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The need of a Local Reference Frame in Greece:
The deficiency of ETRS89 and a new proposed strategy

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1. Introduction

ETRS89 is a conventional terrestrial reference system, **in alignment with the Eulerian rotational motion of the Eurasia plate**, for spatial positioning throughout the European continent. Its primary objective is to provide a stable coordinate system, with minimal horizontal velocities, in support of geodetic, surveying and mapping applications over the various European countries. The key aspect of its realization is based on the ITRS/ITRF velocity reduction to the so-called stable part of Europe, using the following formula (*Boucher and Altamimi, Memo 2008*):

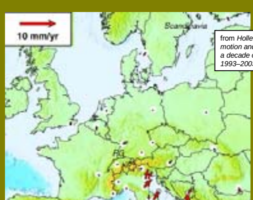
$$\mathbf{v}_i^{ETRFyy} = \mathbf{v}_i^{ITRFyy} + \dot{\mathbf{R}}\,\mathbf{x}_i^{ITRFyy}$$

where the matrix $\dot{\mathbf{R}}$ contains the Cartesian components of the angular velocity vector of the Eurasia plate with respect to ITRFyy

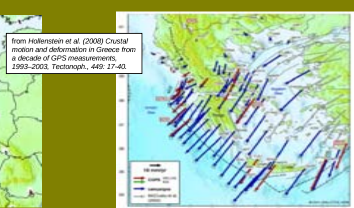
For the actual realization of the European Terrestrial Reference Frames (ETRFyy), the angular velocity of the Eurasia plate is deduced either from a global geophysical model such as AM0-2 or NNR-NUVEL-1/1A (ETRF89 up to ETRF97) or from a geodetically derived velocity field in a network of high-quality ITRF stations within the 'stable part' of Europe (ETRF00).

2. Contrast between ETRF velocities in North and S/E Europe

In general, the magnitude of the ETRF horizontal velocities over the central and northern parts of Europe is at the mm/yr level, providing a rather stable framework for geodetic positioning applications. On the other hand, at the southern part of Europe (and particularly in Greece) the situation is completely different. **In fact, the horizontal velocities wrt. to the stable part of Eurasia in Greece can reach up to several cm/yr** (> 3 cm/yr at TUC2 in Crete), showing also a spatially inhomogeneous behavior. This is caused by the unique geophysical setting of the Hellenic area, which directly affects the behavior of the ETRS89/ETRF-based velocity field in Greece.



Horizontal velocities in Europe with respect to a Eurasia-fixed frame



Horizontal velocities in the Hellenic area with respect to a Eurasia-fixed frame

The above examples show that the horizontal velocity field over Greece is far stronger than in the central and northern Europe, as viewed from an Eurasia-fixed reference frame. Actually, the Hellenic area seems to be divided into two main parts: northern Greece exhibits an apparent consistency (at the few mm/yr level) with the stable Eurasia, whereas the other regions suffer from a clear S/W movement trend at a level of several cm/yr.

3. Motivation of our study

An ETRF-based implementation of a national TRF in Greece (as described, for example, in the *Boucher & Altamimi Memo*) will create a strong velocity field over most of the country, thus canceling out one of the major reasons for adopting ETRS89 as a standard geodetic reference system by the European NMAs!

Our aim is to present an **alternative optimal scheme for implementing a national TRF in Greece** (based on an initial ETRF or ITRF realization consisting of an estimated set of coordinates & velocities over a national network) which will ensure the **'weakest' possible velocity field** throughout the entire part of the country.

4. Mathematical formulation

The key element of our optimization procedure is the implementation of a *Helmert-type velocity transformation* (over a national network of fiducial stations) from a given ITRF or ETRF based regional realization ($\mathbf{x}^{TRF}$, $\mathbf{v}^{TRF}$) to a new local reference frame realization ($\mathbf{x}^{LRF}$, $\mathbf{v}^{LRF}$).

Following *(Altamimi et al. 2002)*, we have the general equations:

$$\mathbf{v}_i^{LRF} = \mathbf{v}_i^{TRF} + \dot{\mathbf{T}} + \dot{D}\mathbf{x}_i^{TRF} + \dot{\mathbf{R}}\mathbf{x}_i^{TRF} \quad \text{for each point } i$$

$$\mathbf{v}^{LRF} = \mathbf{v}^{TRF} + \mathbf{E}^T\dot{\boldsymbol{\theta}} \quad \text{for all network points}$$

where $\mathbf{E}$ is the well-known 'inner-constraint' matrix, and $\dot{\boldsymbol{\theta}}$ is the vector of the adopted transformation parameters (including, in general, 3 shift rates, 3 rotation rates & 1 scale rate; for more details, see below)

The unknown parameters of the above transformation will be determined according to the following optimal criterion for the LRF velocities:

$$\phi = \left(\mathbf{v}^{LRF}\right)^T\left(\mathbf{v}^{LRF}\right) = \min \quad \textbf{LRF will be a frame of minimum 'kinetic energy'}$$

which yields the following solution: $\dfrac{\partial\phi}{\dot{\boldsymbol{\theta}}} = 0 \;\rightarrow\; \dot{\boldsymbol{\theta}} = -\left(\mathbf{E}\mathbf{E}^T\right)^{-1}\mathbf{E}\,\mathbf{v}^{TRF}$

Hence, in the new LRF, the transformed velocities will be given by the 'projective' formula:

$$\mathbf{v}^{LRF} = \left(\mathbf{I} - \mathbf{E}^T\left(\mathbf{E}\mathbf{E}^T\right)^{-1}\mathbf{E}\right)\mathbf{v}^{TRF}$$

IMPORTANT NOTES ON THE LRF OPTIMIZATION

The above optimization/transformation procedure creates a new LRF that differs, **in terms of its temporal evolution**, from the initially given ITRF or ETRF realization ($\mathbf{x}^{TRF}$, $\mathbf{v}^{TRF}$).

The temporal evolution of the new LRF is dictated by a **minimum-norm condition** on its velocity field $\mathbf{v}^{LRF}$ that is obtained, under a Helmert-type transformation scheme, from the existing ITRF or ETRF based velocity field ($\mathbf{v}^{TRF}$).

A critical issue for the **practical implementation of this procedure** is the choice of the optimized transformation parameters $\dot{\boldsymbol{\theta}}$. In fact, a scale-rate parameter **should not be used**, otherwise an artificial scaling distortion is introduced into the new LRF. Moreover, in small geographical areas (such as Greece), the use of shift-rate parameters should be avoided due to their high correlation with the three rotation-rate parameters.

(*) Obviously, the inner-constraint matrix $\mathbf{E}$ in the previous equations should correspond to the chosen structure of the transformation vector $\dot{\boldsymbol{\theta}}$.

Realization of the optimized LRF

A reference epoch (t_o) needs to be conventionally selected, in which the new LRF will be assumed to coincide with the initial TRF

$$\mathbf{x}^{LRF}(t_o) \equiv \mathbf{x}^{TRF}(t_o)$$

At an arbitrary epoch *(t)* the relationship between LRF and TRF will be given in terms of the following coordinate transformation (its derivation is straightforward based on the above equations)

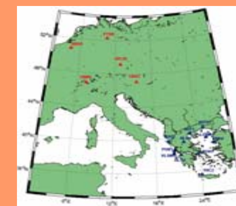
$$\mathbf{x}^{LRF}(t) = \mathbf{x}^{TRF}(t) + (t - t_o)\,\mathbf{E}^T\left(\mathbf{E}\mathbf{E}^T\right)^{-1}\mathbf{E}\,\mathbf{v}^{TRF}$$

where $\mathbf{x}^{TRF}(t)$ is obtained from actual measurements at t, and $\mathbf{v}^{TRF}$ corresponds to the existing velocity field that was used for the LRF optimization procedure.

Alternative LRF optimization based on 2D velocities

The previous transformation procedure can be also implemented using a minimum-norm condition only for the 2D (horizontal) velocities in the new LRF, based on a simple conversion from a geocentric to a topocentric velocity representation (more details will be present in a forthcoming paper).

5. GPS test network for LRF optimization in Greece



- 16 stations (11 NOANET stations in Greece & 5 EPN stations in central Europe), three years of GPS observations (2007-2010)
- Initial reference frame (*TRF*): ETRF00 (2009.0)
- Tight constraints at the 5 EPN stations
- Bernese software (precise IGS orbits, QIF ambiguities, Dry Neil tropospheric model, global ionospheric model)
- Estimated coordinate errors 0.5 mm, estimated velocity errors 0.5 mm/yr (1σ level)

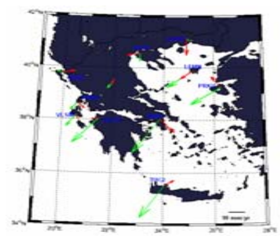
6. Numerical results

The implementation of the previous optimization procedure was based on the sole use of three rotation-rate parameters between ETRF00 and the new LRF. Both 3D and 2D scenarios for the formulation of the 'minimum kinetic energy' criterion were tested.

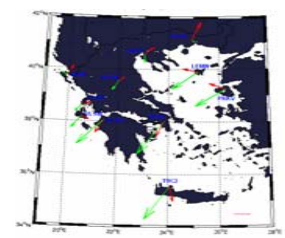
Estimated rotation-rate parameters between ETRF00 and the optimal LRF

	3D scenario	2D scenario
ω_x	-3.264 mas/yr	-3.265 mas/yr
ω_y	-0.982 mas/yr	-0.983 mas/yr
ω_z	-3.101 mas/yr	-3.103 mas/yr

Horizontal velocities with respect to ETRF00 and the optimal LRF



3D optimization scenario



2D optimization scenario

Statistics of the horizontal velocities at the 11 Hellenic GPS stations (in mm/yr)

	mean	σ	max	min
ETRF00	**15.2**	**10.5**	**30.4**	**2.6**
New LRF (2D optimization scenario)	8.2	3.2	13.3	4.4
New LRF (3D optimization scenario)	9.1	3.4	14.1	4.0

7. Conclusions

(i) We have presented a Helmert transformation scheme for implementing an optimal LRF in Greece. (in the sense that its associated velocity field (VF) becomes as small as possible)

(ii) Such an approach is a useful tool towards the establishment of a **national spatial reference framework with maximum temporal stability**. Its realization, however, requires a sufficiently dense and well-modeled VF ($\mathbf{v}^{TRF}$) with respect to an ITRF or ETRF based frame.

(iii) Our numerical tests showed that the optimal LRF has an average VF magnitude of **< 1 cm/yr**. Also, the dispersion of the VF in the optimal LRF drops to ~3 mm/yr, compared to the 10.5 mm/yr dispersion level which is induced by the ETRF00 frame!

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