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Different splitting techniques in the adjustment of large networks: discussion and implementation

HM2009 → REFAG 2010 → **EUREF2011**

Outline

Helmert blocking
and NEQ stacking for geodetic networks

Splitting the adjustment
of permanent GNSS networks:

the standard approach

an alternative approach

A case study

Comparisons and final remarks

Helmert blocking: the mixed model (1/2)

Two independent vectors of observations are given, with both individual and common parameters

$$\mathbf{y}_1 = \begin{bmatrix} \mathbf{F}_1 & \mathbf{A}_1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \boldsymbol{\xi} \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} \mathbf{A}_2 & \mathbf{F}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \mathbf{x}_2 \end{bmatrix},$$

$$\mathbf{C}_{yy} = \sigma_0^2 \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{22} \end{bmatrix}$$

A mixed estimation approach is straightforward

Helmert blocking: the mixed model (2/2)

By pre elimination
the two individual vectors of parameters are estimated

$$\mathbf{y}_1 \Rightarrow \hat{\mathbf{x}}_1, C_{\hat{x}_1\hat{x}_1}, \quad \mathbf{y}_2 \Rightarrow \hat{\mathbf{x}}_2, C_{\hat{x}_2\hat{x}_2}$$

the two "common" NEQ's are generated

$$\mathbf{y}_1 \Rightarrow \hat{\boldsymbol{\xi}}_I, \mathbf{N}_I, \quad \mathbf{y}_2 \Rightarrow \hat{\boldsymbol{\xi}}_I, \mathbf{N}_{II}$$

By NEQ stacking the

common parameters are finally computed

$$\boxed{\hat{\boldsymbol{\xi}} = (\mathbf{N}_I + \mathbf{N}_{II})^{-1}(\mathbf{N}_I\hat{\boldsymbol{\xi}}_I + \mathbf{N}_I\hat{\boldsymbol{\xi}}_{II})}$$

The geodetic case

Two or more networks overlap
(they share several common stations)

They have been independently
surveyed and adjusted

The independent normal systems
can be stacked to
provide a final estimate of the common stations

The GNSS permanent networks case

Permanent networks are adjusted by daily sessions

Big networks (more than 200 stations) require:
the split into subnetworks,
the separate adjustment of the subnetworks,
the stacking of the solutions

Requirements

A reciprocal control of the results

Solution

The subnetworks overlap

The standard (CODOD) approach

The subnetworks configuration is
a priori given and kept fixed

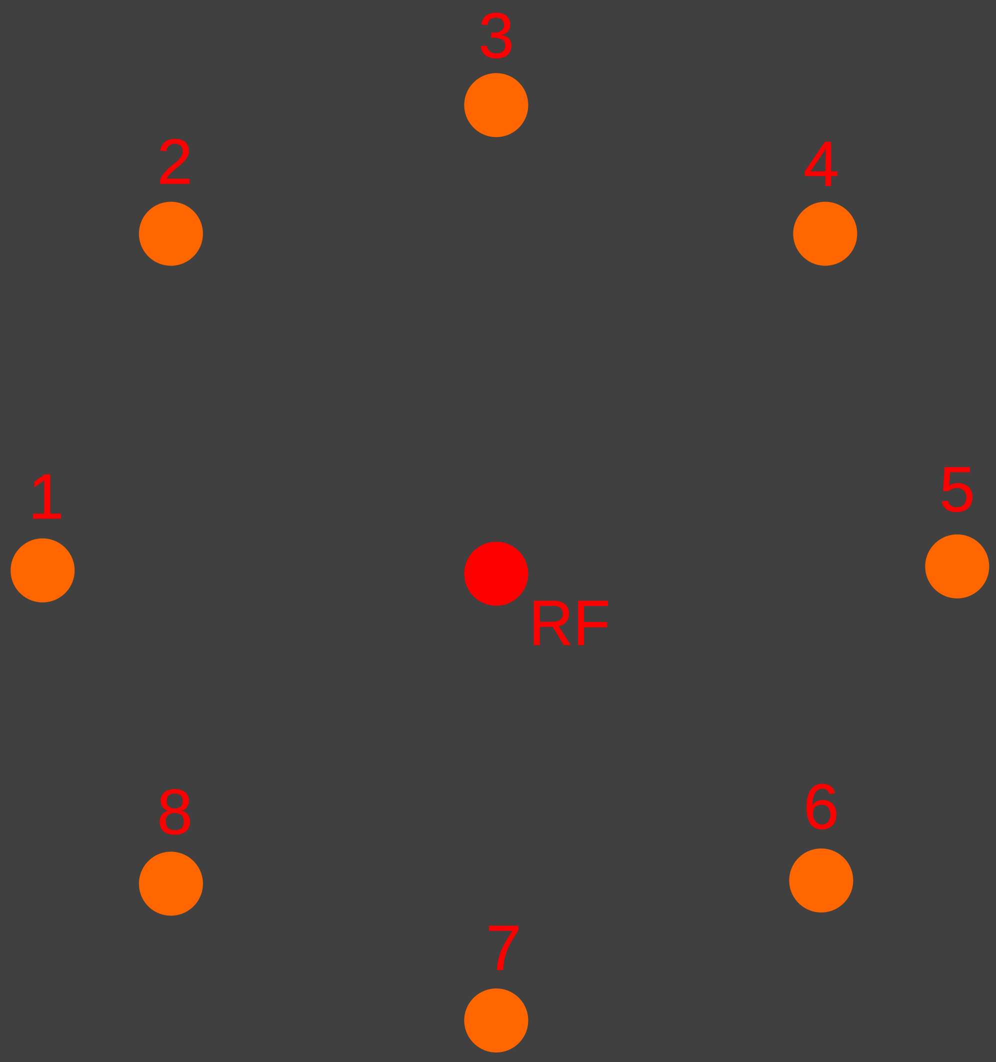
On a daily basis,
each subnetwork is adjusted by a Processing Facility
{an open baselines graph (single differences) is built,
by LS adjustment the subnet coordinates are estimated}

A subnet daily NEQ file is generated

At the daily or weekly basis
all the subnet NEQ's are sequentially combined
to obtain the final network estimates

COnstant in time, Daily Overlapping Distribution

The GNSS networks: an example



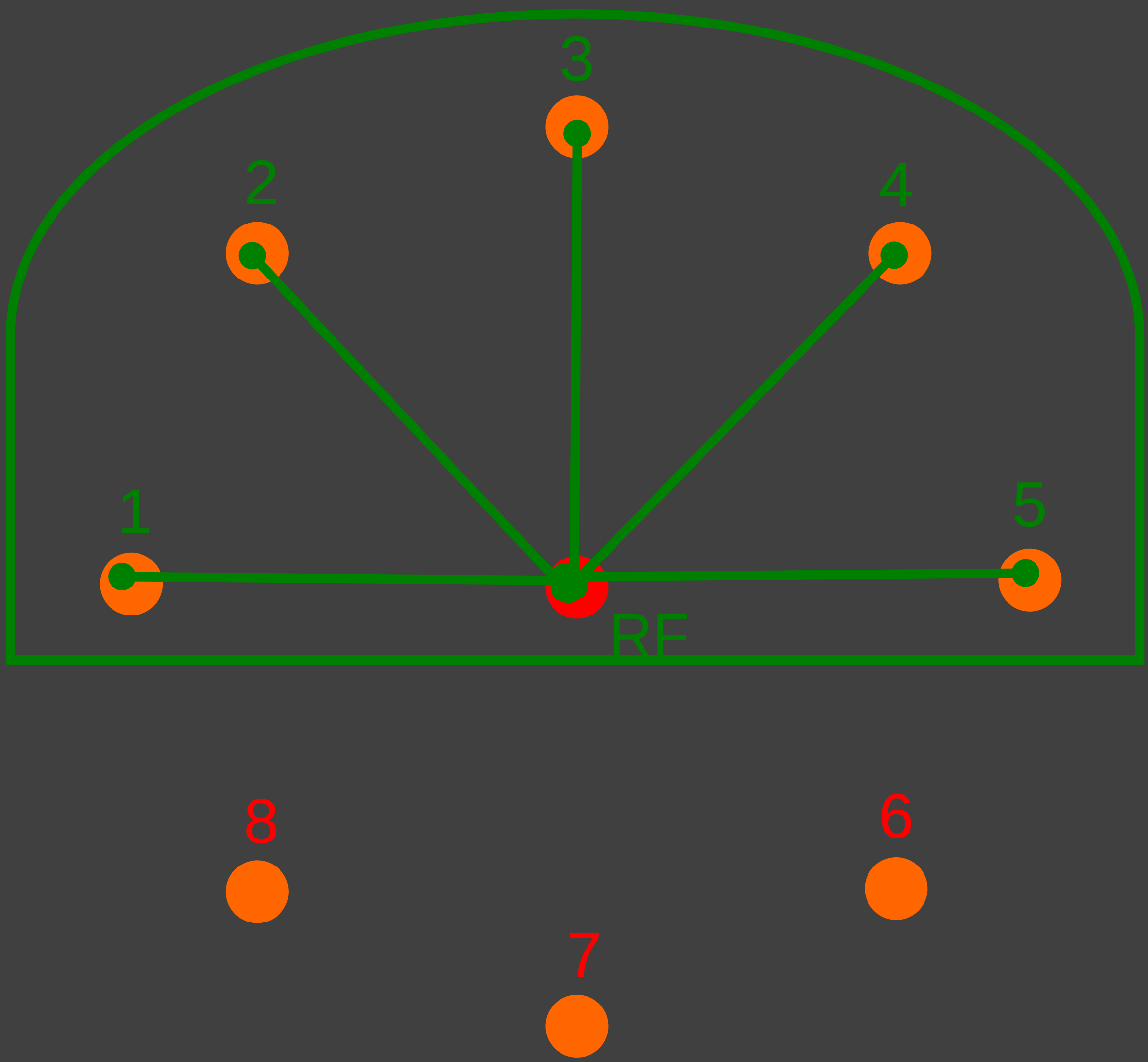
8 Stations to be estimated

1 station defines the (minimal constraints) reference frame

4 Processing Facilities

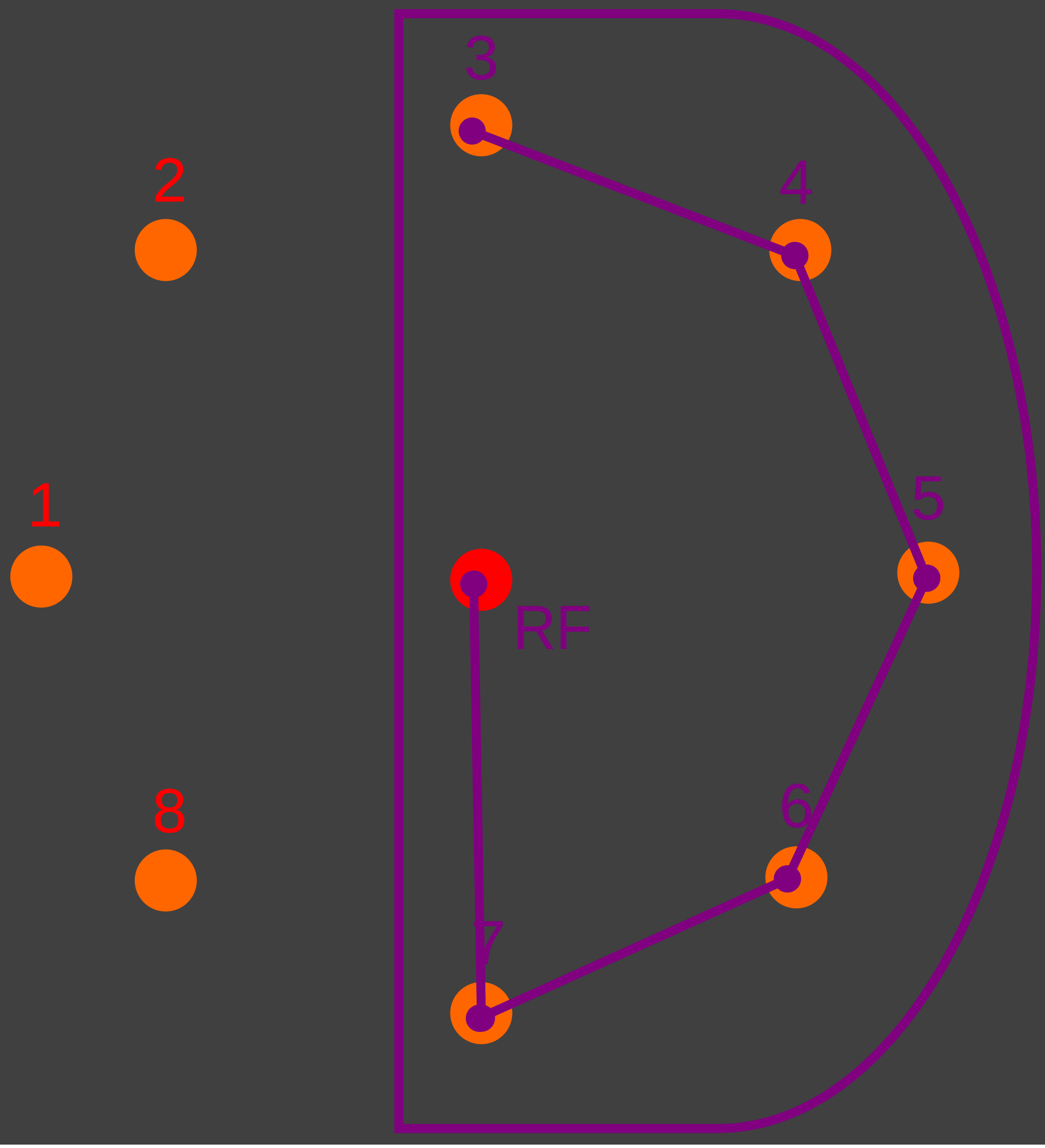
In all the followings:
daily solutions
daily NEQ stacking

The GNSS networks:
COnstant in Time, Daily Overlapping, Distribution



Day D, PF 1:
5 stations
+ RF station
⇓
NEQ D1

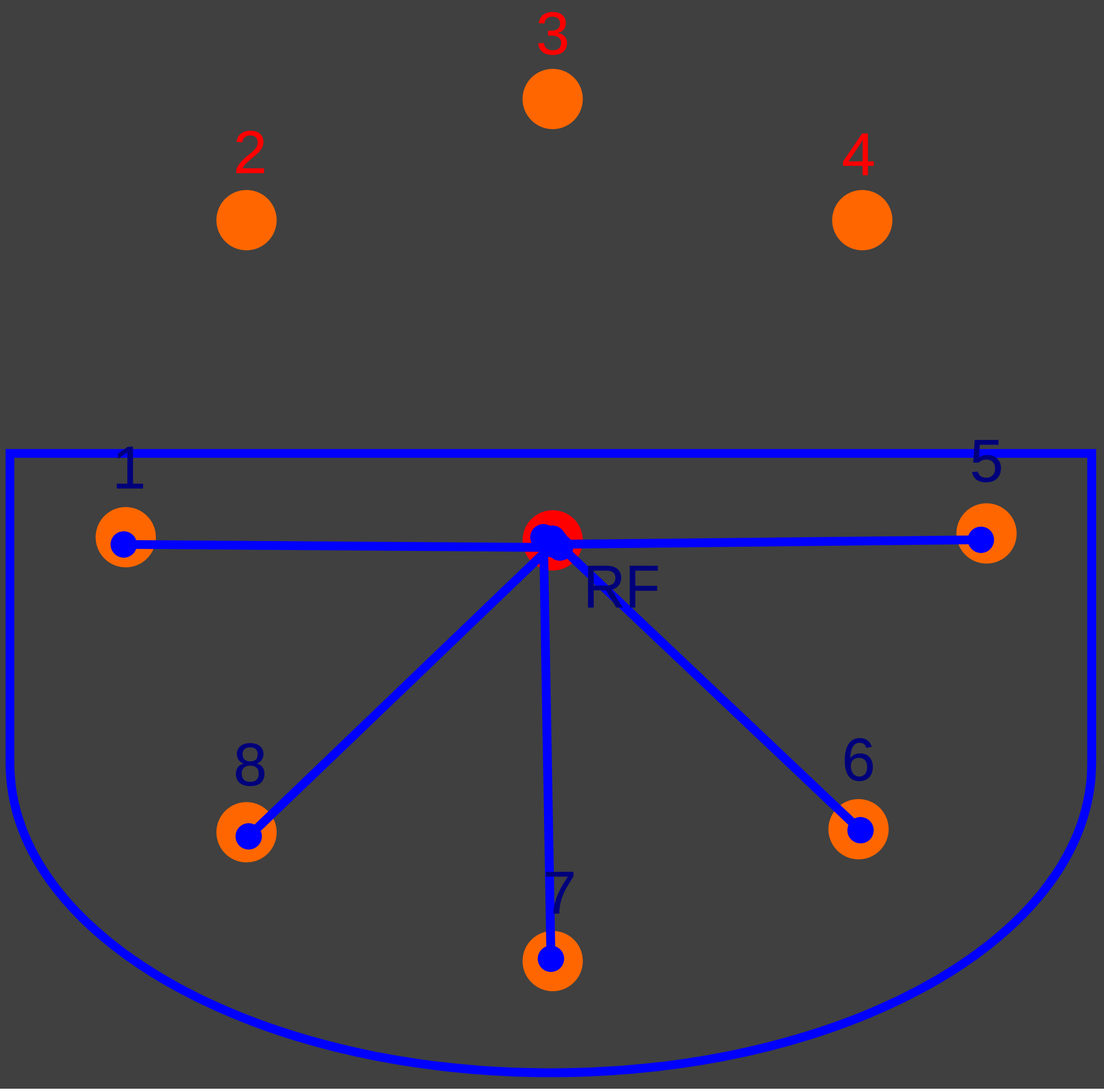
The GNSS networks: an example of CODOD



Day D, PF2:
5 stations
+ RF station
⇓
NEQ D2

4 stations
in common with
D1

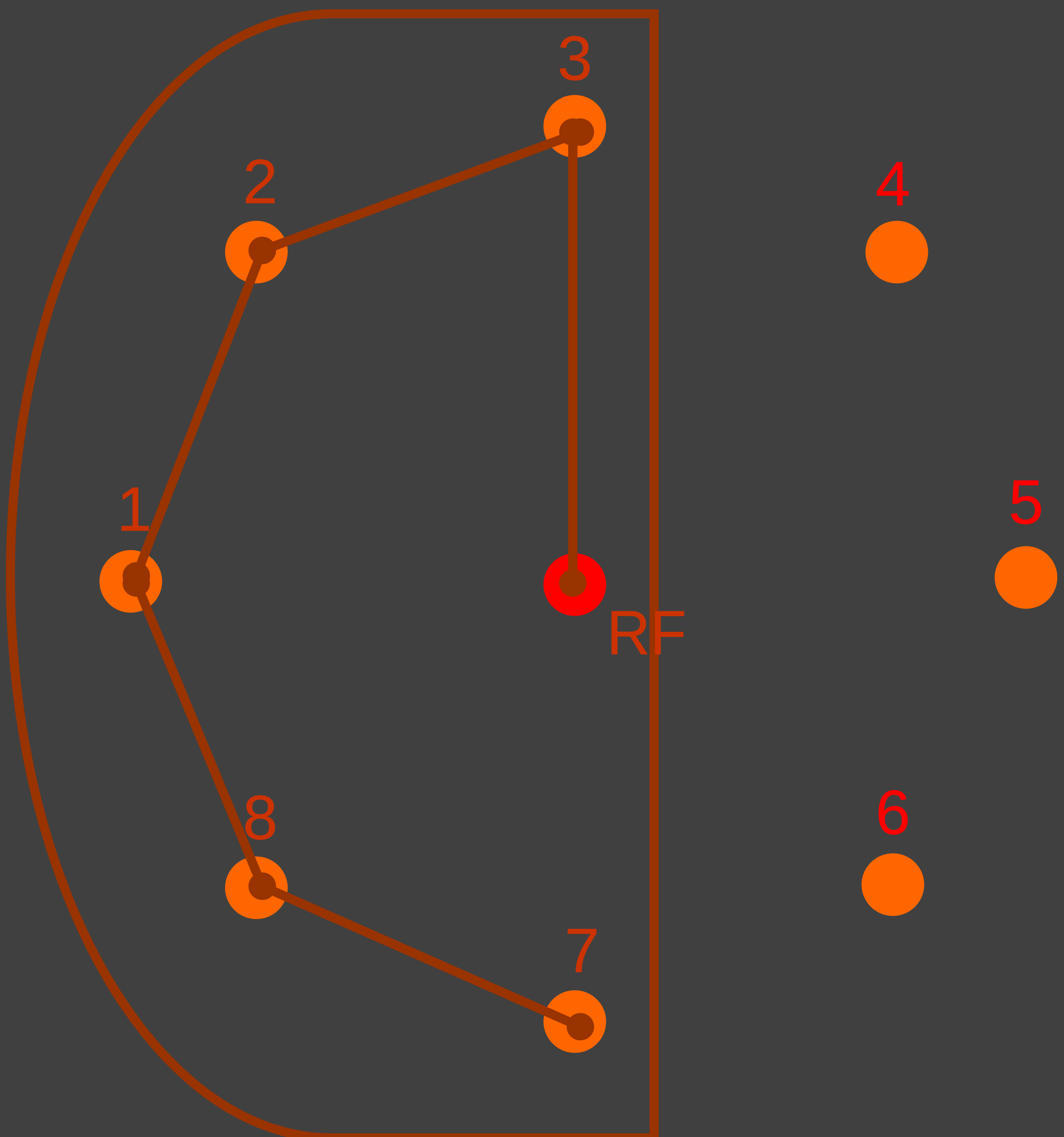
The GNSS networks: an example of CODOD



Day D, PF3:
5 stations
+ RF station
⇓
NEQ D3

3 stations
in common with
D1
4 stations
in common with
D2

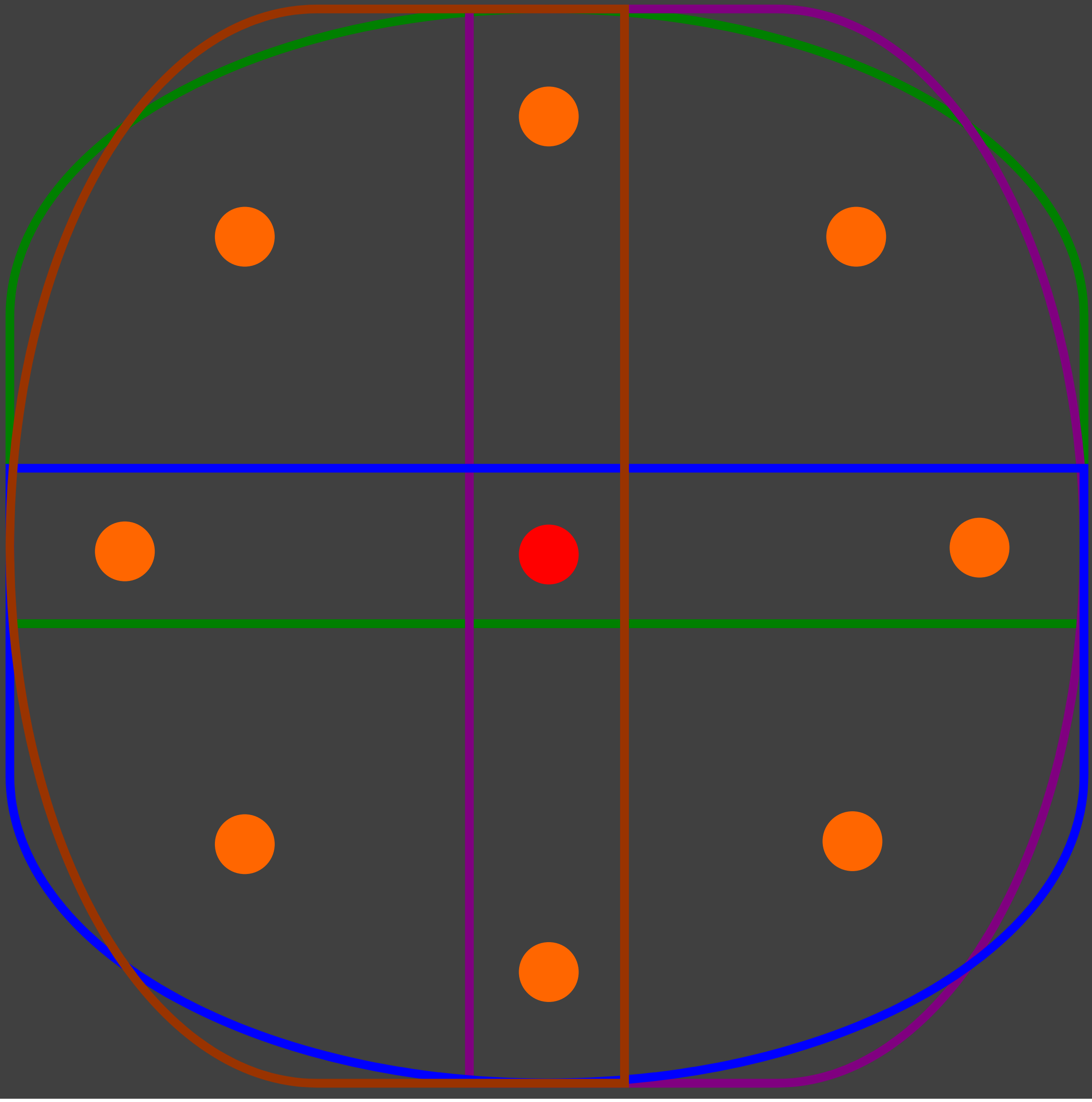
The GNSS networks: an example of CODOD



Day D, PF4:
5 stations
+ RF station
⇓
NEQ D4

4 stations
in common with
D1
3 stations
in common with
D2
4 stations
in common with
D3

The GNSS networks: resumé of CODOD



D1, D2, D3, D4 NEQ's

⇓

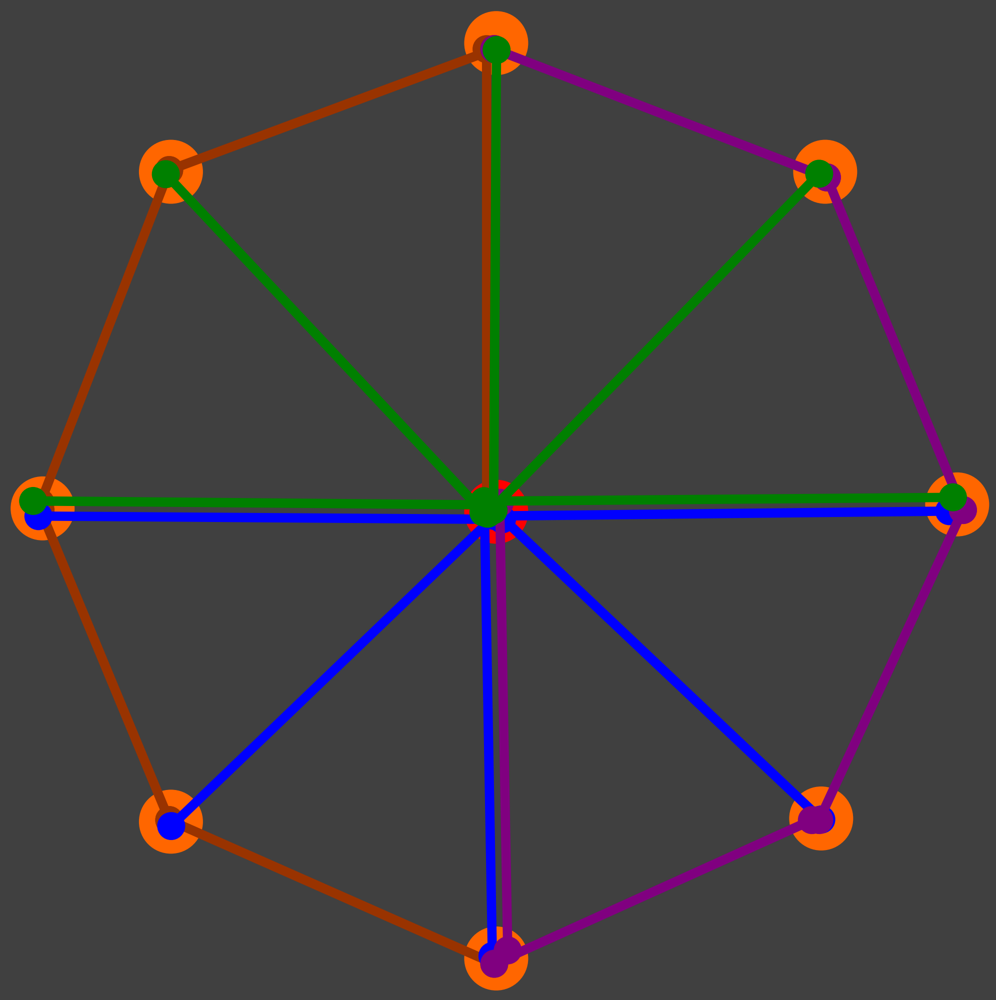
NEQ stacking

⇓

Final daily NEQ

20 baselines adjusted; each daily file is processed at least twice

The GNSS networks: warnings on CODOD



Correlations
of shared observations
cannot be taken into
account

false independently
repeated baselines

false independently
closed polygons

between
different NEQ’s
of different PF’s

Limits of CODOD

In Biagi et Sansò (Hotine Marussi, 2009):

analytical example of a leveling triangle

1. no bias in the estimates of the unknowns
2. the estimates of the variances

   are significantly biased

An alternative approach

In each daily adjustment

one connecting station is shared by all the subnets

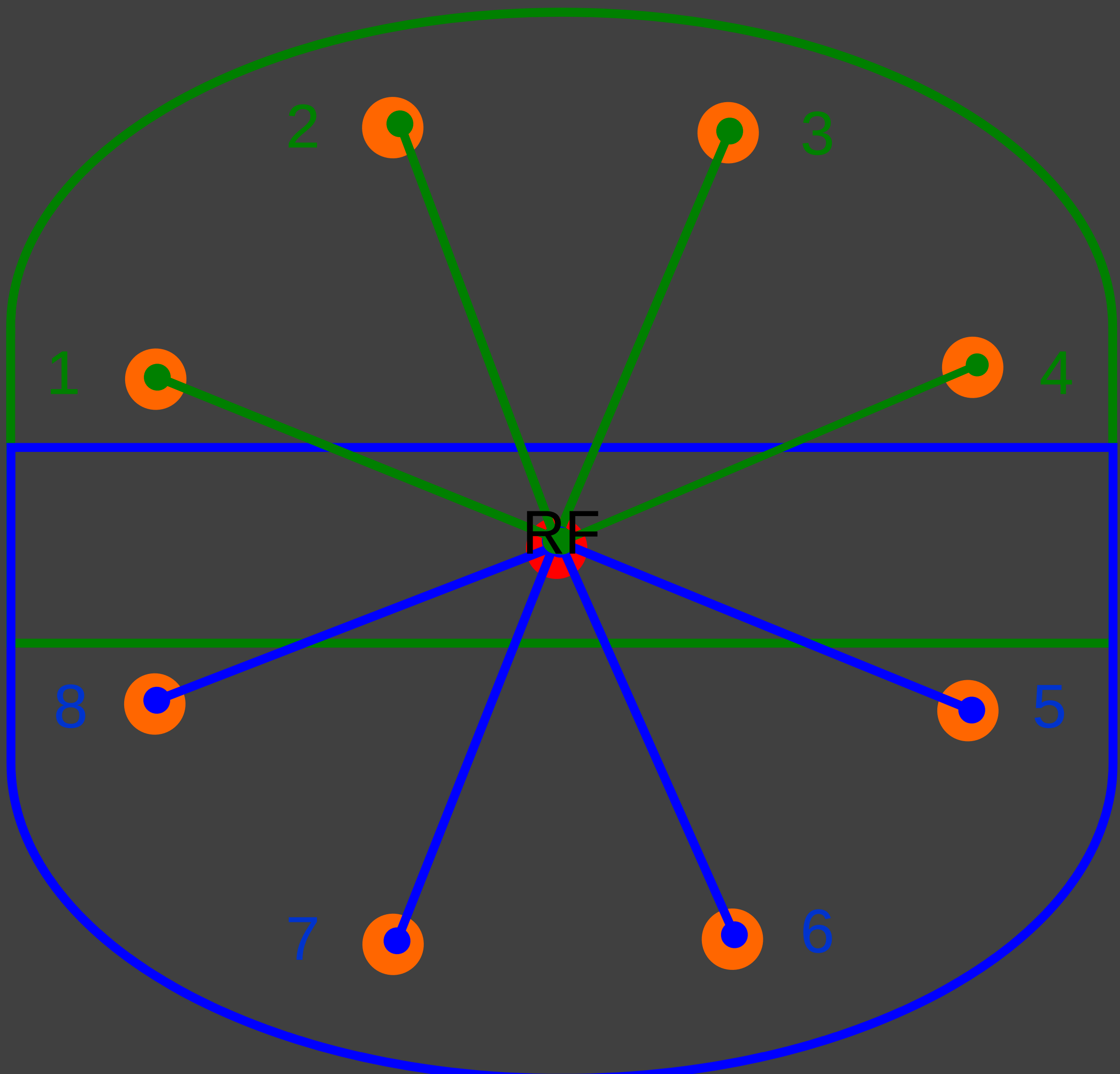
the other daily files are processed only by
one Processing Facility

The configuration of the daily subnets
varies in a cyclic way, to obtain

1. true closures and repetitions
2. cross check for all the stations

VAriable in time, Cyclically Overlapping, Distribution

VACOD: subnets daily solutions

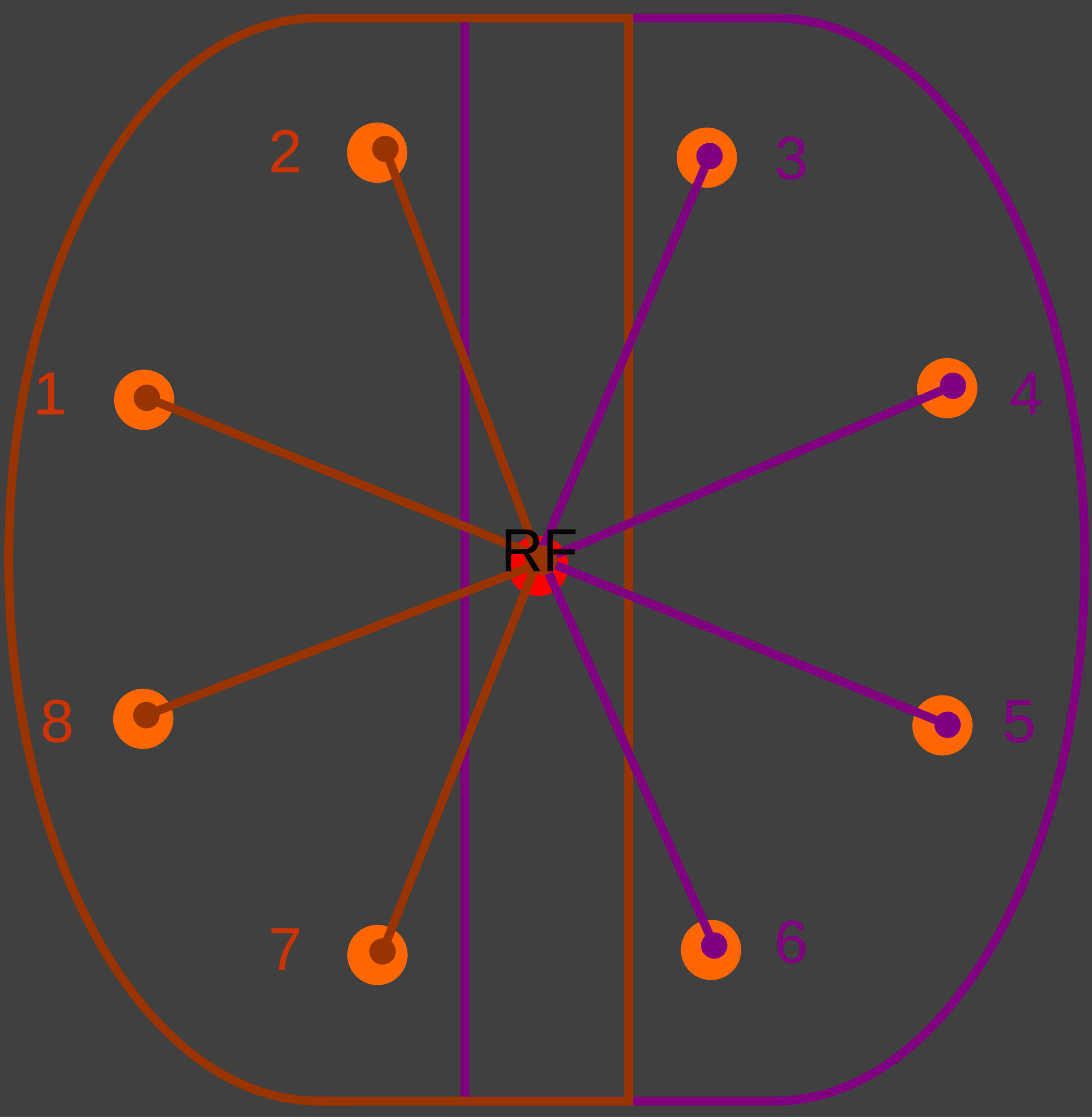


Day odd (D1)

PF1:
4 stations
+ RF station
⇓
NEQ D1-1

PF2:
4 stations
+ RF station
⇓
NEQ D1-2

VACOD: subnets daily solutions

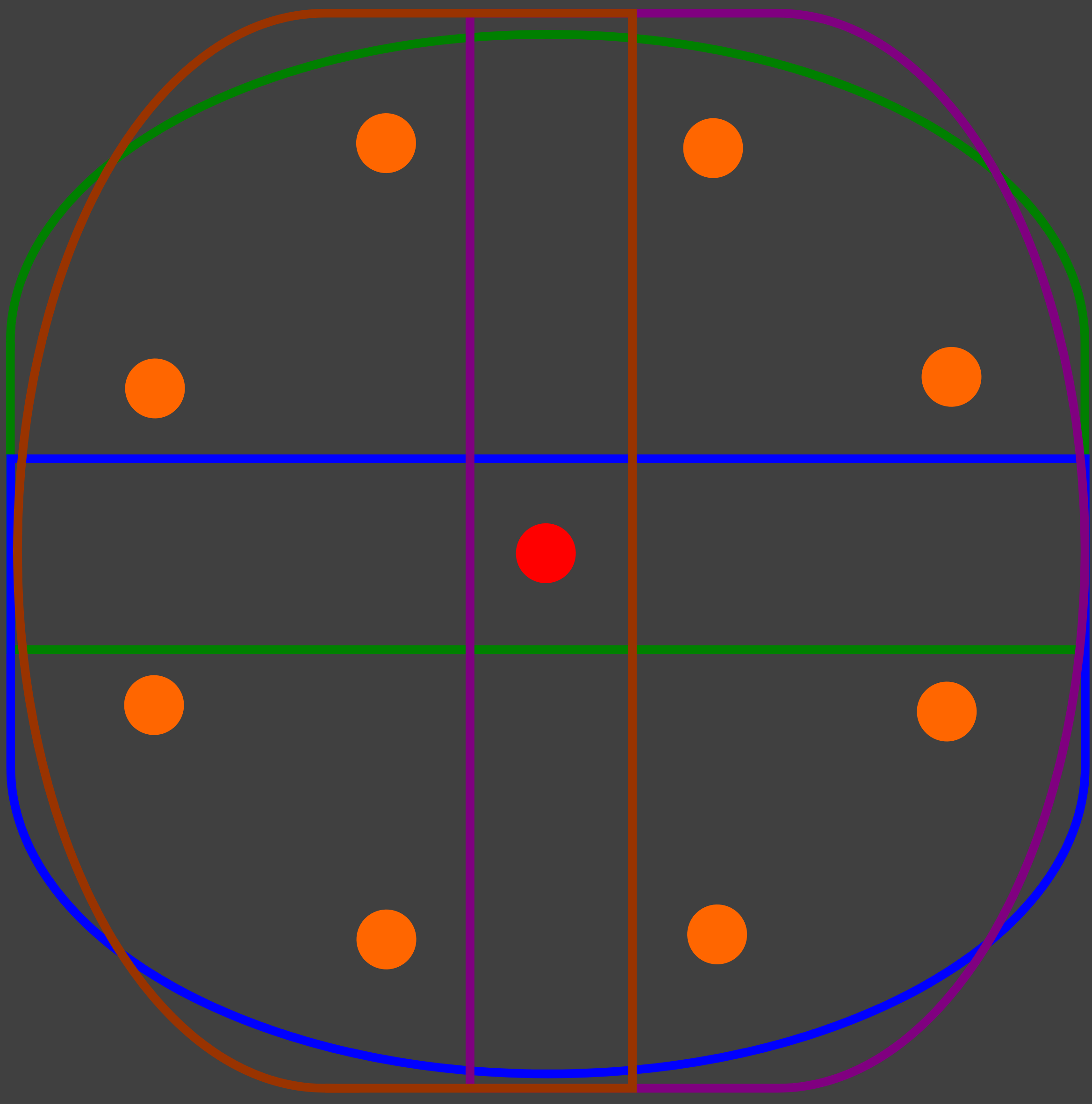


Day even (D2)

PF1:
4 stations
+ RF station
⇓
NEQ D2-1

PF2:
4 stations
+ RF station
⇓
NEQ D2-2

VACOD: final estimate



D1-1, D1-2,
D2-1, D2-2,
NEQ files

⇓

NEQ stacking

⇓

daily D1,D2
NEQ files

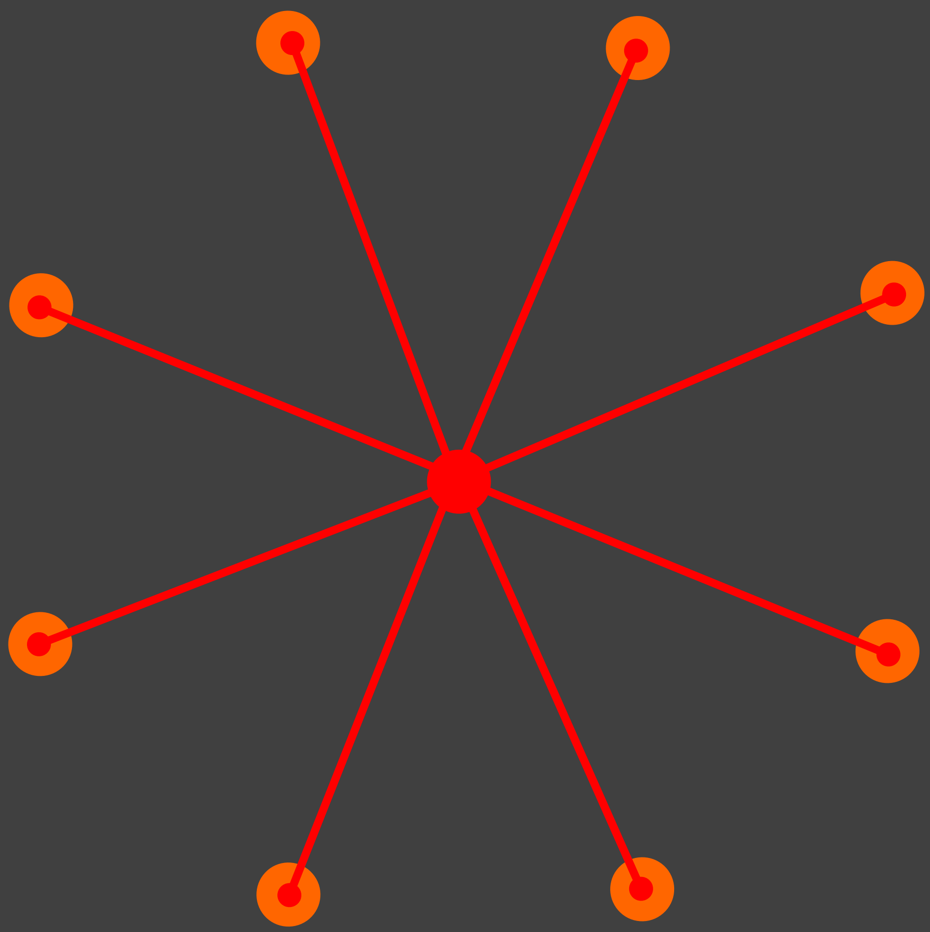
⇓

NEQ stacking

⇓

Final estimate of
the network

VACOD: remarks



In each daily solution
only open graphs

In the stacking
just the correlation
due to the
connecting station
is neglected

In the stacking over
a whole cycle
overlapping subnets
provide required
cross-checks

VACOD implementation

Administrator choices:

1. the length in days of the whole cycle

2. the daily number of subnets and the list of the stations belonging to each subnet

In each day of the cycle:

1. RINEX files are sent to the relevant Processing Facility

2. daily subnets are separately adjusted

3. daily NEQ's are sent back to the Central Center and stacked

4. at end of the cycle, a final solution is generated

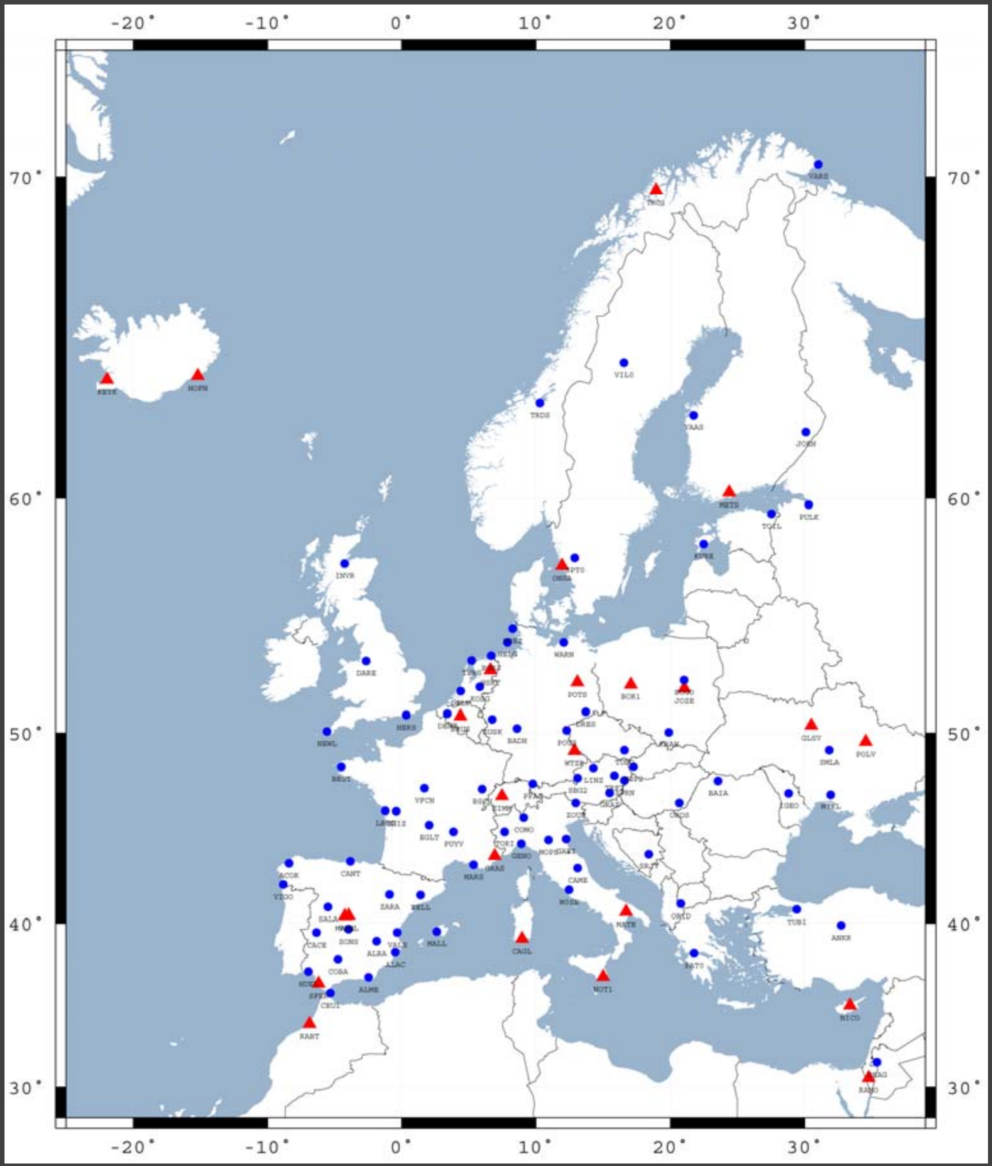
Example: a test network

102 PS's in Europe:
24 IGS stations,
78 other EPN stations
4 weeks (1550-1553),

Daily adjustment (minimal constraints on IGS stations) of the whole network

Weekly stacking
of the daily solutions

Batch benchmark solution



CODOD: the 3 subnetworks

3 daily subnetworks,

each PS in 2 subnetworks

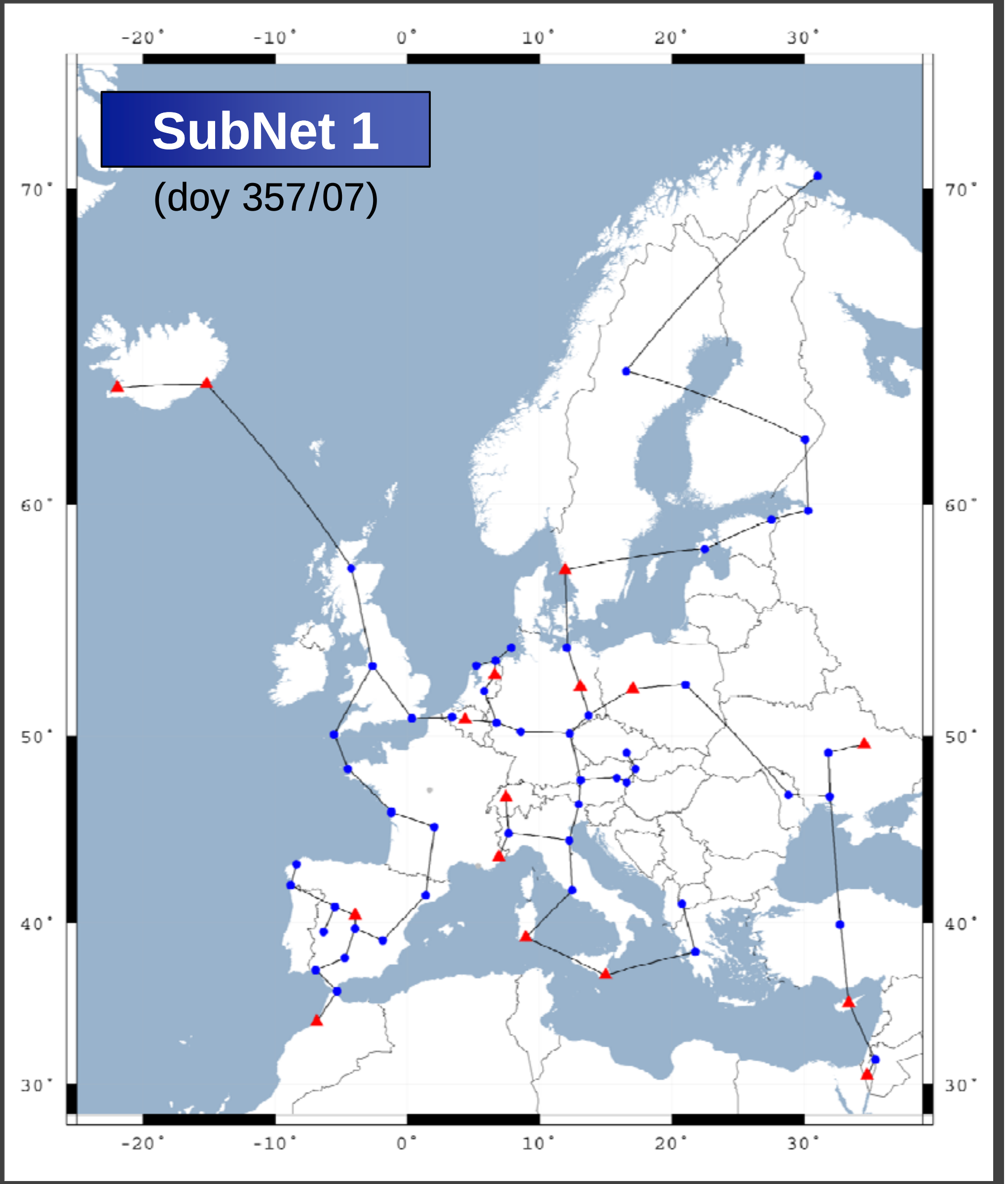
Each subnetwork:

68 PS's: 52 EPN + 16 IGS

Constant configuration of the subnets over the days

Daily stacking (minimal constraints) of the subnets

Weekly stacking of the daily NEQ's



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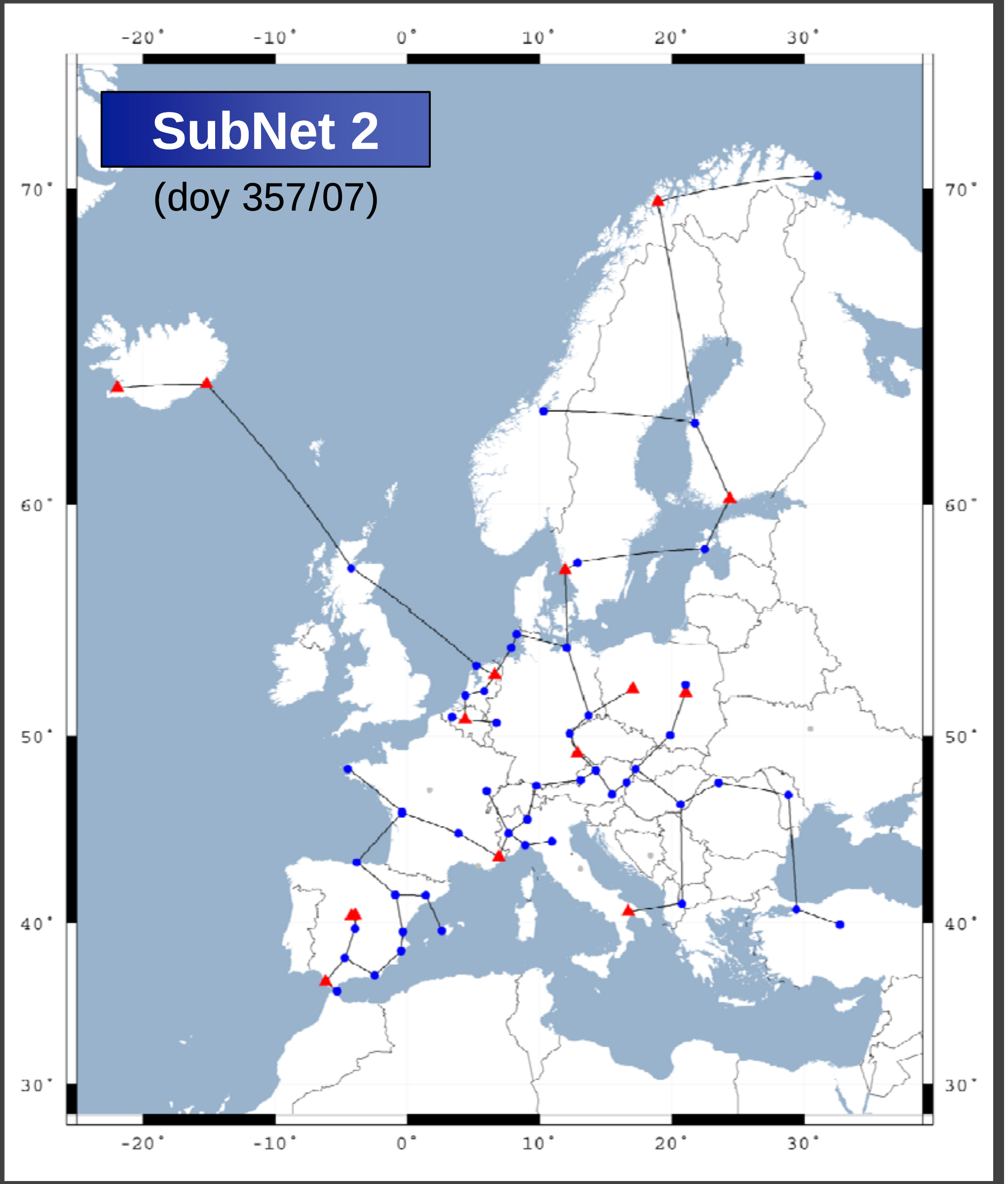

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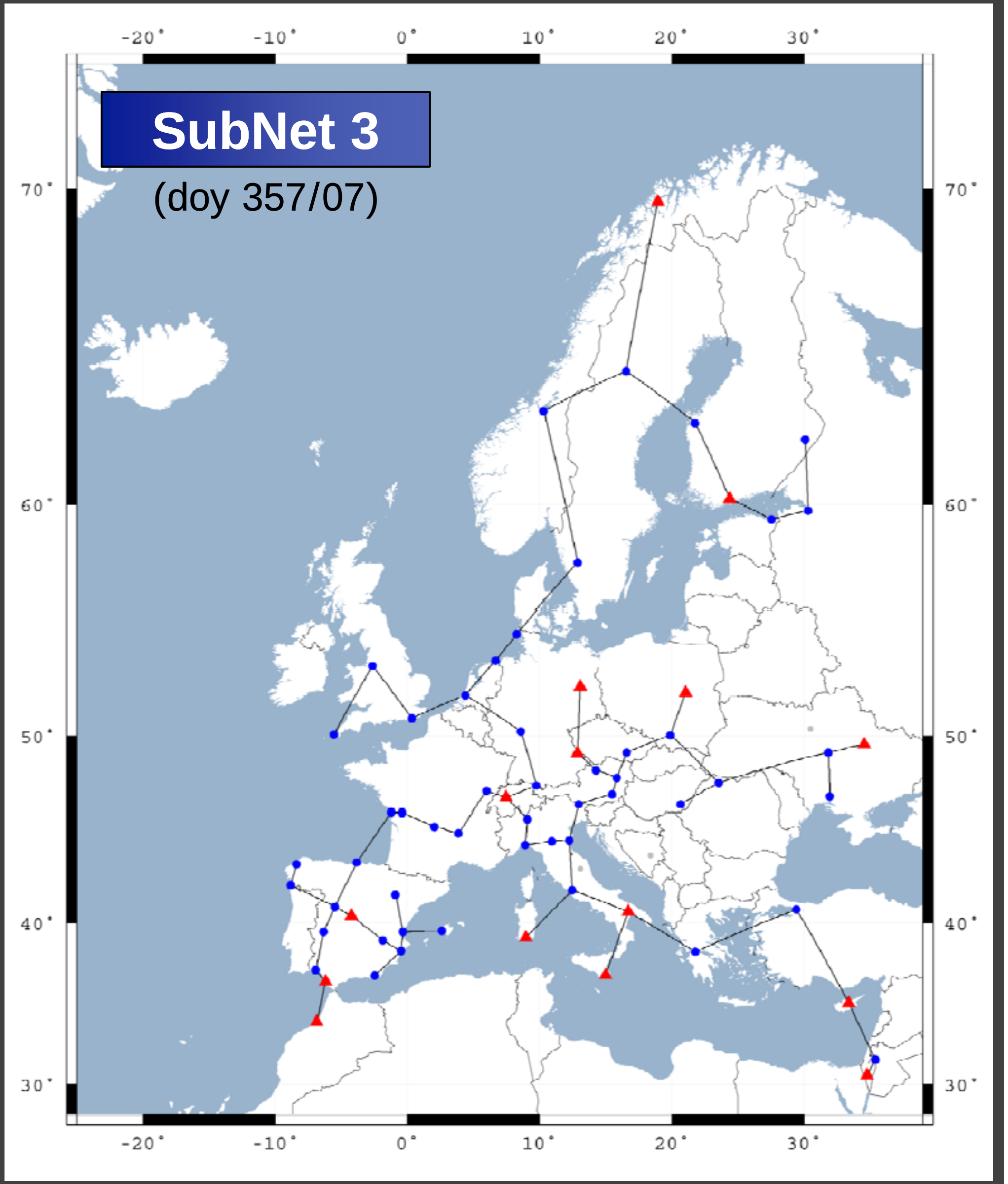


VACOD test

Each day 3 subnets

Cycle over 3 days

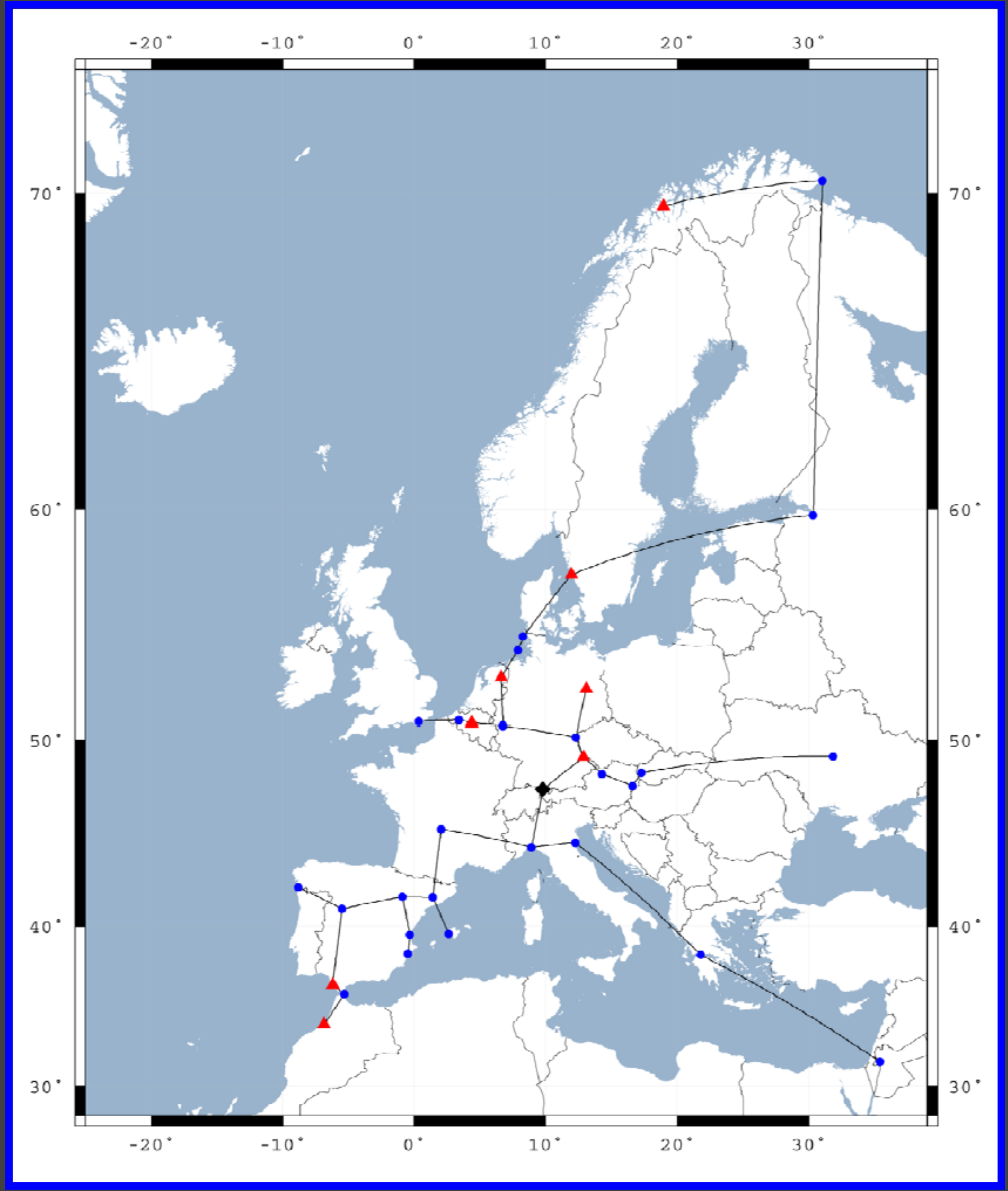
For each subnet:

1 connecting PS,

7-8 IGS PS's,

24-26 other PS's

Figure: one daily example



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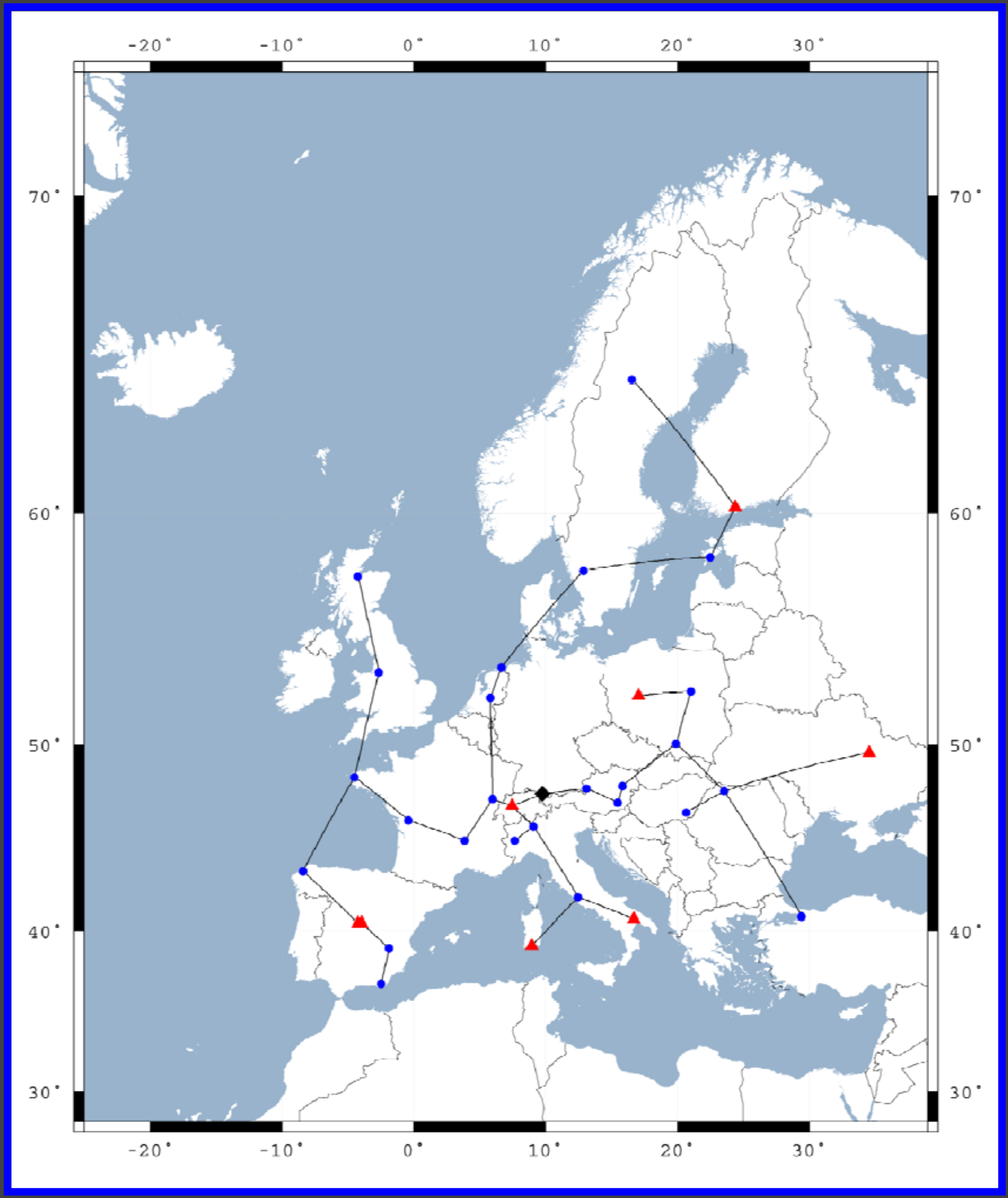

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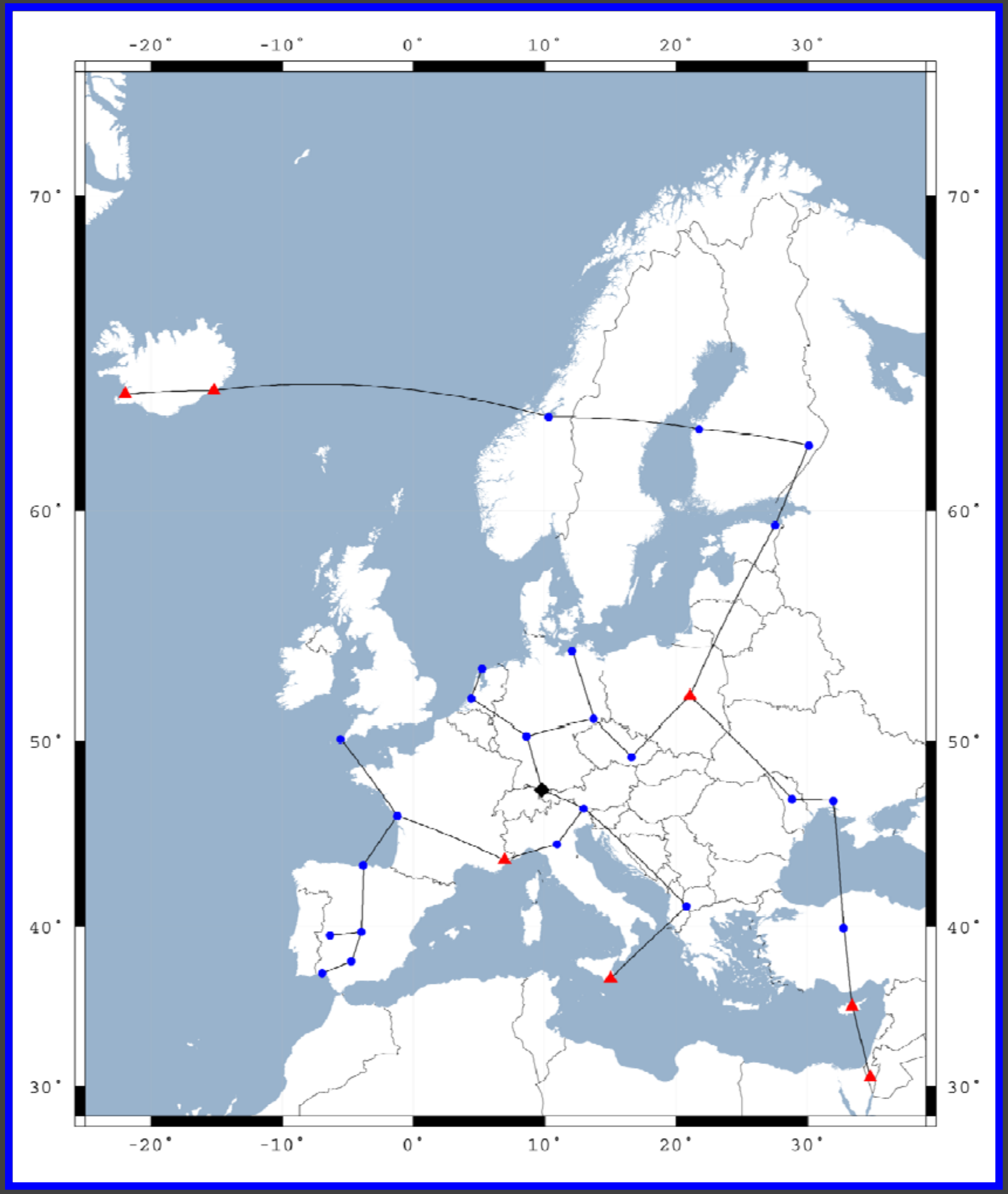


Residuals of the daily solutions

Batch			
(mm)	East	North	Up
StdDev	0.9	1.2	4.2
Worst	6.6	12.9	24.7

	CODOD			VACOD		
(mm)	East	North	Up	East	North	Up
StdDev	0.9	1.1	3.7	1.0	1.3	4.2
Worst	6.5	8.5	24.6	6.1	10.5	24.0

Batch and VACOD repeatabilities almost equal

Some Up improvements in CODOD,

due to the false redoundancies

Comparisons of the weekly results

Mean weekly numbers of observations and unknowns

	Batch	CODOD	VACOD
# Observations	6.946.460	13.765.024	6.836.252
# Unknowns	25.651	51.234	26.196
RMS (mm)	1.25	1.25	1.24

Differences of weekly coordinates wrt Batch results

(mm)	CODOD			VACOD		
	East	North	Up	East	North	Up
Mean	0.1	0.0	0.1	0.1	0.0	0.1
Std	0.2	0.3	0.7	0.2	0.3	1.0
Min	-0.7	-0.4	-2.4	-0.7	-1.0	-3.1
Max	0.7	0.9	2.0	1.3	1.0	2.9

Standard deviations of the final estimates

Batch standard deviations			
(mm)	X	Y	Z
Mean	0.1	0.1	0.1
Max	0.2	0.1	0.2

CODOD/Batch			
	X	Y	Z
Mean	0.7	0.7	0.7
Min	0.5	0.6	0.6
Max	0.8	0.8	0.8

VACOD/Batch		
X	Y	Z
1.0	1.0	1.0
0.8	0.8	0.8
1.0	1.1	1.0

CODOD depresses covariances,
VACOD provides correct estimates
but for connecting stations

Conclusions

CODOD approach (standard) to split
the adjustment of big networks provide
biased estimates of the covariances

An alternative approach (VACOD) to split
a network into overlapping subnetworks
provides correct covariances estimates

A prototype has been
implemented and tested on a realistic example

Future work

Is it possible to tune the CODOD covariances?

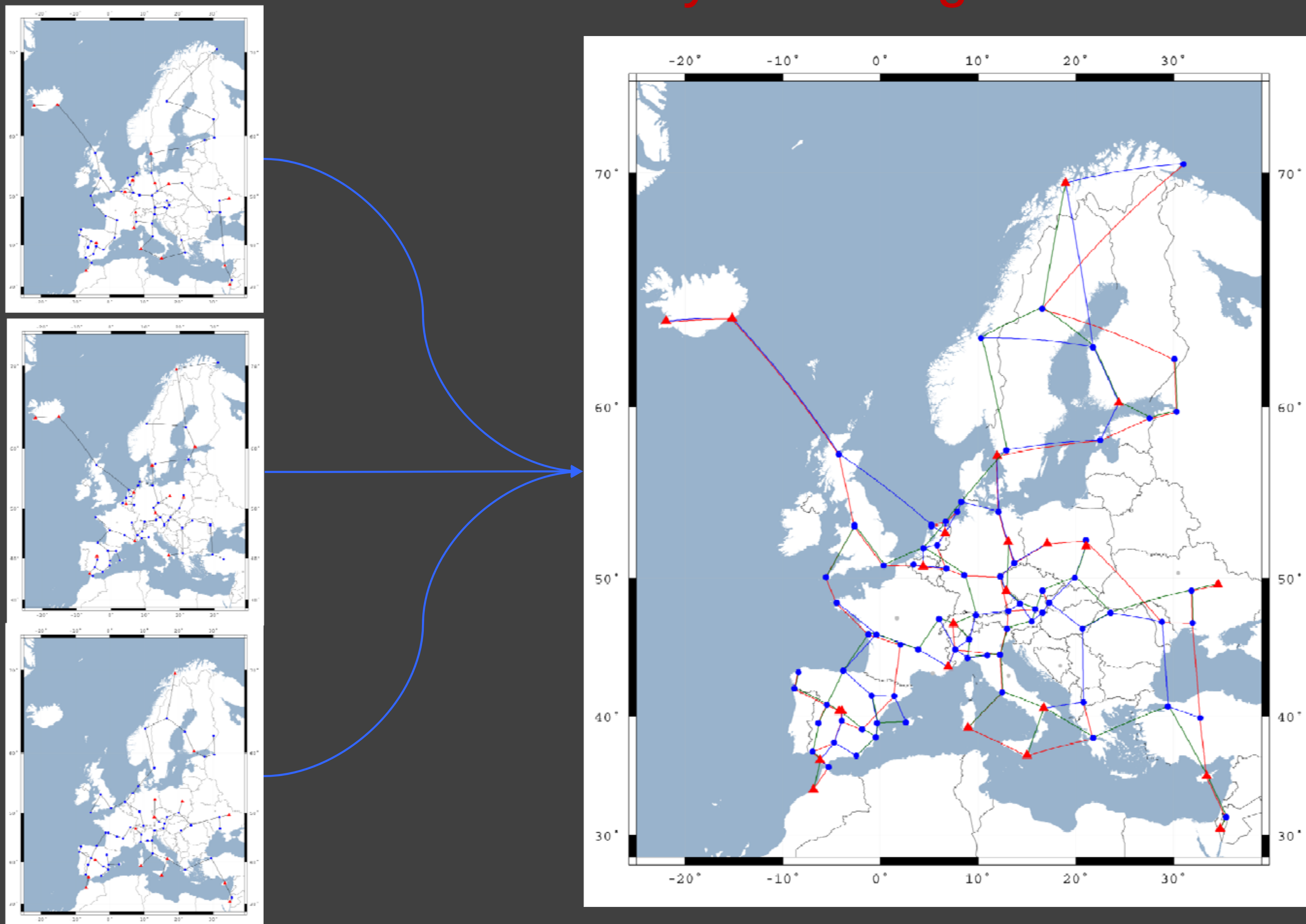
For example

$$C_{\hat{x}\hat{x}} = \mathbf{\Lambda} C_{\hat{x}\hat{x}}^{CODOD} \mathbf{\Lambda}$$

$$\lambda_{ij} = \begin{cases} k_i, i = j \\ 0, i \neq j \end{cases}, \quad k_i = \sqrt{\frac{r_i^{CODOD}}{r_i^{TRUE}}}$$

Future investigations are needed!

CODOD: the daily stacking



VACOD implementation

Implemented 2/2

user choices: the length in days of the whole cycle and the number of subnets

On each day of the cycle:

1.a the connecting (IGS) station is the barycentric one

1.b the other stations are randomly attributed to one and only one subnet

2.a RINEX files are sent to the proper Processing Facility

2.b daily subnets are separately adjusted

2.c daily NEQ’s are sent back to the Central Facility and stacked

3. at end of the cycle, a final solution is generated