# A combined geoid model for Hungary based on the spectral properties of gravity related data

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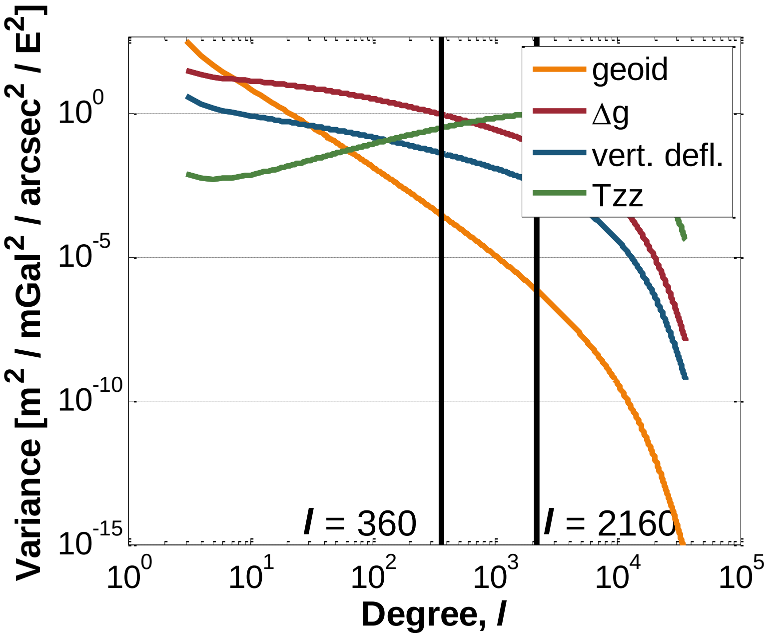
EUREF2013 Symposium
29-31, May2013 Budapest

# Spectral sensitivity of gravity field quantities




Tscherning- Rapp model

$$\sigma_l^2(\Delta g) = \frac{A\,(l-1)}{(l-2)(l+B)}\,\sigma_0^{l+2}$$

$$A = 425.28\ \mathrm{mGal}^2$$

$$B = 24$$

$$\sigma_0 = 0.999617$$

| | 3 - 36 | 37 - 360 | 361-2160 | 2161-36000 |
|---|---|---|---|---|
| N | 99.2 % | 0.8 % | 0.0 % | 0.0 % |
| $\Delta g$ | 22.5 % | 42 % | 28.6 % | 6.9 % |
| vertical defl. | 29.5 % | 38.6 % | 25.6 % | 6.3 % |
| $T_{zz}$ | 0.0 % | 0.8 % | 19.1 % | 80.1 % |

# Spectral combination

$$T(r,\phi,\lambda) = T^M(r,\phi,\lambda) + T^G(r,\phi,\lambda) + T^E(r,\phi,\lambda)$$

GGM

EGM2008

gravity anomaly

horizontal grav. gradients

optimal combination in the spectral domain

n-th degree surface harmonics of *T*

$$T_n = w_n^M \; T_n^M + w_n^G \; T_n^G + w_n^E \; T_n^E \qquad w_n^M + w_n^G + w_n^E = 1, \qquad \forall n$$

$$T^{M,G,E} = \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} w_n^{M,G,E} \; T_n^{M,G,E}$$

# Spectral combination

**$T^M$ from GGM**

$$T^M(r,\phi,\lambda) = \sum_{n=2}^{n_{max}} w_n^M \ T_n^M$$

**$T^G$ from gravity anomalies**

$$T^G(r,\phi,\lambda) = \frac{R}{4\pi} \iint_\sigma S^*(\psi)\, \Delta g \, d\sigma \qquad S^*(\psi) = \sum_{n=2}^{n_{max}} \frac{2n+1}{n-1} w_n^G \, P_n(cos\psi)$$

**$T^E$ from the combination of *Tzx* and *Tzy* gradients**

$$T^E(r,\phi,\lambda) = \frac{R^2}{4\pi} \iint_\sigma E^*(\psi) \left(T_{zy} \, cos\alpha^* + \ T_{zx} \, sin\alpha^*\right) d\sigma$$

$$E^*(\psi) = \sum_{n=2}^{n_{max}} \frac{1}{n\,(n+1)} w_n^E \, P_n^1(cos\psi)$$

# Spectral weights

**weighting scheme**

**deterministic**

not involve error estimates

**stohastic**

full error knowledge

**e.g. EGG2008**

**quasi-deterministic (Kern et al.)**

$$\tilde{\sigma}_n = \sigma_n + \varepsilon_n \approx \sigma_n^{model} + \varepsilon_n$$

from data

"true" unknown

$\sigma_n / \varepsilon_n$ : amount of signal /error contained in each degree

$$\varepsilon_n = \left| \tilde{\sigma}_n - \sigma_n^{model} \right|$$

data

GGM or analytical model (Kaula, ...)

Kern et al. (2007).: A study on the combination of satellite, airborne, and terrestrial gravity data. J Geod. 77: 217-225.

# Spectral weights – QD method

Error degree variances

**GGM**

$$\varepsilon_n^{GGM} = \sum_{m=1}^{n} (\varepsilon_{Cnm}^2 + \varepsilon_{Snm}^2)$$

**terrestrial data**

$i$ –*th* measurement ($i$ = 1, 2, …)

$$\varepsilon_n^i = \left| \tilde{\sigma}_n^i - \left(\sigma_n^{model}\right)^i \right|$$

**Weights for $\forall\, \boldsymbol{n}$**

| | n ≤ $n_{max}$(GGM) | n > $n_{max}$(GGM) |
|---|---|---|
| GGM | $\boldsymbol{w} \propto \mathbf{1}/\boldsymbol{\varepsilon}_{\boldsymbol{n}}^{\boldsymbol{GGM}}$ | $\boldsymbol{w} = \mathbf{0}$ |
| i-th data set | $\boldsymbol{w} \propto \mathbf{1}/\left\|\tilde{\boldsymbol{\sigma}}_{\boldsymbol{n}}^{\boldsymbol{i}} - \left(\boldsymbol{\sigma}_{\boldsymbol{n}}^{\boldsymbol{GGM}}\right)^{\boldsymbol{i}}\right\|$ | $\boldsymbol{w} \propto \mathbf{1}/\left\|\tilde{\boldsymbol{\sigma}}_{\boldsymbol{n}}^{\boldsymbol{i}} - \left(\boldsymbol{\sigma}_{\boldsymbol{n}}^{\boldsymbol{model}}\right)^{\boldsymbol{i}}\right\|$ |

# Degree variances from PSD

**2D gridded data** → **2D PSD** → **1D radial PSD**

planar power spectrum, PSD(w)

$$\sigma_n = \frac{n+0.5}{2\pi R^2} PSD(w) \qquad w \approx \frac{n+0.5}{R}$$

spherical power spectrum

**degree variances**

**resolvable spectral band**

$$n_{min} = \frac{40000\ \ km}{D\ [km]}$$

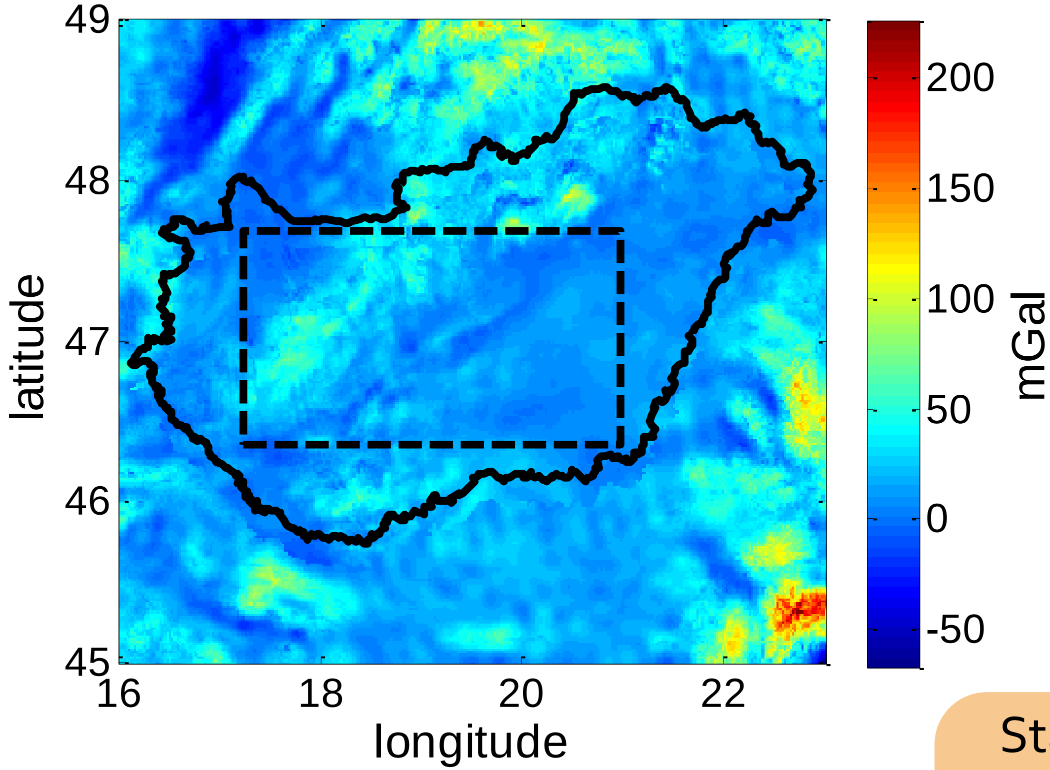
***D:*** extension of the data set

$$n_{max} = \frac{20000\ km}{d\ [km]}$$

***d:*** grid spacing

# Data sets - Gravity

RTM reduced free-air anomaly




sample cut for PSD estimate (area: 160 km x 300 km)

- Inside Hungary:

1′ x 1.5′ mean free-air Δg

- Outside

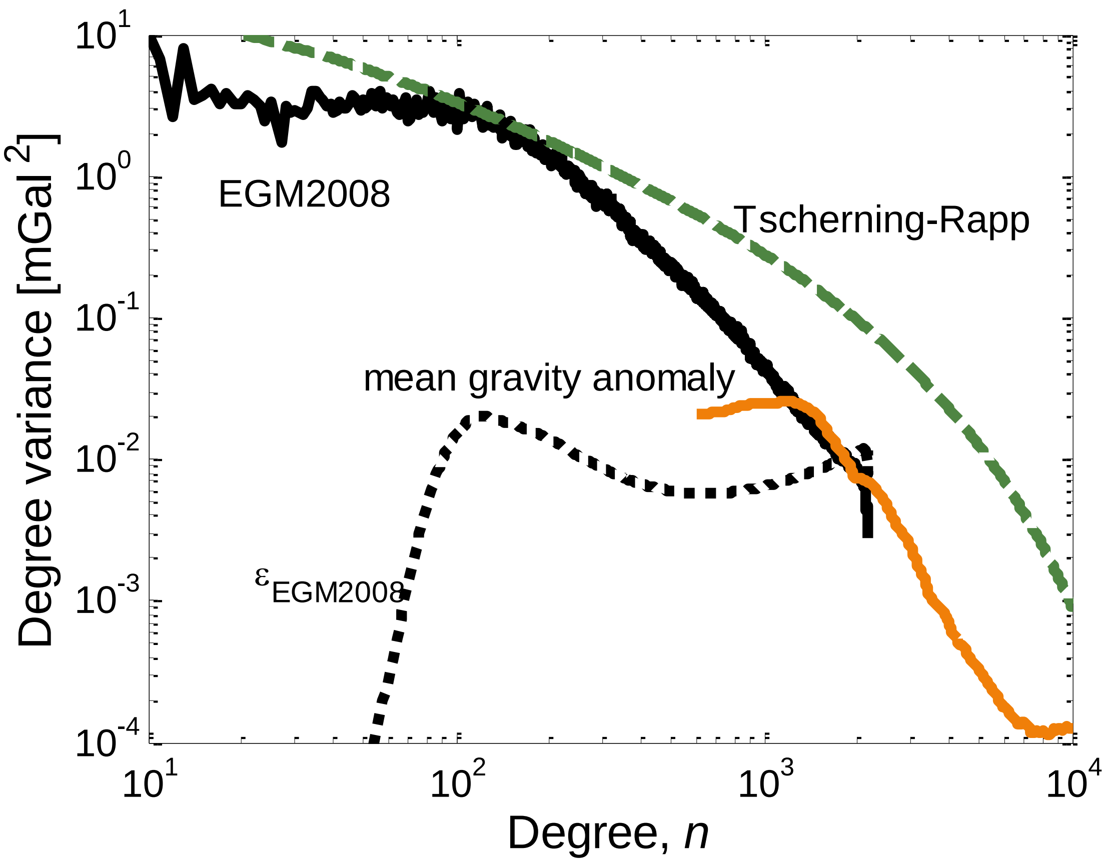
synthetic Δg (EGM2008 n/m 2-2160)

- RTM Δg n/m ≥ 2160 (GRAVSOFT-TC)

Statistics of RTM reduced Δg (Hungary) [mGal]

| mean | std. | min. | max. |
|---|---|---|---|
| 15.0 | 12.8 | -20.6 | 108.1 |

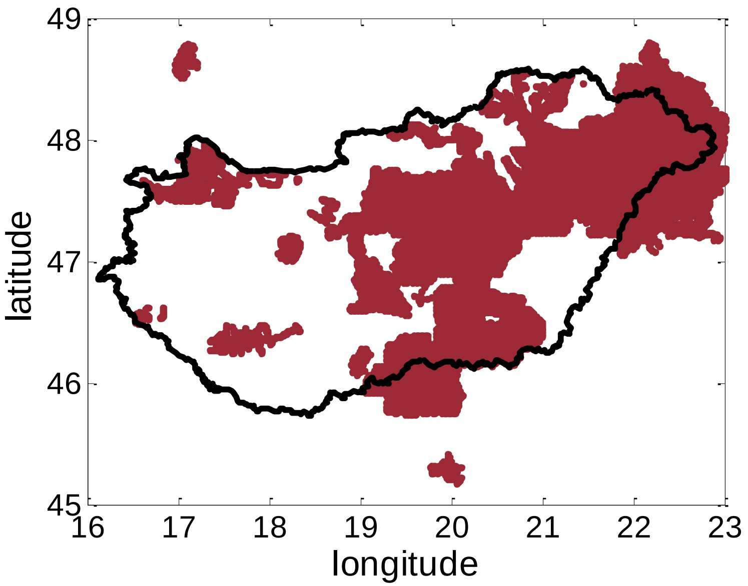
# Gravity degree variances




- EGM2008 to d/o 600 was removed
- recoverable spectral bands

$n_{min}$ = 600
$n_{max}$ = 10,000
( ~ 2km grid spacing)

# Data sets – Horizontal gradients

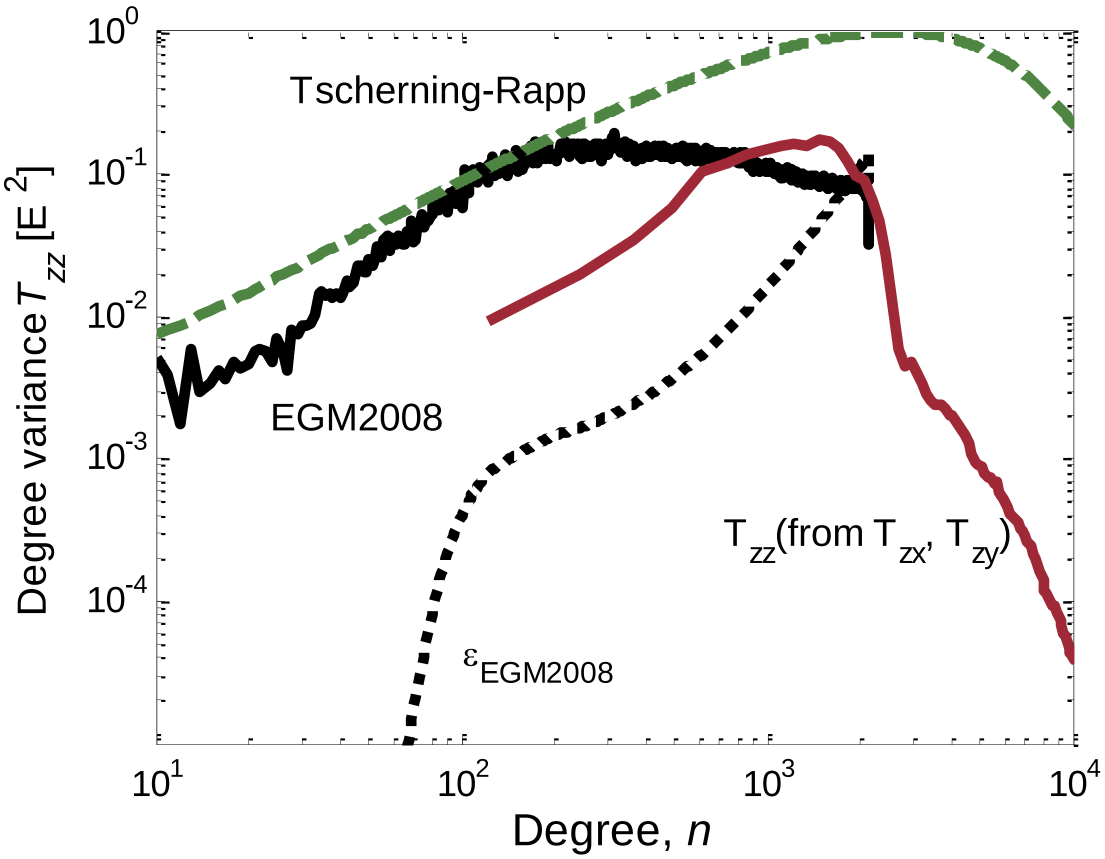



**Statistics of terrain effect reduced gradients [E]**

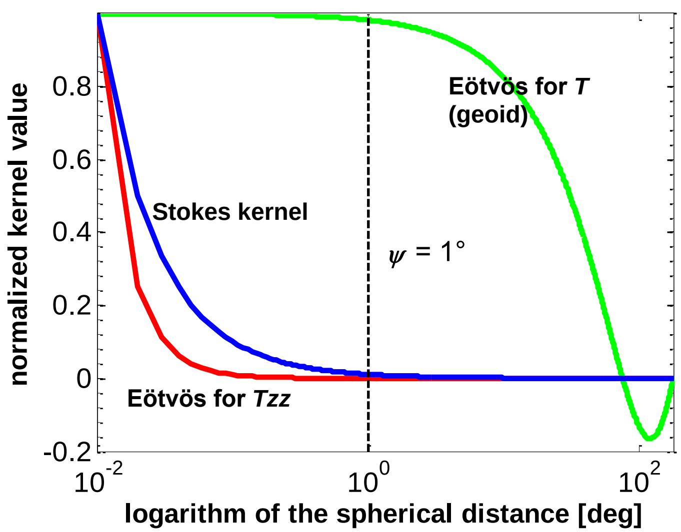
| data set | mean | std. | min. | max. |
|---|---|---|---|---|
| *Tzy* | 0.1 | 9.7 | -88.4 | 72.9 |
| *Tzx* | 0.0 | 8.9 | -67.4 | 71.3 |

- 26,000 torsion balance sites
- nearby terrain effect from levelling <u>not</u> RTM gradients
- grid gradients onto the grid of gravity data
- fill-in gaps: EGM2008 n/m 2- 2160

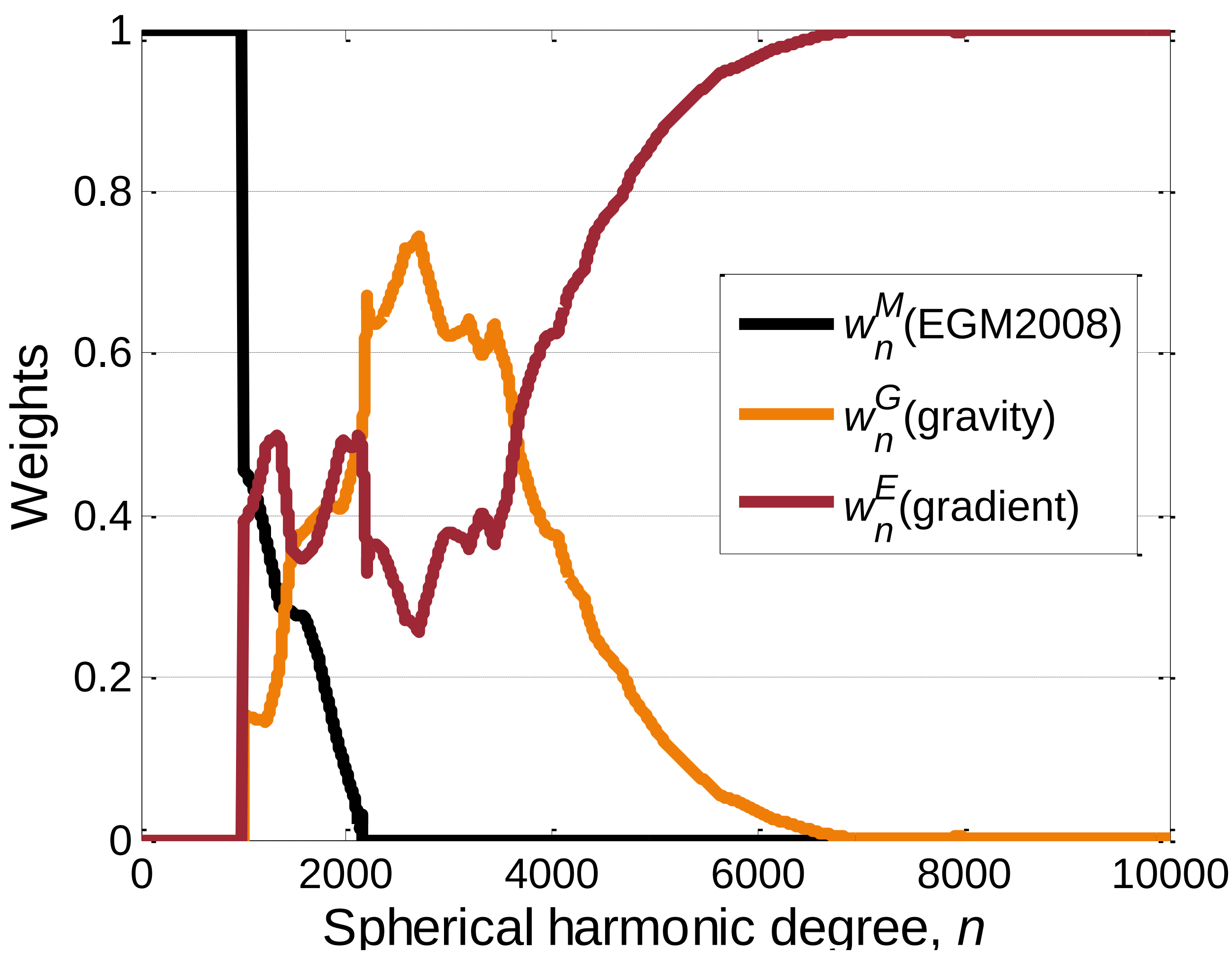
# Vertical grad. degree variance




*Tzx* and *Tzy* were transformed to *Tzz* to determine spectral weights.

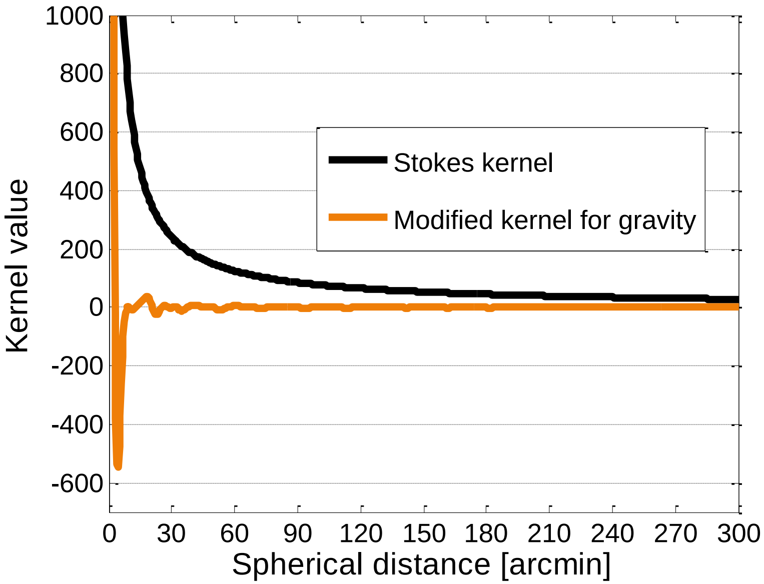


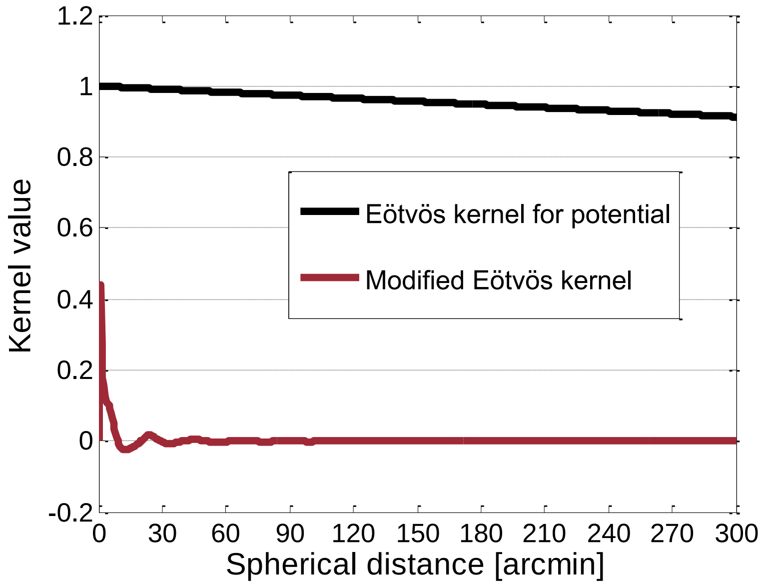
# Spectral weights



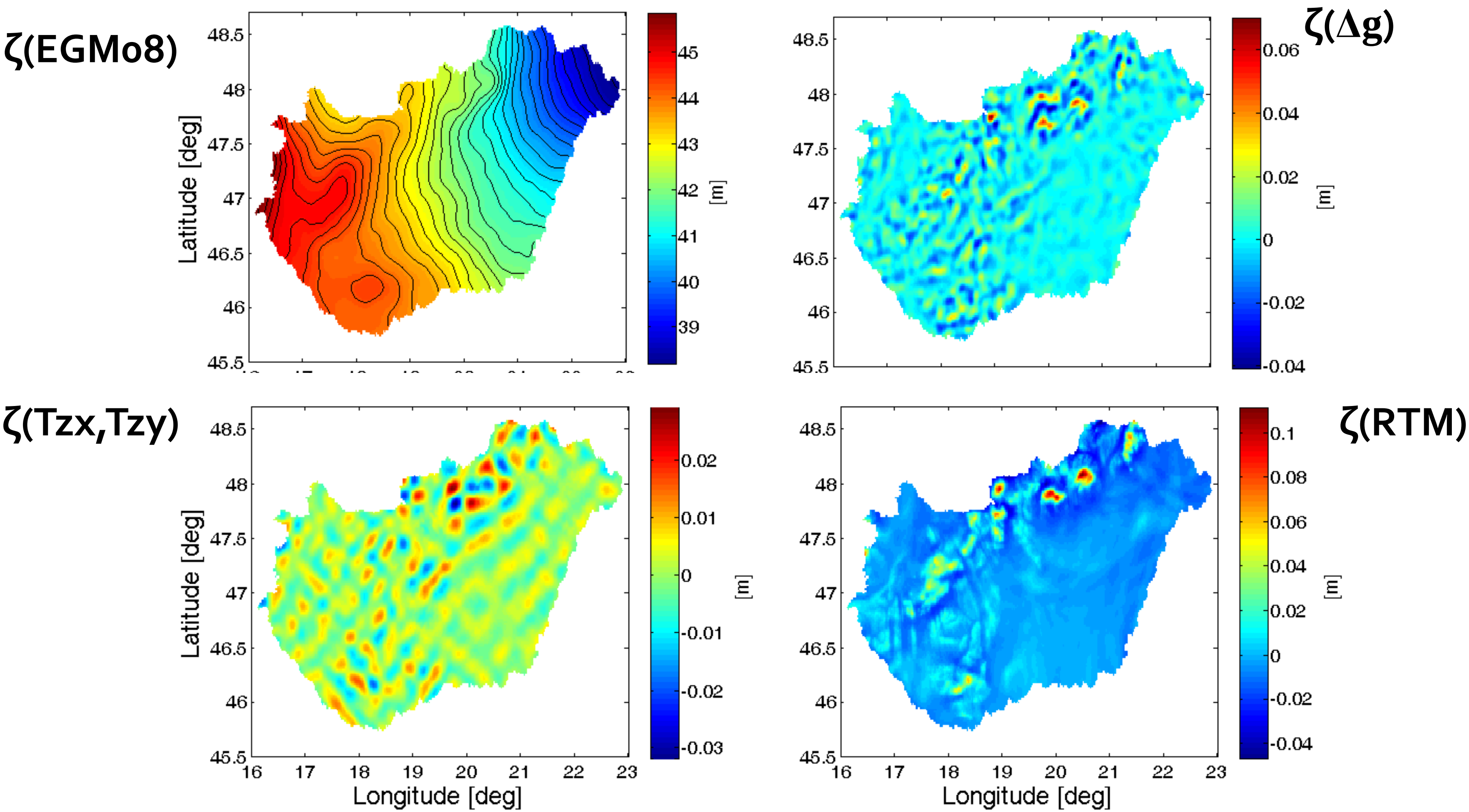
# Evaluation of surface integrals

- local data solution
- 1D spherical FFT (Matlab)
- modified kernels are bounded for $\psi = 0°$
- mean kernel based on Gauss-Legendre quadrature



# Quasi-geoid model contributions from different data sources



# Statistics of different quasi-geoid height contribution [m]

| parameter | mean | std. | min. | max. |
|---|---|---|---|---|
| $\zeta$ (EGM2008) | 42.530 | 1.706 | 38.189 | 45.841 |
| $\zeta$ (gravity) | 0.000 | 0.008 | -0.040 | 0.067 |
| $\zeta$ (grav. grad.) | 0.000 | 0.005 | -0.032 | 0.029 |
| $\zeta$ (RTM) | -0.003 | 0.011 | -0.047 | 0.111 |
| $\zeta$ (combined) | **42.527** | **1.709** | **38.175** | **45.868** |

# Validation of the model

- LSC various geoid models: inconsistencies between GPS/lev. and gravimetric, astrogeodetic data sets
- re-measurement some part of the levelling network – significant surface subsidence ( ~ 10 cm, 30 years)

- Future investigations:
  - using synthetic data from EGM & topography
  - topographic effect for torsion balance data from high-resolution DTM

Mean gravity and torsion balance data sets provided by the Geological and Geophysical Institute of Hungary are gratefully acknowledged.

This study was supported by the National Scientific Research Fund OTKA K 101603.

# Thank you for your attention!