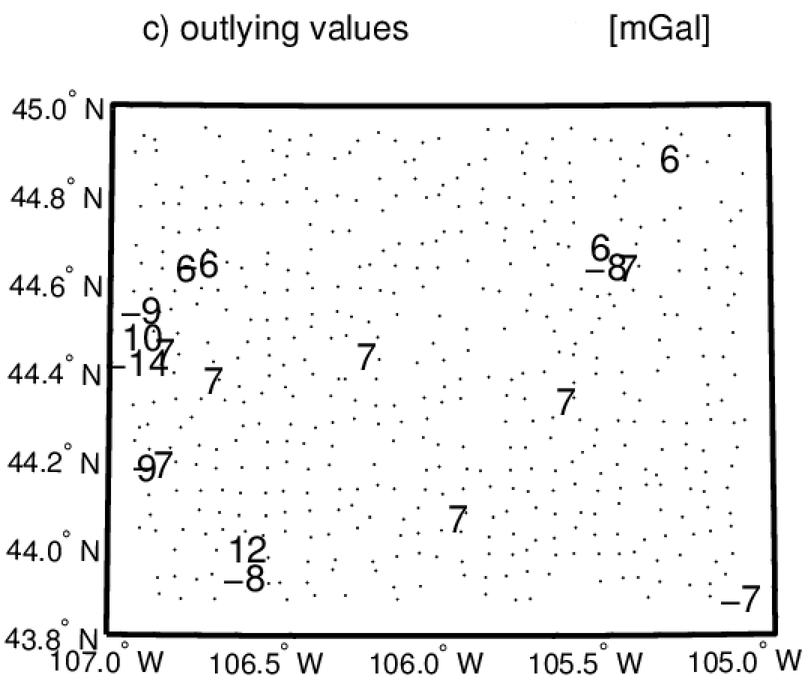
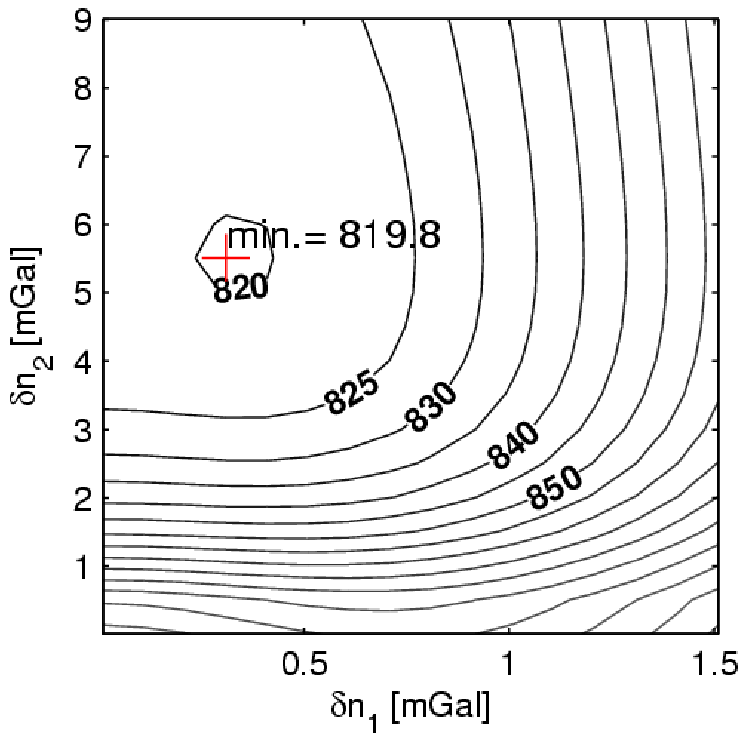
REML estimation of a priori noise (δn_i) for two subsets (C_0 and CL are fixed)

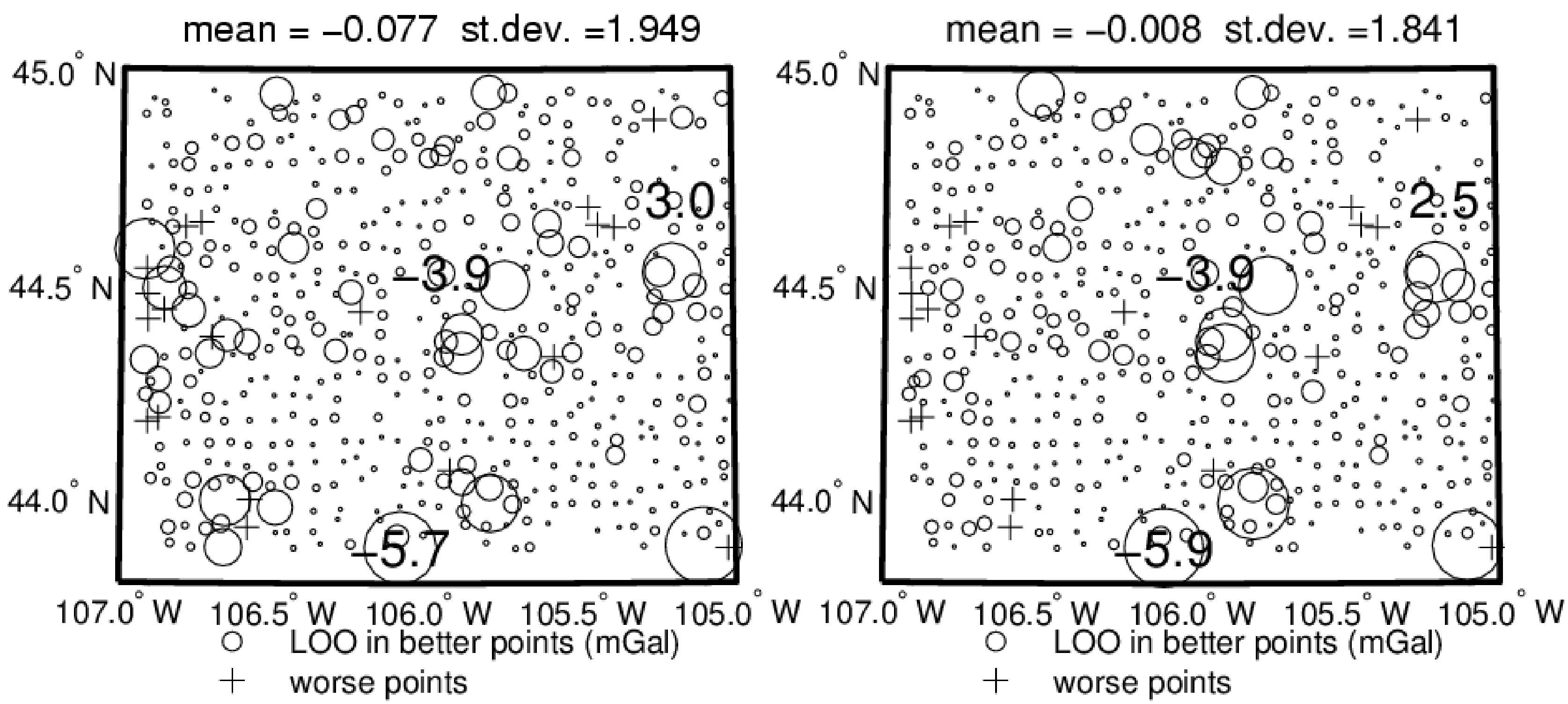
Data are split using cross-validation (LOO) values



REML for different
a priori noise standard deviations



… and validation afterwards (also LOO)



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Conclusions

LOO validation confirms REML estimates

REML estimation of CL can differ in relation to CL from the covariance function fitting

REML minimum indicating noise standard deviation varies with the change of resolution
(different highest frequency – different contribution of uncorrelated part?).