Estimation of the strain rate tensor from GPS observations in the Ukraine area

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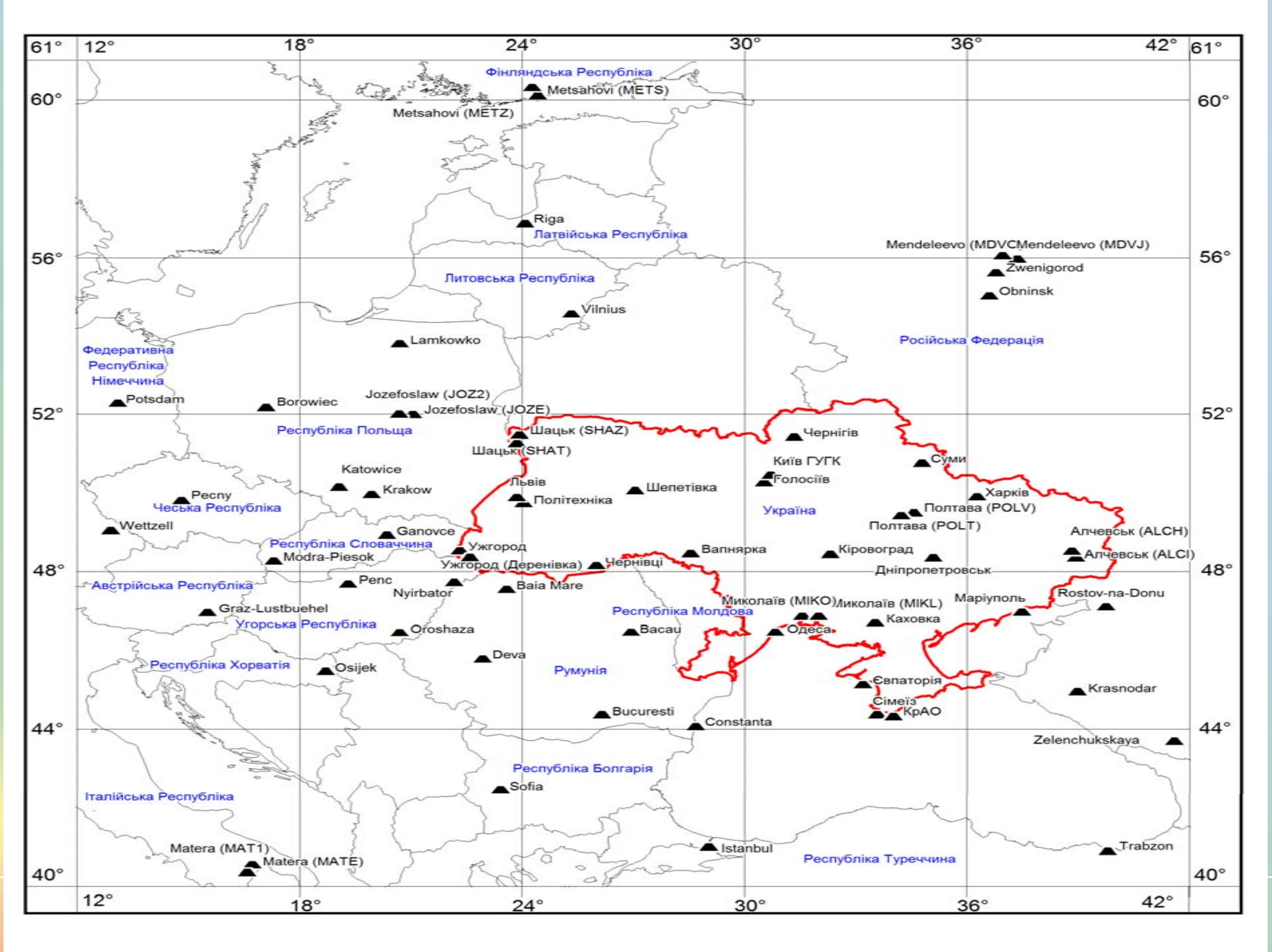
Introduction

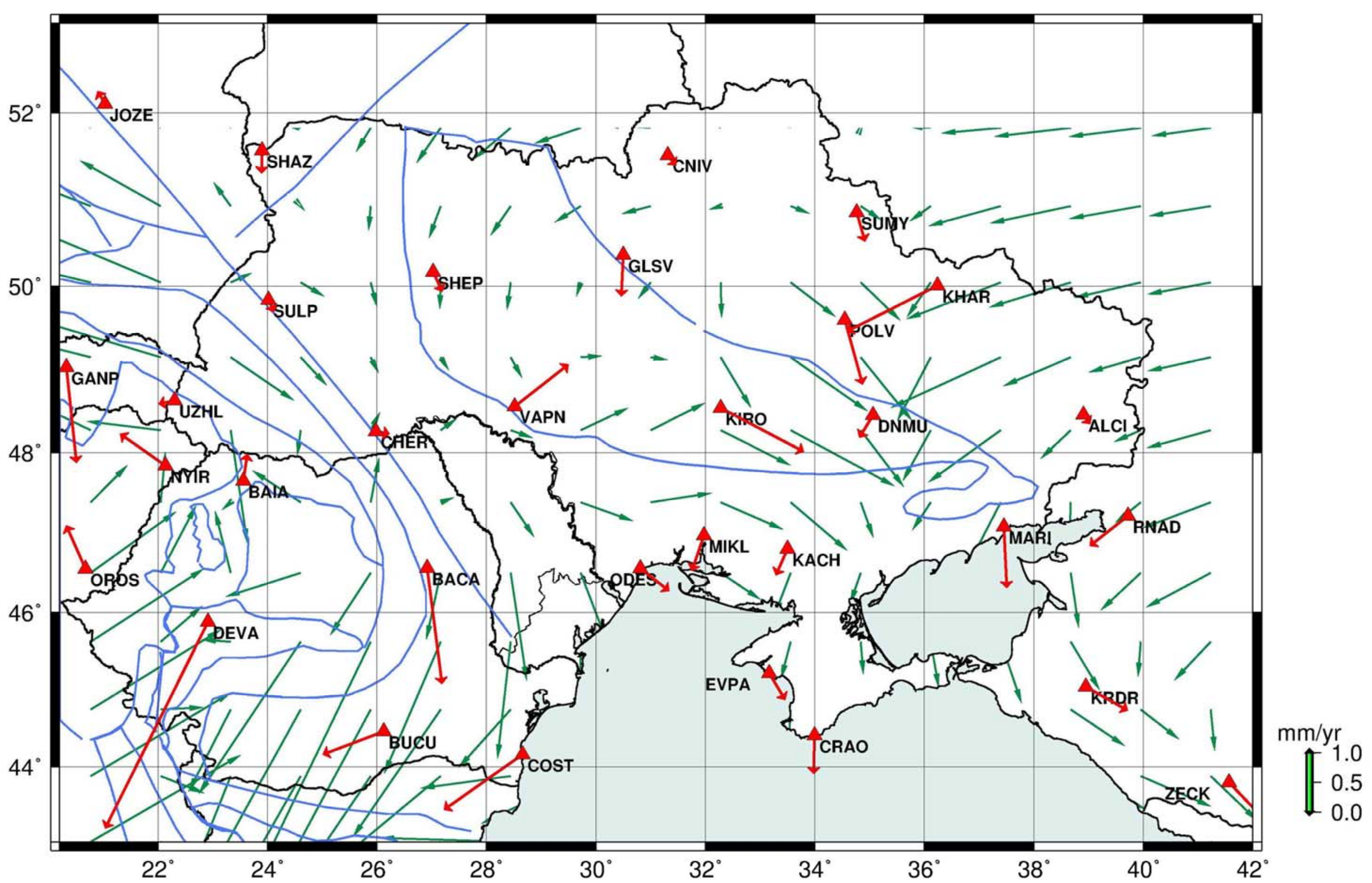
2D velocities field and strain rate tensor in the Ukraine area were developed in the following steps:

- ***first one is based on the finite element approach for the approximation by basis bicubic spline functions as well as collocation method for the densification of velocity field,***
- ***the second one represents the inversion of velocities from GPS observations to the strain rate tensor,***
- ***finally the eigenvalue/eigenvector problem was solved for the estimation of the strain rate tensor from GPS observations in the Ukraine area.***

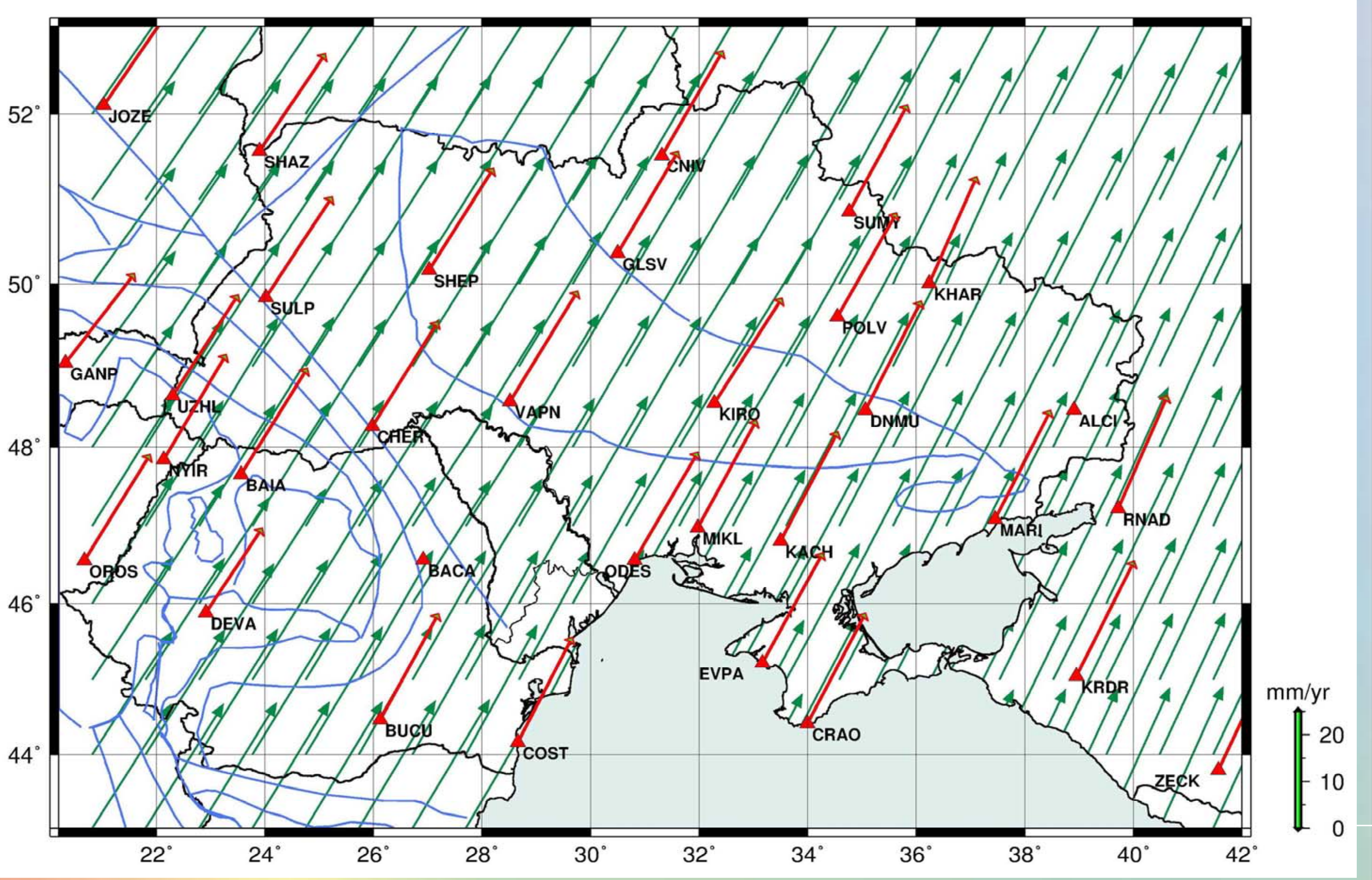
Ukrainian network of permanent and periodically observed GNSS-stations (R. Vysotenko, 2010)



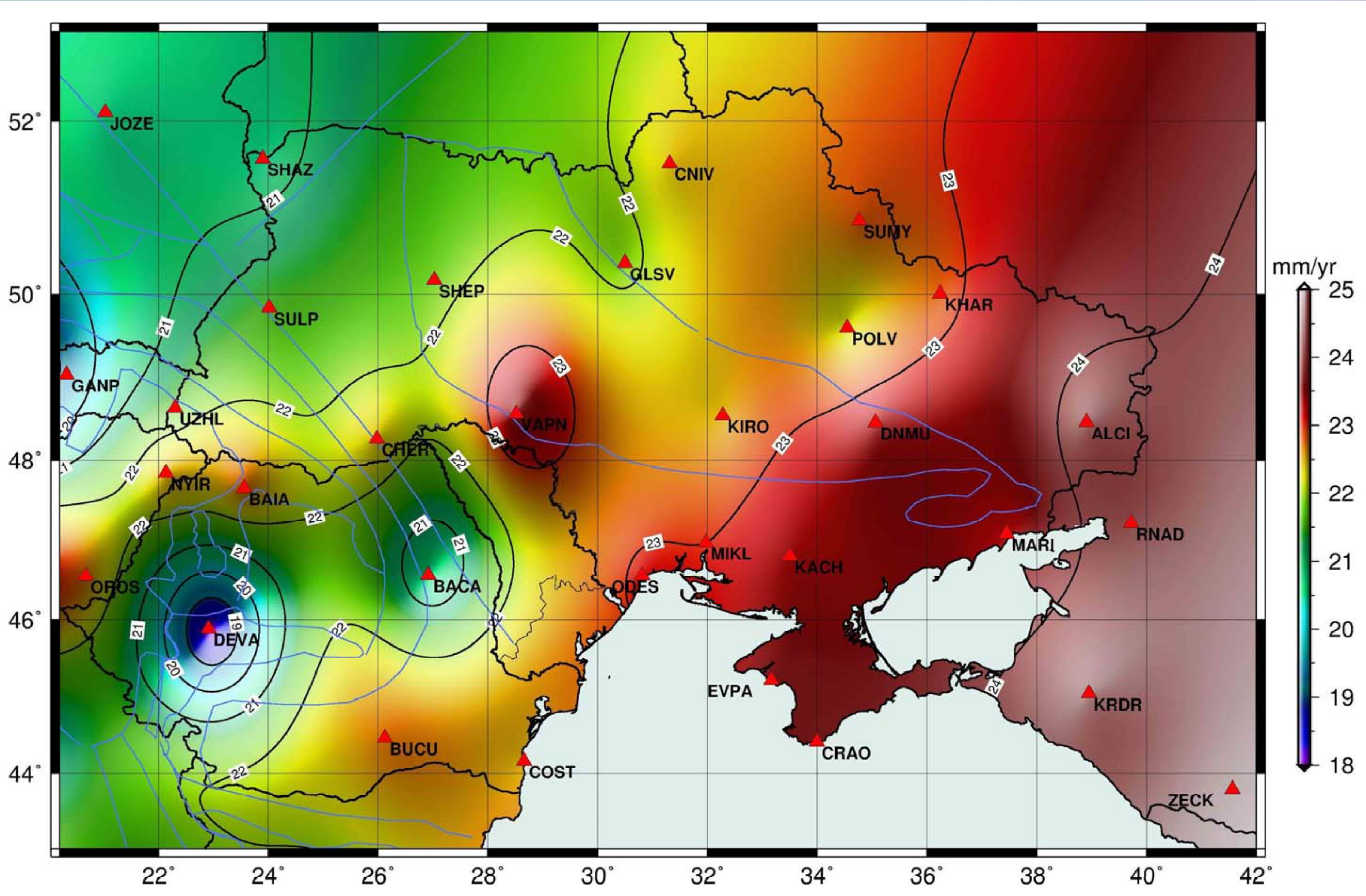
Velocities in the Ukraine area (ETRS89 system); (-) basic geological structures and borders



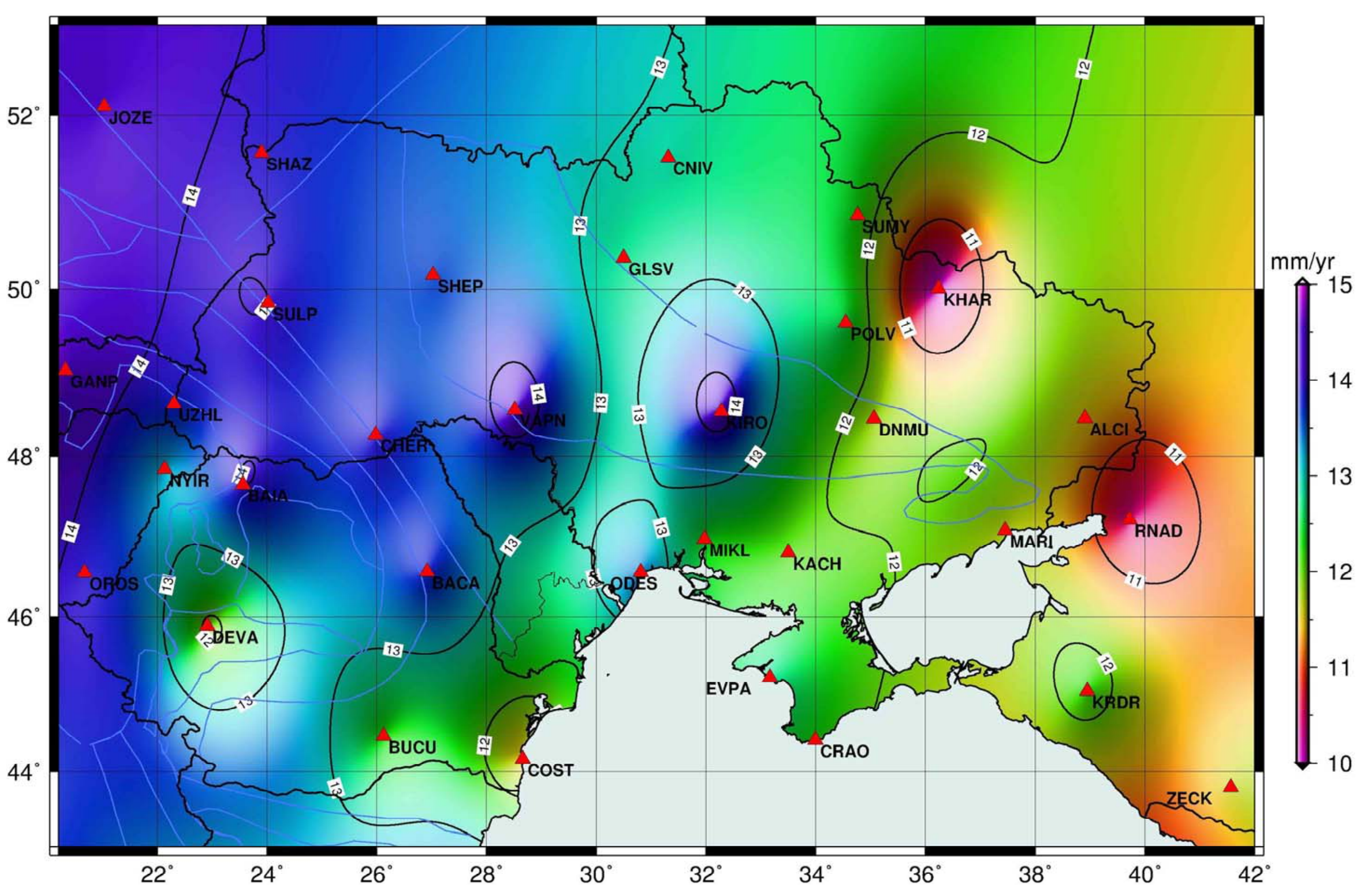
Velocities in the Ukraine area (ITRF2005 system); (-) basic geological structures and borders



Eastern component V_E of velocities in the Ukraine area (ITRF2005 system); (-) basic geological structures



Northern component V_N of velocities in the Ukraine area (ITRF2005 system); (-) basic geological structures



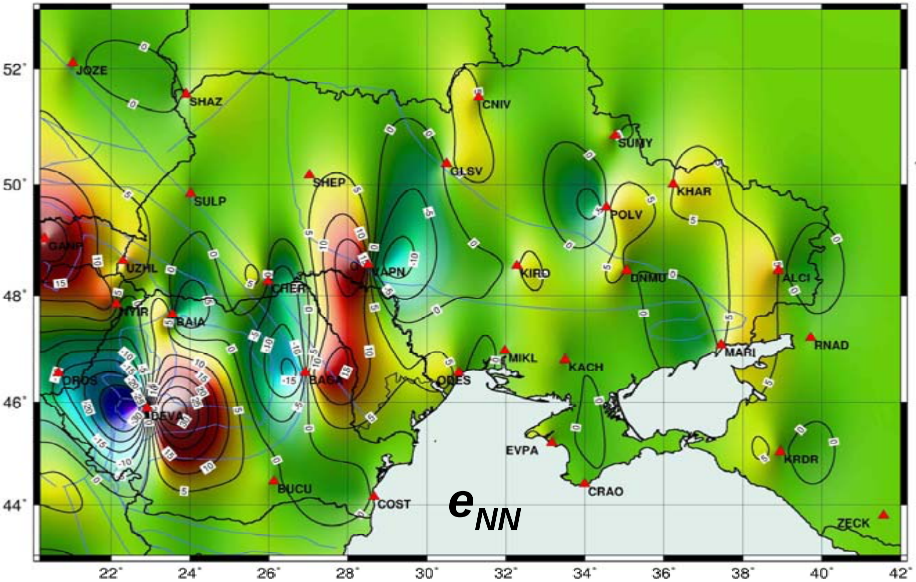
2D strain rate tensor and rotation tensors (Haines, Holt, 1993; England, Molnar, 1997; Kreemer, 2000)

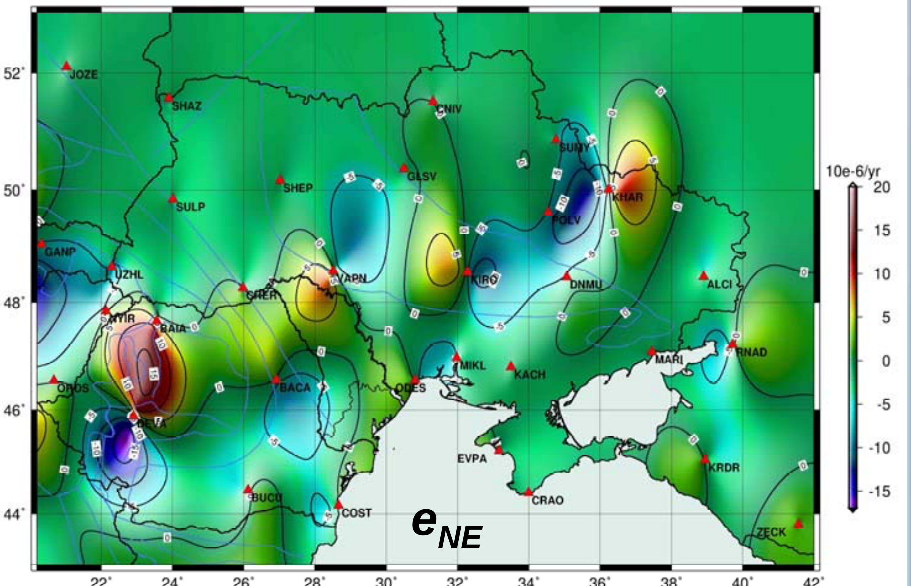
$$\mathbf{S}_V = \begin{bmatrix} \dfrac{\partial V_N}{\partial \varphi} & \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} + \dfrac{\partial V_E}{\partial \varphi}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial V_N}{\partial \lambda} + \dfrac{\partial V_E}{\partial \varphi}\right) & \dfrac{\partial V_E}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \dot{\varepsilon}_{\varphi\varphi} & \dot{\varepsilon}_{\varphi\lambda} \\ \dot{\varepsilon}_{\varphi\lambda} & \dot{\varepsilon}_{\lambda\lambda} \end{bmatrix}$$

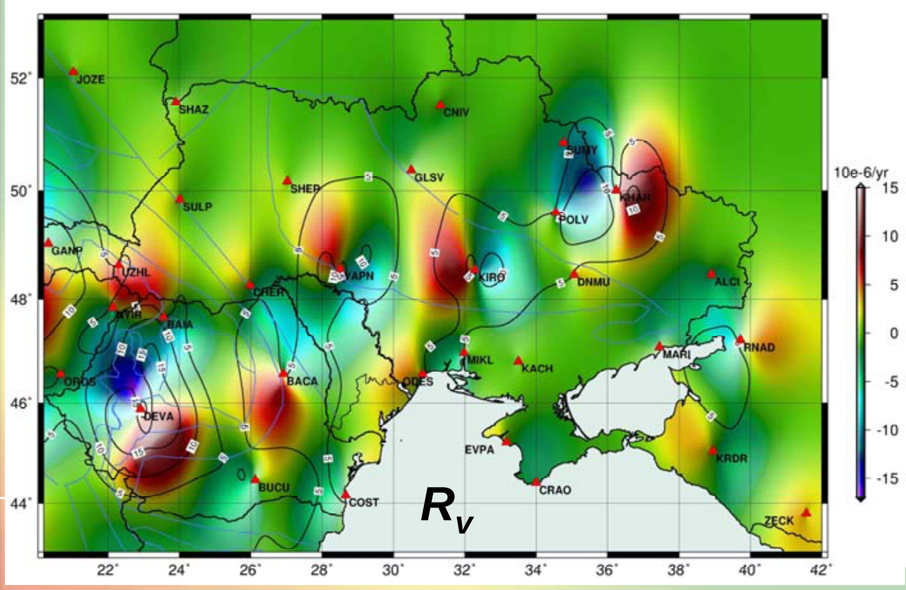
$$\mathbf{R}_V = \begin{bmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{bmatrix} = \dot{\omega} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

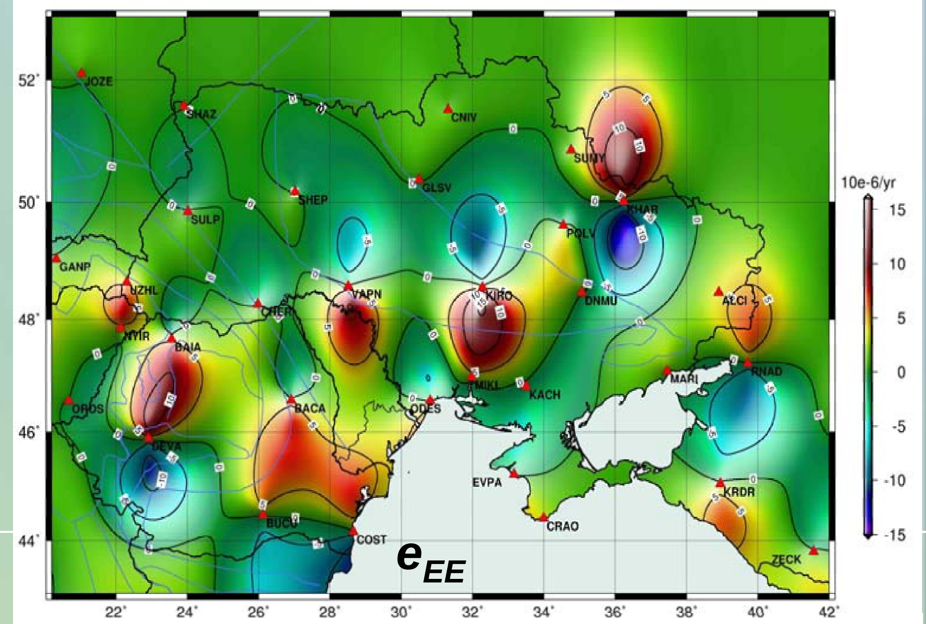
$$\left.\begin{aligned} \dot{\varepsilon}_{\lambda\lambda} &= \frac{\tilde{\mathbf{n}}}{\cos\varphi}\frac{\partial \mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial \lambda}, \\ \dot{\varepsilon}_{\varphi\varphi} &= -\tilde{\mathbf{e}}\frac{\partial \mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial \varphi}, \\ \dot{\varepsilon}_{\varphi\lambda} &= \frac{1}{2}\left(\tilde{\mathbf{n}}\frac{\partial \mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial \varphi} - \frac{\tilde{\mathbf{e}}}{\cos\varphi}\frac{\partial \mathbf{\Omega}(\tilde{\mathbf{r}})}{\partial \lambda}\right), \end{aligned}\right\}$$

2D strain rate and rotation tensors









3D strain tensor and its invariants

$$\mathbf{T} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{12} & t_{22} & t_{23} \\ t_{13} & t_{23} & t_{33} \end{bmatrix} = \begin{bmatrix} t_{xx} & t_{xy} & t_{xz} \\ t_{xy} & t_{yy} & t_{yz} \\ t_{xz} & t_{yz} & t_{zz} \end{bmatrix}$$

$$\mathbf{T} = \tilde{\mathbf{T}} + \mathbf{D} \qquad \tilde{\mathbf{T}} = \chi \cdot \mathbf{I} = \frac{\mathrm{trace}\mathbf{T}}{3} \cdot \mathbf{I}$$

$$\mathrm{trace}\mathbf{T} = t_{xx} + t_{yy} + t_{zz}$$

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} = \begin{bmatrix} \frac{2t_{xx} - t_{yy} - t_{zz}}{3} & t_{xy} & t_{xz} \\ t_{xy} & \frac{-t_{xx} + 2t_{yy} - t_{zz}}{3} & t_{yz} \\ t_{xz} & t_{yz} & \frac{-t_{xx} - t_{yy} + 2t_{zz}}{3} \end{bmatrix}$$

$$I_2 = -\frac{1}{2} \left\| \mathbf{D} \right\|_{R^3}^2 < 0$$

$$\left\| \mathbf{D} \right\|_{R^3}^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} d_{ij}^2$$

$$I_3 = \det \mathbf{D}$$

Solution of eigenvalue/eigenvector problem

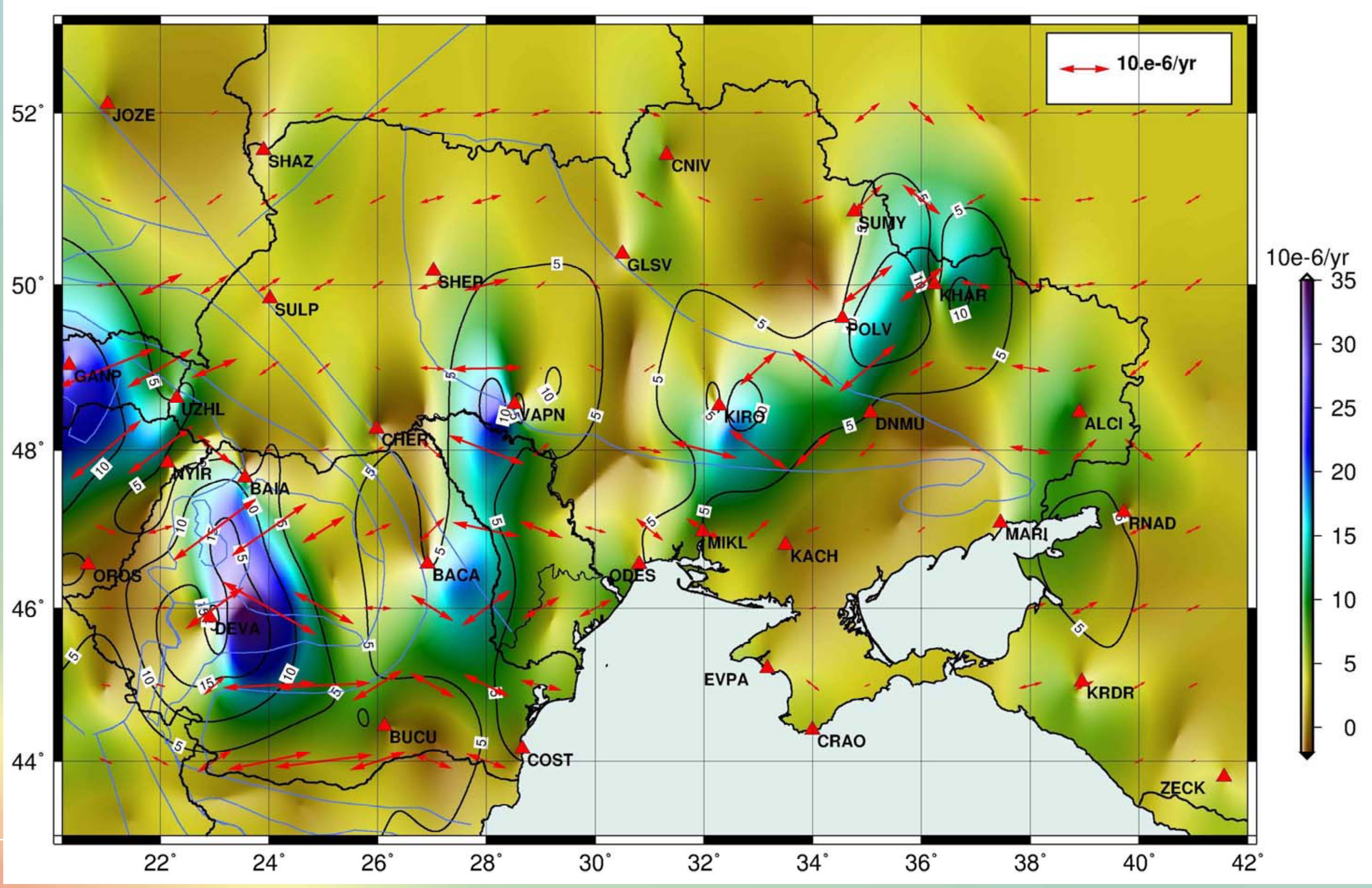
$$\left.\begin{matrix}\Lambda_1\\ \Lambda_2\\ \Lambda_3\end{matrix}\right\} = \chi + \left\{\begin{matrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{matrix}\right. \qquad \left.\begin{matrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{matrix}\right\} = \sqrt{\frac{2}{3}}\cdot\left\|\mathbf{D}\right\|_{R^3}\cdot\left\{\begin{matrix}\tilde{\lambda}_1\\ \tilde{\lambda}_2\\ \tilde{\lambda}_3\end{matrix}\right. \qquad \left.\begin{matrix}\tilde{\lambda}_1\\ \tilde{\lambda}_2\\ \tilde{\lambda}_3\end{matrix}\right\} = \left\{\begin{matrix}\sin\left(\frac{\varphi}{3}-\frac{\pi}{3}\right)\\ -\sin\frac{\varphi}{3}\\ \sin\left(\frac{\varphi}{3}+\frac{\pi}{3}\right)\end{matrix}\right.$$

$$\varphi = \sin^{-1}\left(\frac{3\sqrt{6}I_3}{\left\|\mathbf{D}\right\|_{R^3}^3}\right) \qquad \mathbf{X}_j = \frac{1}{\sqrt{\mathbf{Z}_j^{\mathrm{T}}\mathbf{Z}_j}}\mathbf{Z}_j \qquad \mathbf{Z}_j = \mathbf{P} + \lambda_j\mathbf{S} + \lambda_j^2\mathbf{E}$$

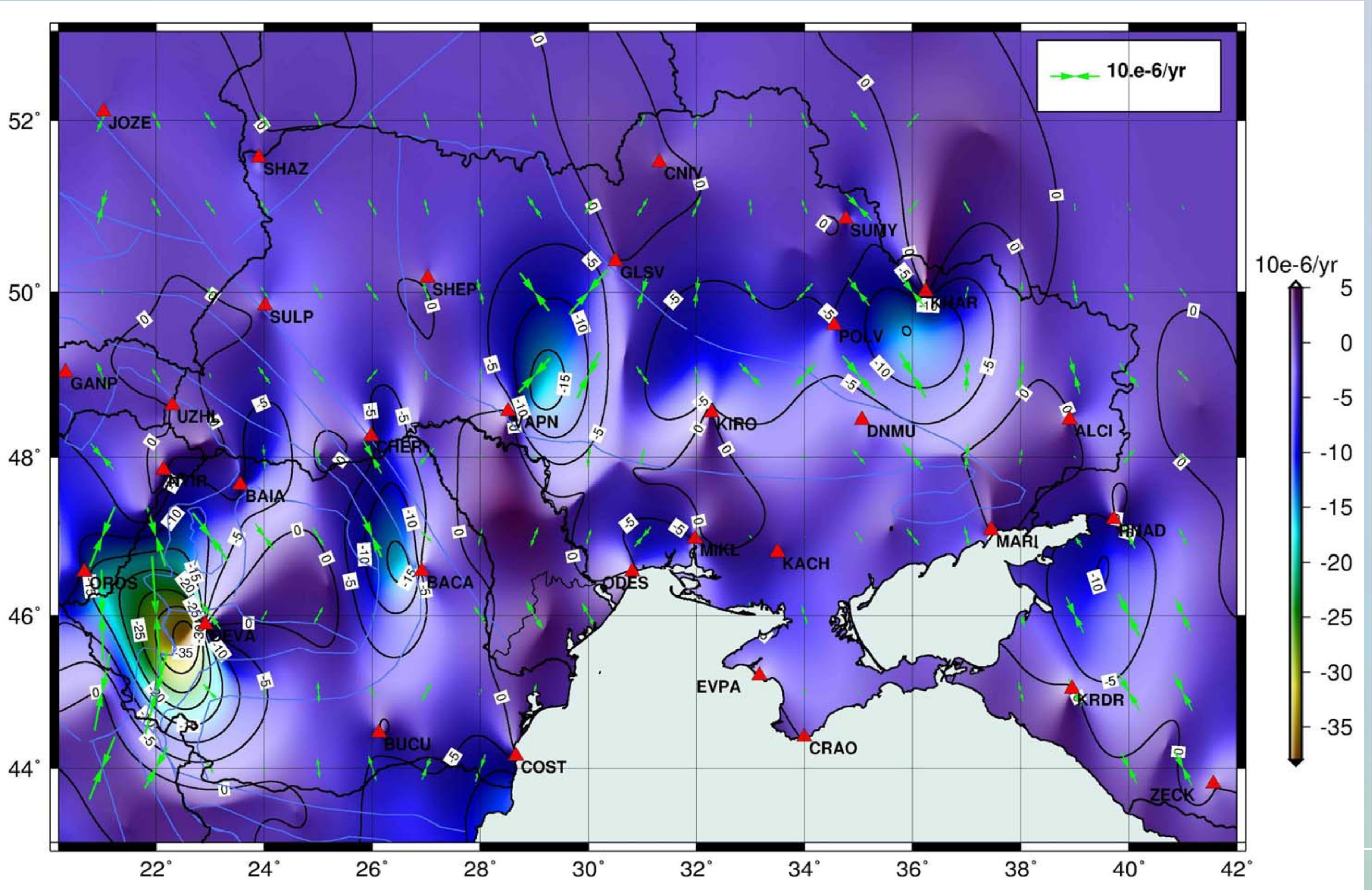
$$\mathbf{P} = \mathbf{t}_1\times\mathbf{t}_2 + \mathbf{t}_2\times\mathbf{t}_3 + \mathbf{t}_3\times\mathbf{t}_1 \qquad \mathbf{S} = \mathbf{t}_1 + \mathbf{t}_2 + \mathbf{t}_3 \qquad \mathbf{E} = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{t}_1 = \begin{bmatrix}\dfrac{2t_{xx}-t_{yy}-t_{zz}}{3}\\ t_{xy}\\ t_{xz}\end{bmatrix} \qquad \mathbf{t}_2 = \begin{bmatrix}t_{xy}\\ \dfrac{2t_{yy}-t_{xx}-t_{zz}}{3}\\ t_{yz}\end{bmatrix} \qquad \mathbf{t}_3 = \begin{bmatrix}t_{xz}\\ t_{yz}\\ \dfrac{2t_{zz}-t_{xx}-t_{yy}}{3}\end{bmatrix}$$

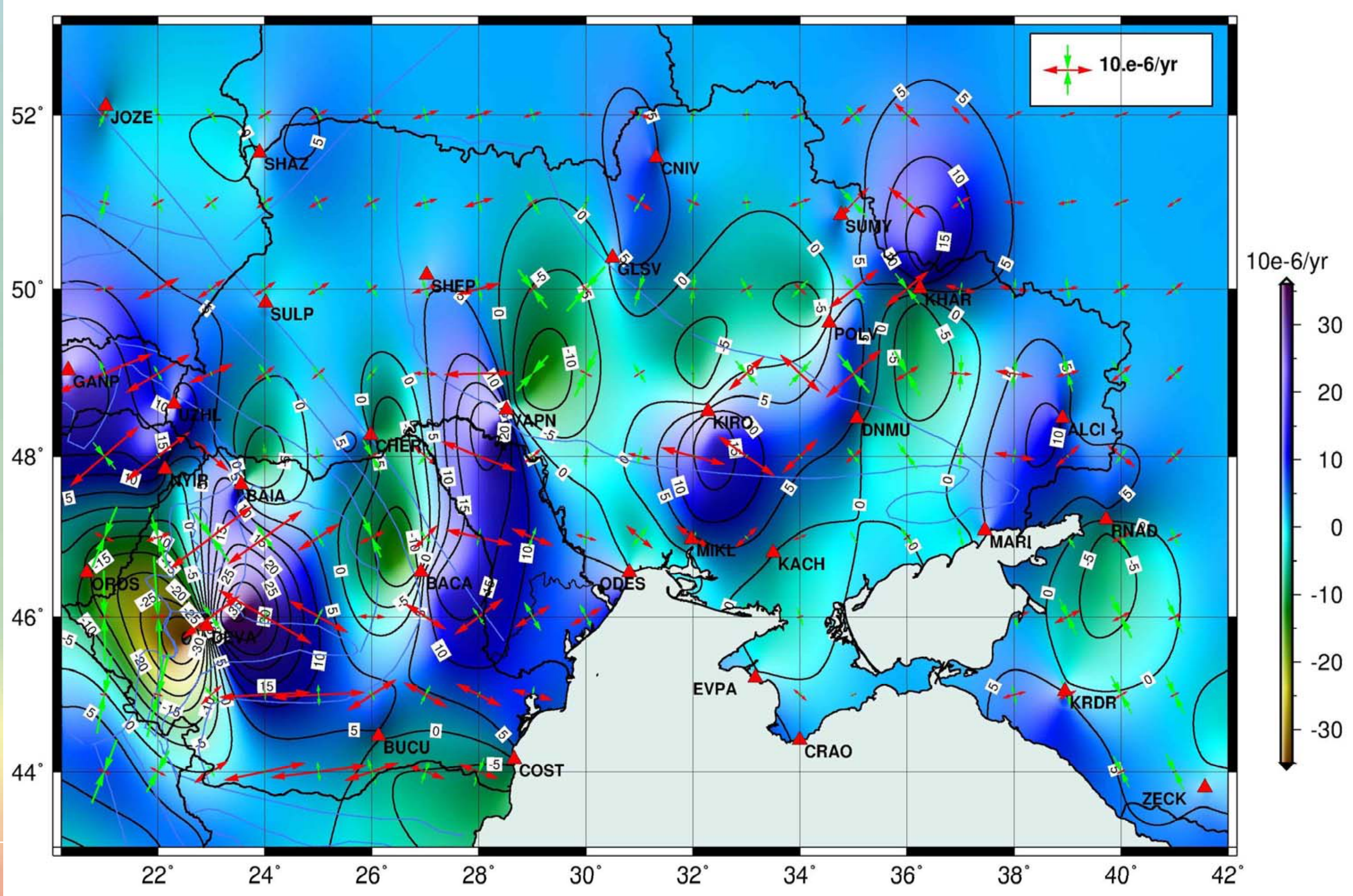
Maximum eigenvalue-eigenvector Λ_1 (extension)



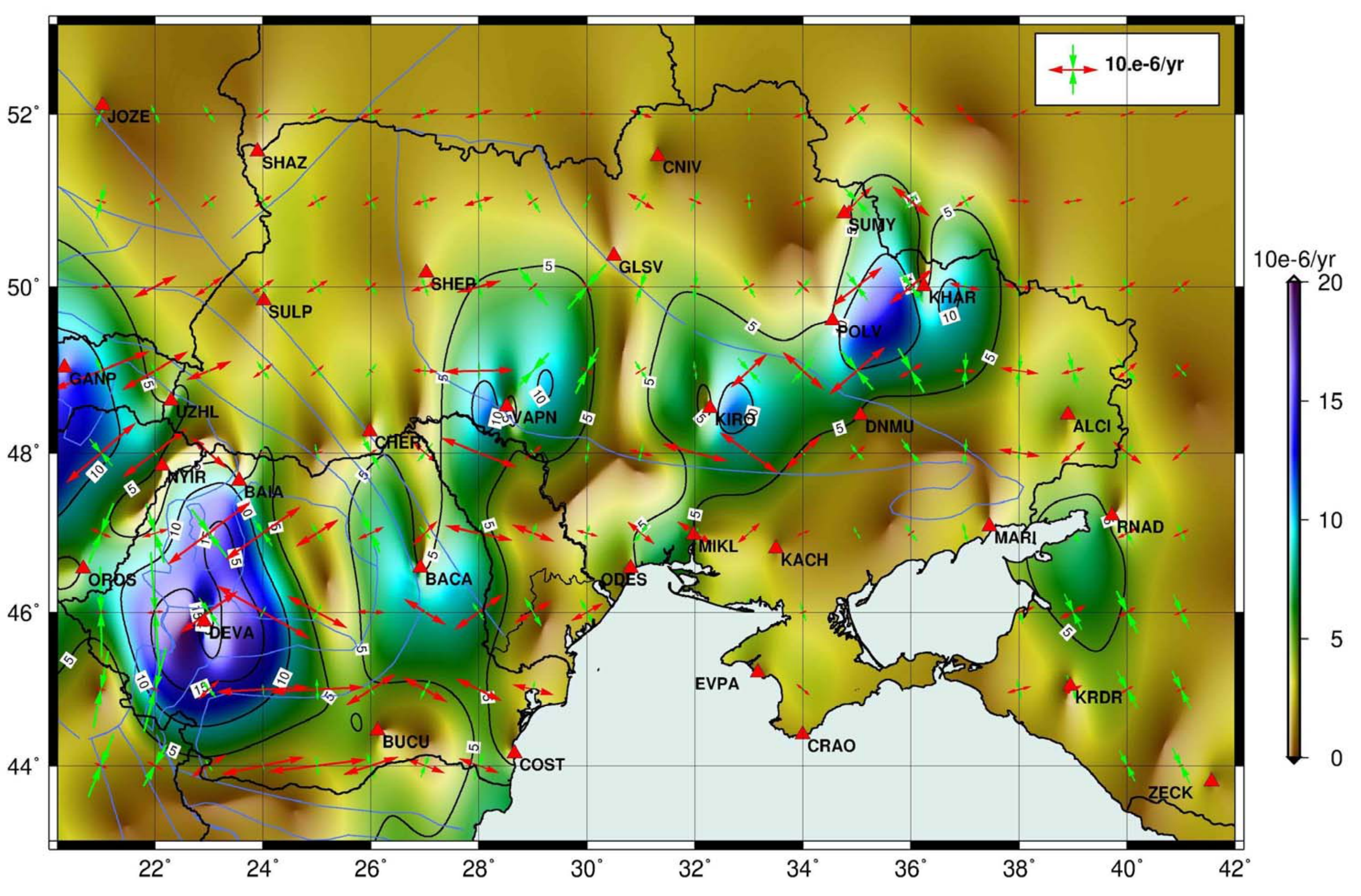
Minimum eigenvalue-eigenvector Λ_2 (compression)



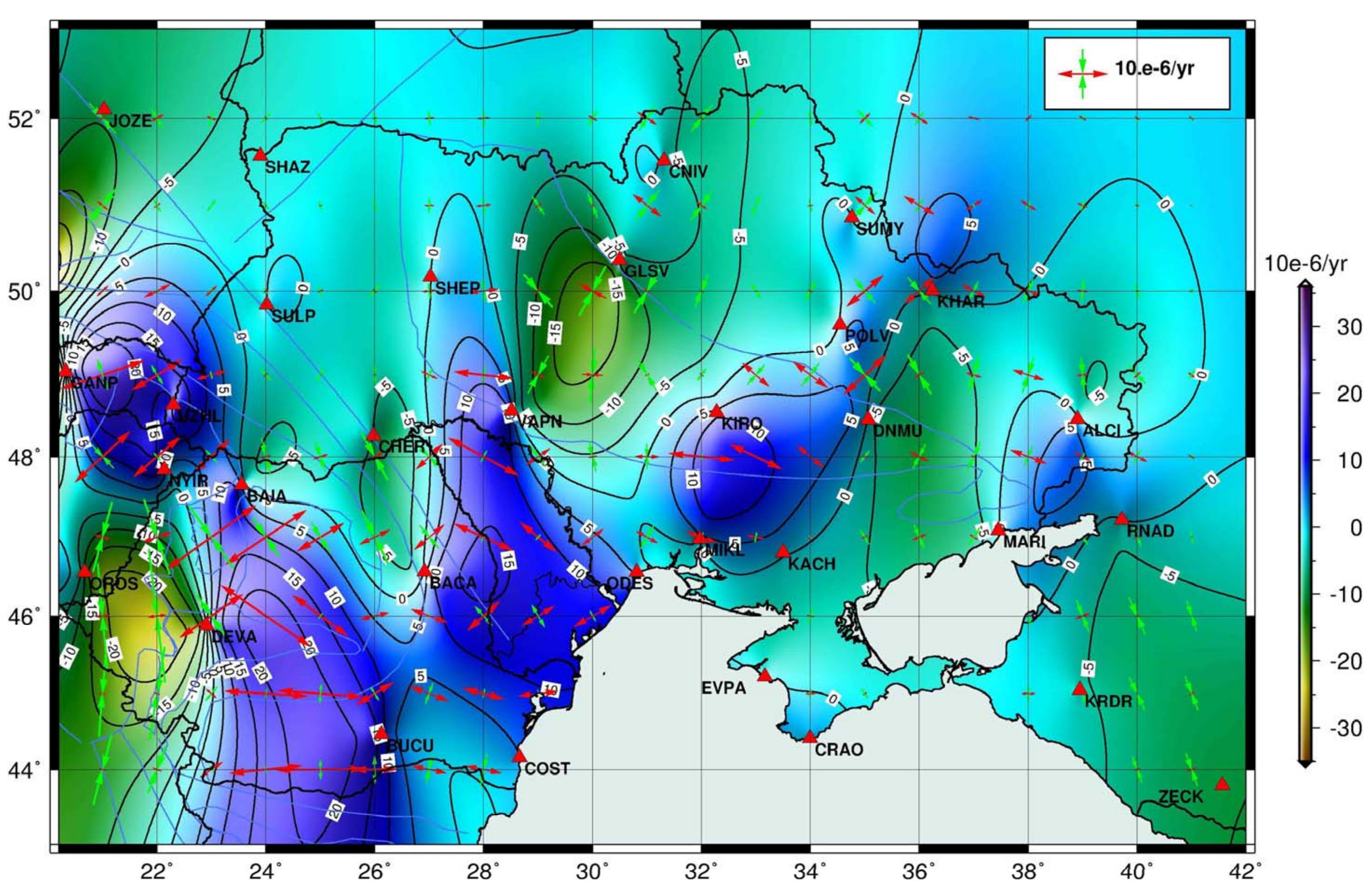
Dilatation rate in the Ukraine area (ITRF2005)



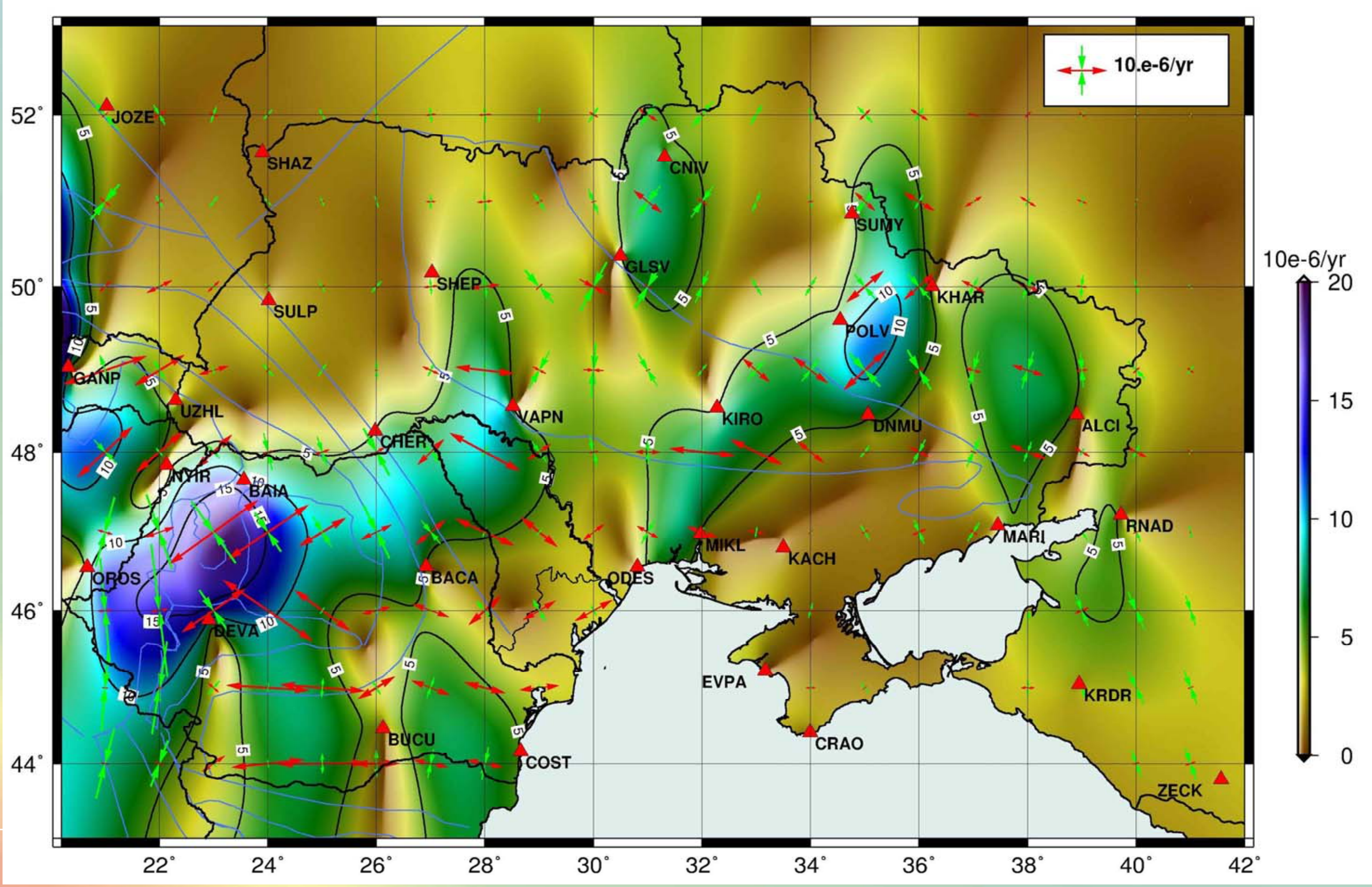
Maximum shear rate in the Ukraine area (ITRF2005)



Dilatation rate in the Ukraine area (ETRS89)



Maximum shear rate in the Ukraine area (ETRS89)



Conclusions

- **Analytical solution for the eigenvalue-eigenvector problem of 3D strain tensor (rate) was proposed. As a partial case the obtained formulas allow the solution of 2D eigenvalue-eigenvector problem.**
- **Densification of GPS-derived velocities data were tested by the least squares collocation method and direct approximation by smoothing bicubic spline functions.**
- **Small variations between dilatation rate and maximum shear rate in the ITRF2005 and ETRS89 systems are caused by various influence of the plate motion in this region because differences between velocities in ITRF2005 and ETRS89 systems are slightly changed.**

Thank you

for your attention!