Different splitting techniques in the adjustment of large networks: discussion and implementation

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Extended abstract of: Biagi L., Caldera S., Permanent networks monitoring: remarks and case studies, Appl. Geomat., 2011, 3

Let suppose that two independent observations vectors, $\mathbf{y}_1, \mathbf{y}_2$, depend both on individual ($\mathbf{x}_1, \mathbf{x}_2$) and common ($\boldsymbol{\xi}$) parameters:

$$\mathbf{y}_1 = \begin{bmatrix} \mathbf{F}_1 & \mathbf{A}_1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \xi \end{bmatrix} + \mathbf{v}_1, E\{\mathbf{v}_1\} = 0, \mathbf{C}_{y_1 y_1} = \sigma_0^2 \mathbf{Q}_{11},$$

$$\mathbf{y}_2 = \begin{bmatrix} \mathbf{A}_2 & \mathbf{F}_2 \end{bmatrix} \begin{bmatrix} \xi \\ \mathbf{x}_2 \end{bmatrix} + \mathbf{v}_2, E\{\mathbf{v}_2\} = 0, \mathbf{C}_{y_2 y_2} = \sigma_0^2 \mathbf{Q}_{22}$$

The two systems can be combined:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{v}, \ \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{F}_2 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \xi \\ \mathbf{x}_2 \end{bmatrix}, \ \mathbf{C}_{yy} = \sigma_0^2 \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{22} \end{bmatrix}$$

The Least Squares solution is given by:

$$\mathbf{N}\mathbf{x} = \mathbf{A}^T \mathbf{Q}^{-1} \mathbf{y}, \ \mathbf{N} = \mathbf{A}^T \mathbf{Q}^{-1} \mathbf{A} \\ \Rightarrow \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{A}^T \mathbf{Q}^{-1} \mathbf{y} \quad (1)$$

In the following, eq. (1) will be called batch solution. The combination of independent solutions represents an alternative way. By pre-elimination the individual parameters vectors are estimated

$$\mathbf{y}_1 \Rightarrow \hat{\mathbf{x}}_1, C_{\hat{x}_1 \hat{x}_1}, \qquad \mathbf{y}_2 \Rightarrow \hat{\mathbf{x}}_2, C_{\hat{x}_2 \hat{x}_2} \quad (2)$$

The two normal systems relevant to the common parameters vector are generated

$$\mathbf{y}_1 \Rightarrow \hat{\xi}_I, \mathbf{N}_I, \qquad \mathbf{y}_2 \Rightarrow \hat{\xi}_{II}, \mathbf{N}_{II} \quad (3)$$

Finally, the common parameters are estimated

$$\hat{\xi} = (\mathbf{N}_I + \mathbf{N}_{II})^{-1} (\mathbf{N}_I \hat{\xi}_I + \mathbf{N}_{II} \hat{\xi}_{II}) \quad (4)$$

Let suppose that two geodetic networks have been independently surveyed and adjusted and that they share several stations or, in other words, they overlap. The application of (2), (3) and (4) is the obvious way to adjust them. In the geodetic framework, the combination of independent solutions of overlapping networks is called also Normal Equation (NEQ) stacking.

In the adjustment of permanent GNSS networks, the normal system to be inverted has size equal to the square of the unknowns number. Moreover, a rigorous adjustment must correctly take into account the correlations between the baselines and the resulting covariance matrix of the observations is not diagonal. On a standard PC, the daily processing of a network of about 150 stations is still possible in less than four hours but the processing time increases exponentially with the stations number. From a practical point of view, a limit exists to the number of stations that can be simultaneously processed. Actually, hardware evolves very quickly, and the processing limits are continuously changing. Several networks in the world exceed the dimension that requires a distributed adjustment: IGS (igscb.jpl.nasa.gov), some regional networks (for example EPN in

Europe, www.epncb.oma.be) and other local networks (for example, the Japan National network, www.gsi.go.jp). In any case, the distributed adjustment is a popular choice also for smaller networks. To apply it, the following process is usually implemented on a daily basis:

1. the network is split into overlapping subnetworks,
2. each subnetwork is processed by one Processing Facility (PF),
3. all the daily subnetwork solutions (NEQ files) are transmitted to a coordination center,
4. finally, all the daily subnetworks solutions are combined by NEQ stacking to produce a network solution.

The final solution can be obtained either by a daily or by a weekly stacking. Note that in our notation, a Processing Facility (PF) can be one analysis center in a globally or regionally distributed structure like IGS or EPN. More simply, it can be a set of procedures installed on the server of the control center of a local network.

In the usual approach, the overlaps are such that all the stations belong to more subnetworks and the configuration of the subnetworks is constant in time, i.e. over all the days. For example, in the present praxis, EPN (about 250 stations) is split into 16 subnetworks and each one of them is adjusted by one Local Analysis Center. Subnetworks dimensions range from 30 stations to 70 stations and each station belongs to 3 subnetworks at least. This approach can be defined COnstant in time, Daily Overlapping, Distribution (CODOD). CODOD introduces duplicated observations because the same daily files are processed by more PF’s. In particular, it builds false independent repeated baselines and closed polygons among different subnetworks.

Generally, if just one observation is duplicated in the observations vector $\mathbf{y}$, the resulting $\mathbf{Q}_{yy}$ is singular, because its columns/rows are no more linearly independent. Therefore, the Least Squares system (3) cannot be solved. In CODOD, the combination is numerically only possible because the duplications and the correlations of the subnetworks solutions are neglected and the adopted stochastic model is wrong. A network solution can be obtained also by combining the solutions of subnetworks that are reciprocally connected by one station (simply connected subnetworks). However, the combination of overlapping subnetworks allows to cross check the procedures applied by the different PF's and the data quality. An alternative approach is proposed, allowing the rigorous combination of overlapping networks by stacking solutions that are really independent.

1. Each day the network is split into simply connected subnetworks.
2. The daily configuration of the splitting varies in a cycle over more days. In different days, each station is included in different subnetworks, in such a way that the subnetworks overlap at the end of the cycle.
3. The daily subnetworks solutions can be combined to provide one daily network solution.
4. At the end of the cycle, the daily solutions of all the subnetworks are combined in a final solution.

This approach is called VAriable in time, Cyclically Overlapping, Distribution (VACOD).

In order to manage the daily variability of the subnetworks configuration, VACOD imposes a significant coordination between the PF's. However, less computational effort is needed with respect to CODOD. On a daily basis, VACOD is comparable to the batch adjustment of the whole network and no false repeated baselines and closed polygons are built. At the end of the cycle it allows the rigorous combination and the cross check of subnetworks that overlap and have been independently estimated. In the combination, only the correlations of the connecting stations are neglected and the adopted stochastic model is almost correct.

In order to numerically compare the results provided by CODOD and VACOD, a test network of 102 stations has been analyzed. The network (Fig. 1) is composed of 24 IGS stations in Europe and 78 EPN stations. Four weeks of data, GPSWeeks from 1550 to 1553, have been processed for a total of 2839 RINEX daily files. The threshold on the daily data completeness has been set to 2760 epochs and 84 files have been consequently rejected.

All the network adjustments here described have been performed by the Bernese Processing Engine (BPE) of the Bernese GPS Software 5.0 (in the following BSW5.0), developed at the Astronomical Institute of University of Bern. The processing options follow the international IERS/IGS conventions and the technical suggestions in BSW5.0 manual. In particular, the data are processed by daily sessions and the final solution of the local network is aligned to IGS by imposing a No Net Translation (NNT) condition on the barycenter of a set of fiducial IGS stations, whose a priori coordinates are computed by interpolating long time series of the official weekly IGS solutions.

At first, 28 daily solutions by a batch adjustment of the whole network (in the following BA) have been computed. The baselines of the network span from 25 Km to 960 Km, while the mean length is about 230 Km. In the processing, few baselines provide mediocre results but nothing is really worrying. All the days, more than 85% of the ambiguities are resolved and the network adjustment RMS's are always in the range 1.1-1.4 mm. At the end, the daily solutions have been stacked into 4 weekly solutions.

To apply CODOD and VACOD approaches, three PF's have been hypothesized. In CODOD all the three subnetworks are homogeneously spread over the whole area. Each subnetwork contains 16 IGS stations and 52 EPN stations and each station belongs to two subnetworks. In VACOD, three daily configurations alternate. Each day the network is split into three simply connected subnetworks, composed of about 8 IGS and 26 EPN stations. The first and second daily configurations (CFI and CFII) apply a regional clustering, respectively from West to East and from North to South. In the last configuration (CFIII), the central-southern Europe in which EPN is denser, is split into two homogeneously spread subnetworks, while the other stations are in the third subnetwork. On a standard Windows PC (~1000 Euros) the batch processing of the network requires about 80 minutes per day. The daily processing of one CODOD subnetwork requires about 50 minutes, therefore two hours and half are needed for the complete daily processing. The daily processing of one VACOD subnetwork requires 20 minutes, for a total time of one hour. All the daily quality indexes of CODOD and VACOD are similar to those of BA.

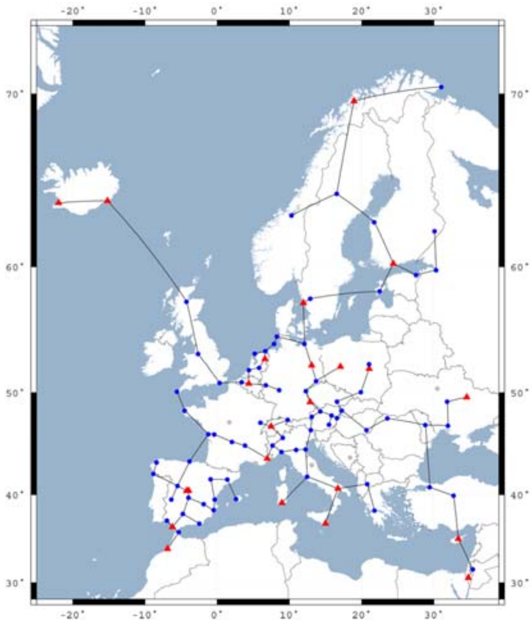

Figure 1. The analyzed network and the graph of the baselines generated in the batch adjustment: example of day 2009-263. Red triangles: reference IGS stations, blue circles: EPN stations.

Finally, the subnetworks solutions have been combined, both in daily and in weekly solutions, and the results have been compared with those provided by BA.

At first, the time series of the daily coordinates are analyzed, in term of repeatabilities and daily differences. The repeatabilities of VACOD and BA time series are almost equal. The CODOD repeatabilities are slightly better and the reason is quite clear. False repeated baselines and false closed polygons are built among different subnetworks. Consequently, the individual errors are in some way "averaged" in the processing and in the following combination. It should be remembered that the combination is numerically possible only because a wrong stochastic model is adopted. The second comparison regards the differences of the daily estimates of CODOD and VACOD with respect to those of BA. The CODOD standard deviations seem better. In any case, the VACOD results are similar at the mm level, are completely satisfactory and no biases exist with respect to BA.

Then, the weekly solutions are presented, in term of stations coordinates and formal covariances.

As the weekly coordinates are concerned, the three approaches produce almost equal results and the differences are negligible.

The standard deviations of the coordinates of the BA weekly solutions have been extracted. Then, the analogous standard deviations provided by CODOD and VACOD have been compared with them.

As expected, CODOD duplications and false redundancies significantly bias all the estimated variances, that are meaningless. VACOD provides correct estimates and only the variances of the connecting stations are slightly underestimated, as their correlations in the subnetworks are not taken into account.

The results demonstrate that VACOD provides estimates and accuracies completely satisfactory and almost equal to those obtained by the batch adjustment. CODOD seems to provide slightly better statistics. However, and this is the point, CODOD daily combinations are possible only because a wrong stochastic model is adopted. The approximation does not affect the estimates of the coordinates but significantly biases their covariances, that are significantly underestimated and completely unrealistic. For this reason VACOD approach is preferable.

Full Paper: Biagi L., Caldera S., Permanent networks monitoring: remarks and case studies, Appl. Geomat., 2011, 3