Invariant deformation parameters from GPS permanent networks using stochastic interpolation

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Outline

Starting hypotheses

Methodology:

>>with particular attention to the new concepts

A case study:

a dense network in Japan

Preliminary results on EPN analysis

The temporal scale

Permanent networks provide continuous time series;

deformations are computed at discrete epochs:

a time model is needed.

The spatial scale

Permanent networks provide discrete in space observations;
deformations are represented by a field:

a spatial model is needed.

To study large areas deformations should be analyzed
on the ellipsoid surface (curvilinear coordinates)
and not on some planar projection:

>>a metric deformation function is required
to restore orthogonal deformation parameters
from curvilinear analyses.

The estimation process

Preprocessing

Estimation of the linear trend for each station P.
Clustering of one region in homogeneous networks.
Estimation of the Tisserand parameters
for homogeneous networks.

Processing

Within each homogeneous network,
modeling of the deformation at any point, by:

finite elements (interpolation),
and/or collocation (prediction).

The temporal scale: a linear interpolation

Coordinates on a short time span are linear functions of time.

Observation model for each station:

$$\boldsymbol{\eta}(P_k, t_i) = \boldsymbol{\eta}_{0k} + \mathbf{v}_k t_i + \boldsymbol{\varepsilon}_{ki}; \;\; \boldsymbol{\eta} = [E, N]^T / [\lambda, \varphi]^T, \; \mathbf{v} = [v_E, v_N]^T / [v_\lambda, v_\varphi]^T$$

Stochastic model, for the network:

At first iteration

$$\mathbf{C}(t_i) = \underset{3N \times 3N}{\mathbf{I}} \qquad \forall \quad i$$

Estimation of $\boldsymbol{\eta}_{0k}, \mathbf{v}_k, \boldsymbol{\varepsilon}_{ik}$ by Least Squares;

empirical estimation of $\mathbf{C}$.

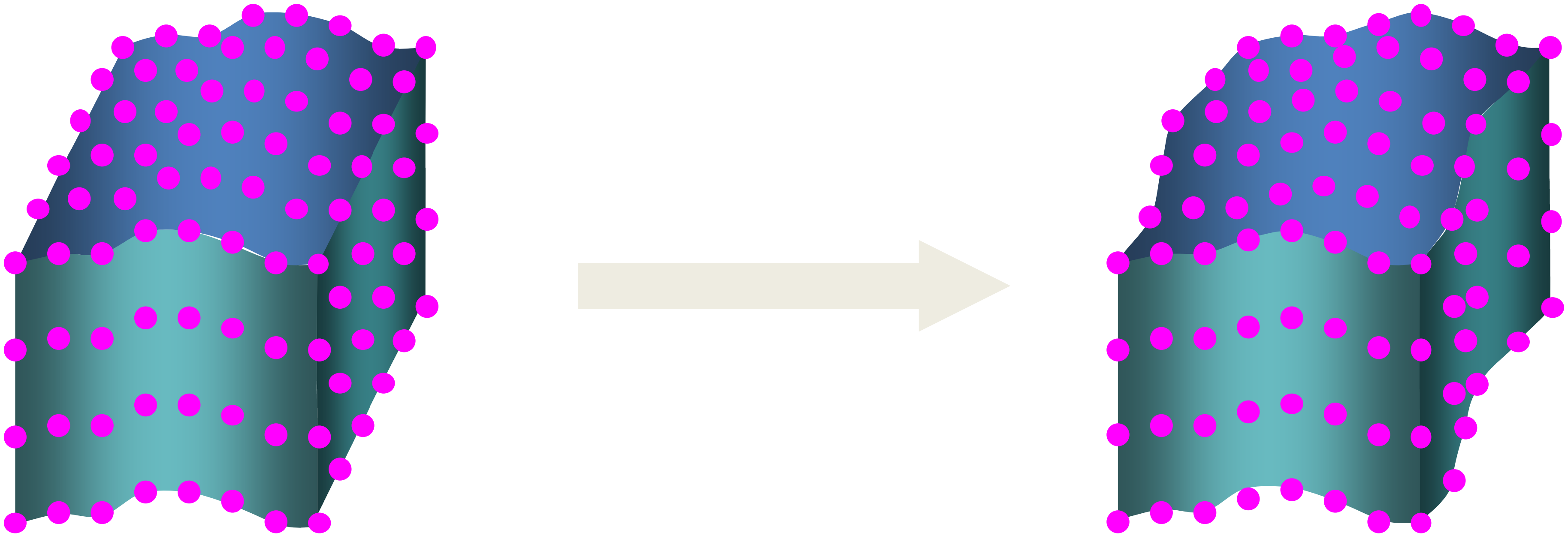
identical for each epoch,
no correlations between different epochs.

$$\mathbf{C}(t_i) = \begin{bmatrix} \mathbf{C}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_1}(t_i) & \mathbf{C}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}(t_i) & \dots & \mathbf{C}_{\boldsymbol{\eta}_1\boldsymbol{\eta}_p}(t_i) \\ \mathbf{C}^T_{\boldsymbol{\eta}_1\boldsymbol{\eta}_2}(t_i) & \mathbf{C}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_2}(t_i) & \dots & \mathbf{C}_{\boldsymbol{\eta}_2\boldsymbol{\eta}_p}(t_i) \\ \dots & \dots & \dots & \dots \\ \mathbf{C}^T_{\boldsymbol{\eta}_p\boldsymbol{\eta}_1}(t_i) & \mathbf{C}^T_{\boldsymbol{\eta}_p\boldsymbol{\eta}_2}(t_i) & \dots & \mathbf{C}_{\boldsymbol{\eta}_p\boldsymbol{\eta}_p}(t_i) \end{bmatrix} = \mathbf{C} \quad \forall \quad i$$

>>simplified but stochastically rigorous covariance estimation
>> no iterative approach required.

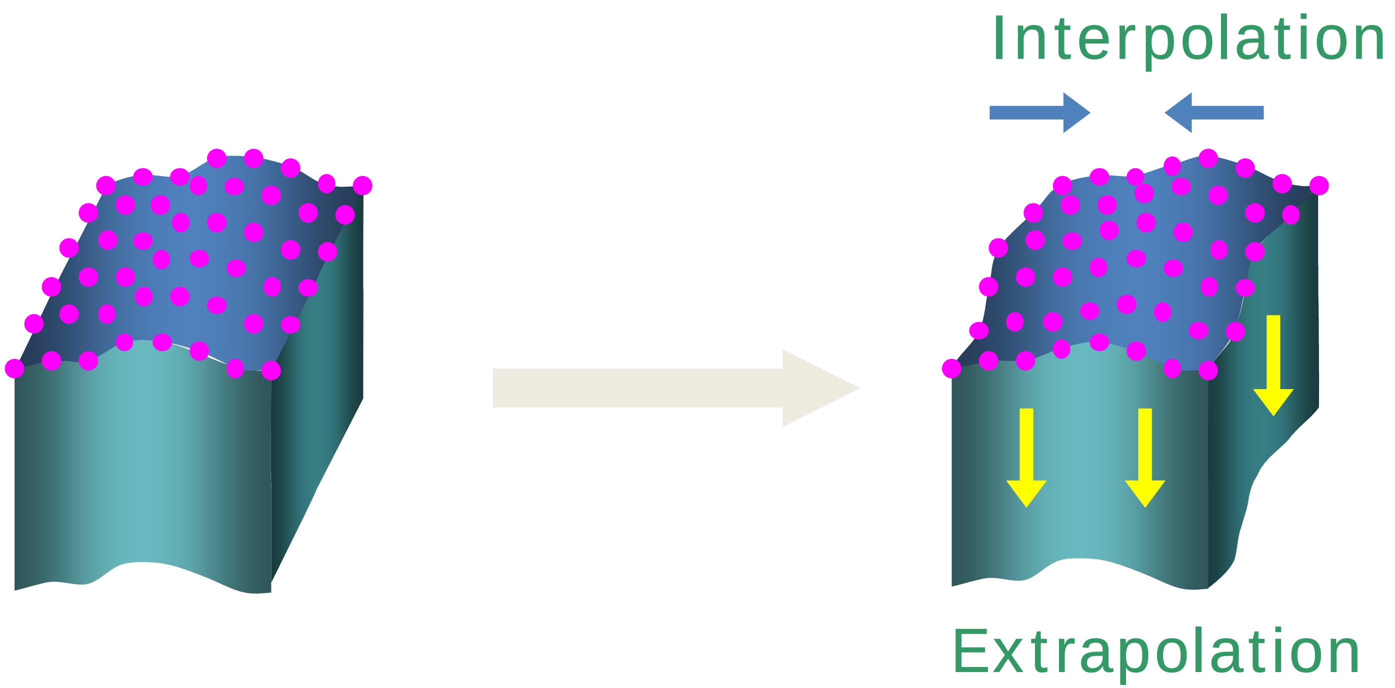
Horizontal deformation on the surface of the reference ellipsoid

Horizontal deformation on ellipsoidal surface



Actual deformation is 3-dimensional

But we can observe only on 2-dimensional earth surface!

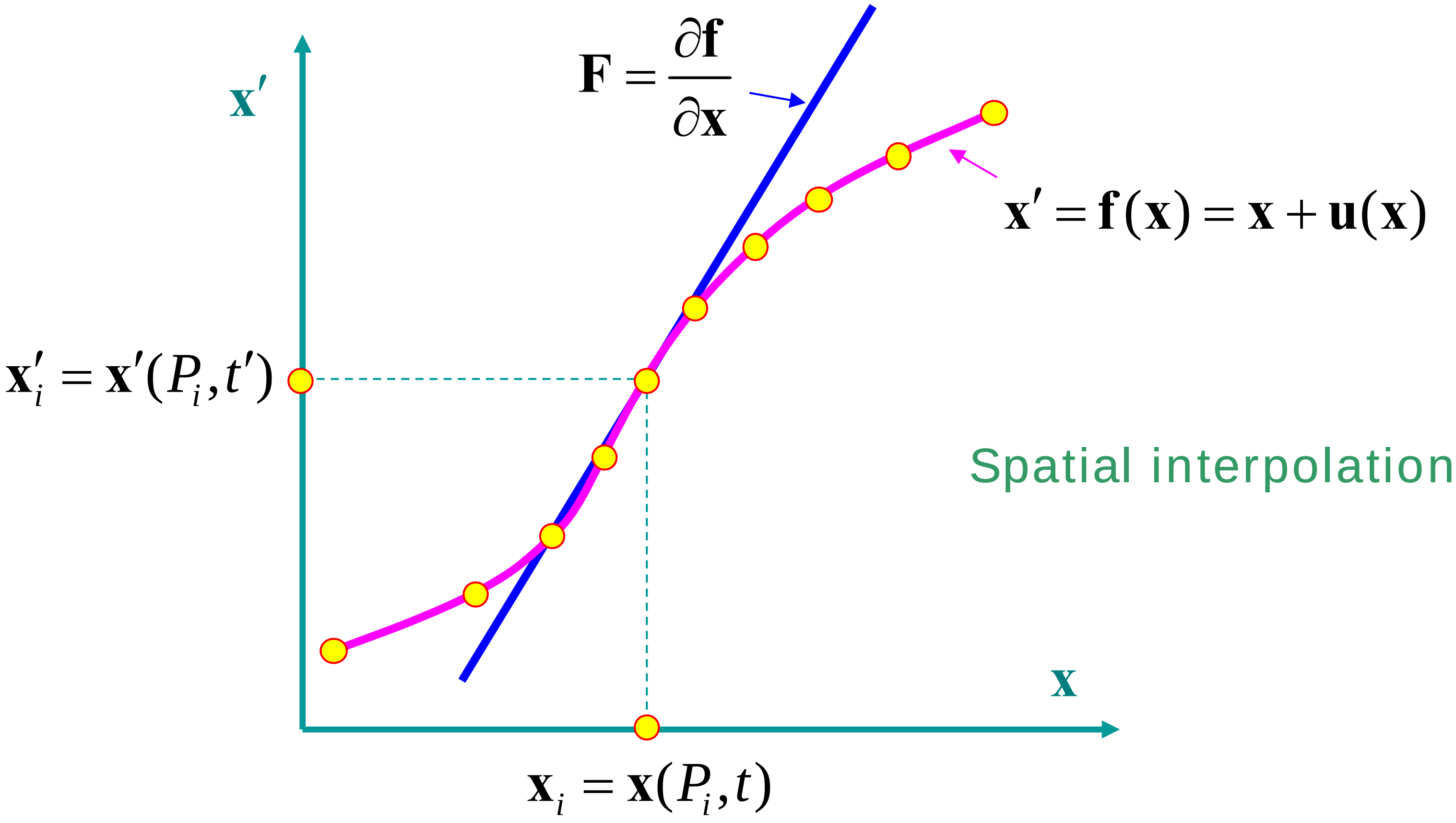


Why not 3D deformation?

3D deformation: interpolation and extrapolation.

Extrapolation from surface geodetic data is not reliable if no additional geophysical data are available.

Horizontal deformation analysis



Spatial interpolation

Discrete geodetic information at GPS permanent stations $\mathbf{x}_i \to \mathbf{x}'_i$

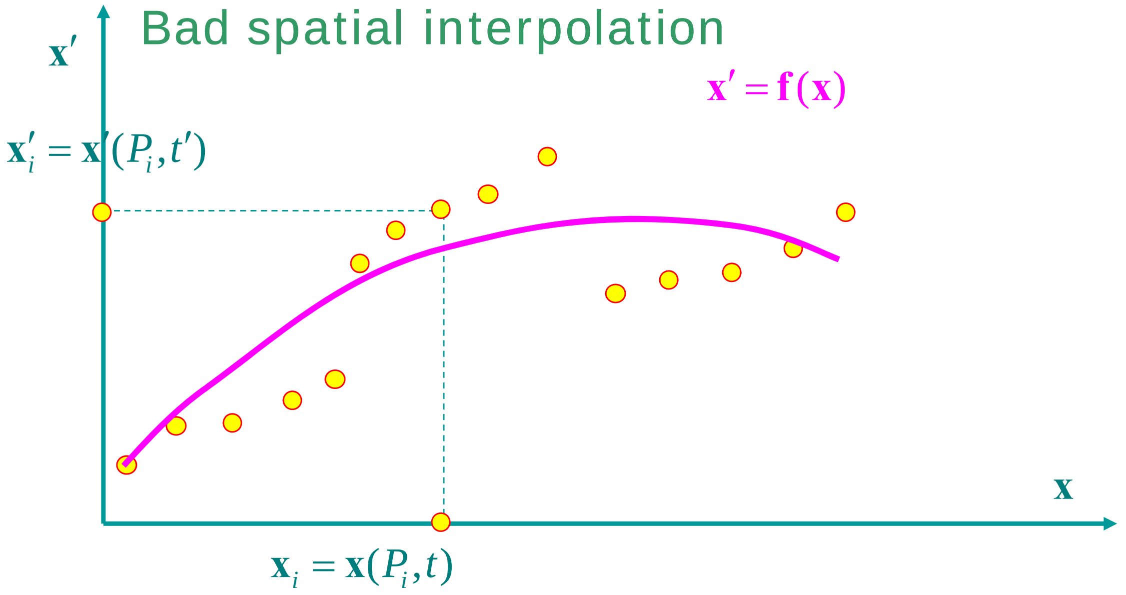
Interpolation to obtain continuous displacement information

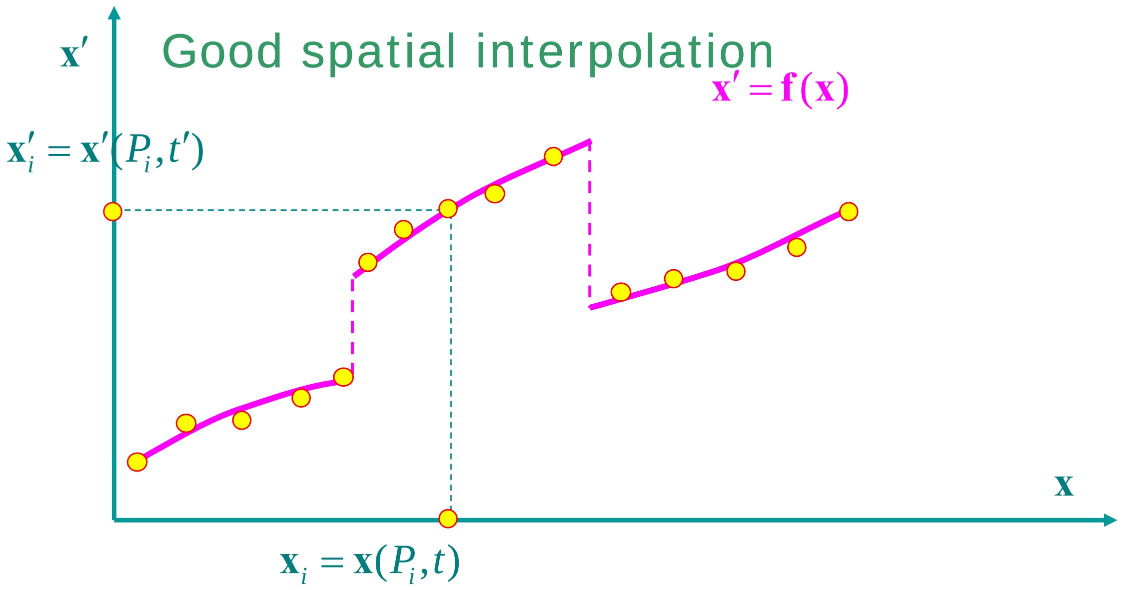
Differentiation to obtain the deformation gradient F

Interpolation in presence of discontinuities

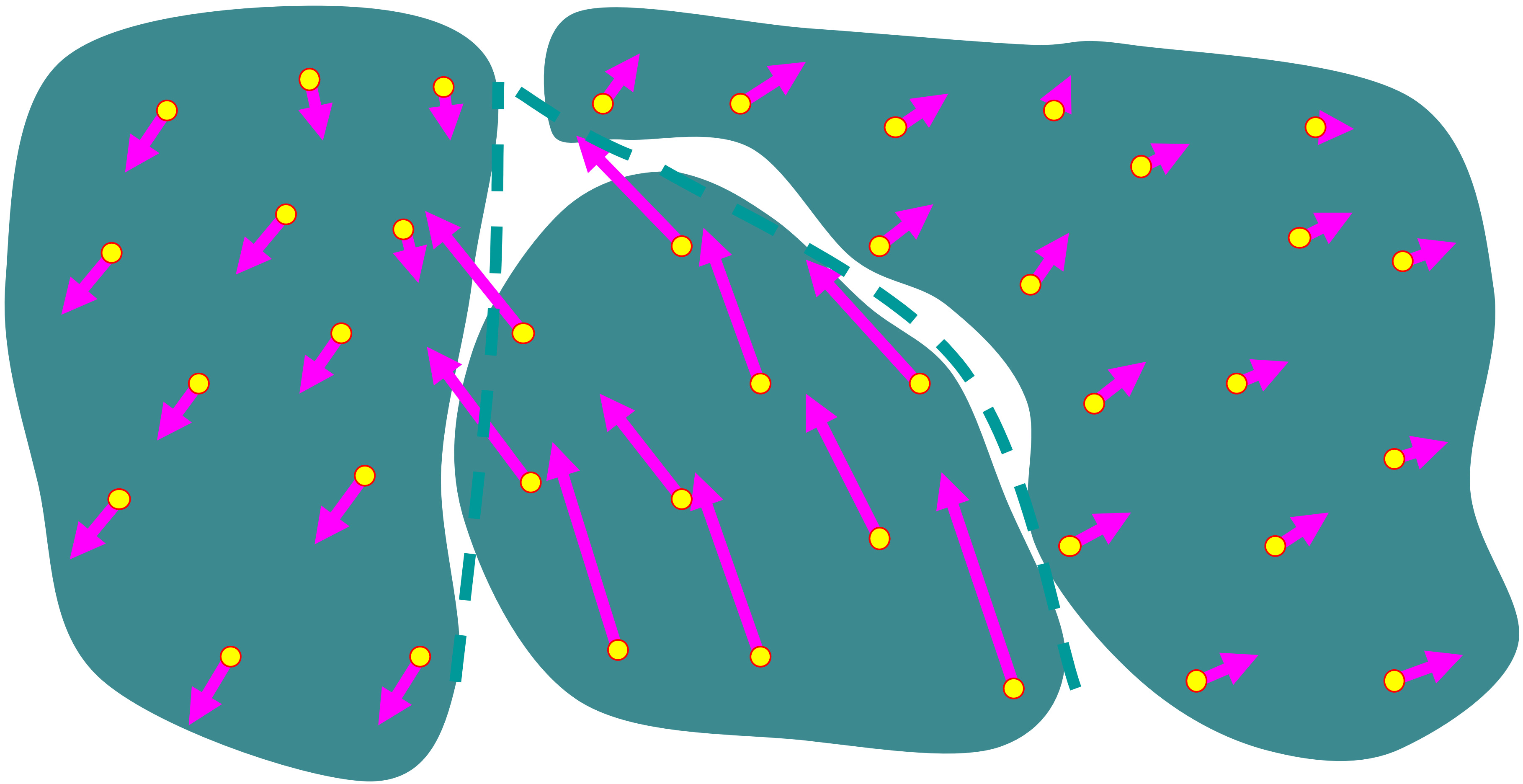
Separation of rigid motion from deformations:

piecewise analysis is needed before interpolation.



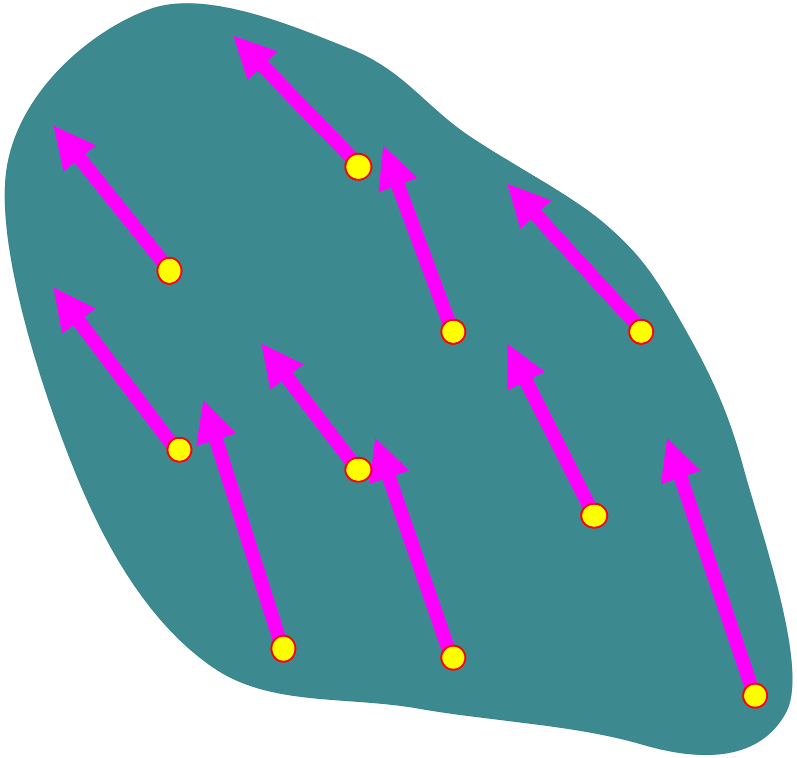


Separation of rigid motion from deformation

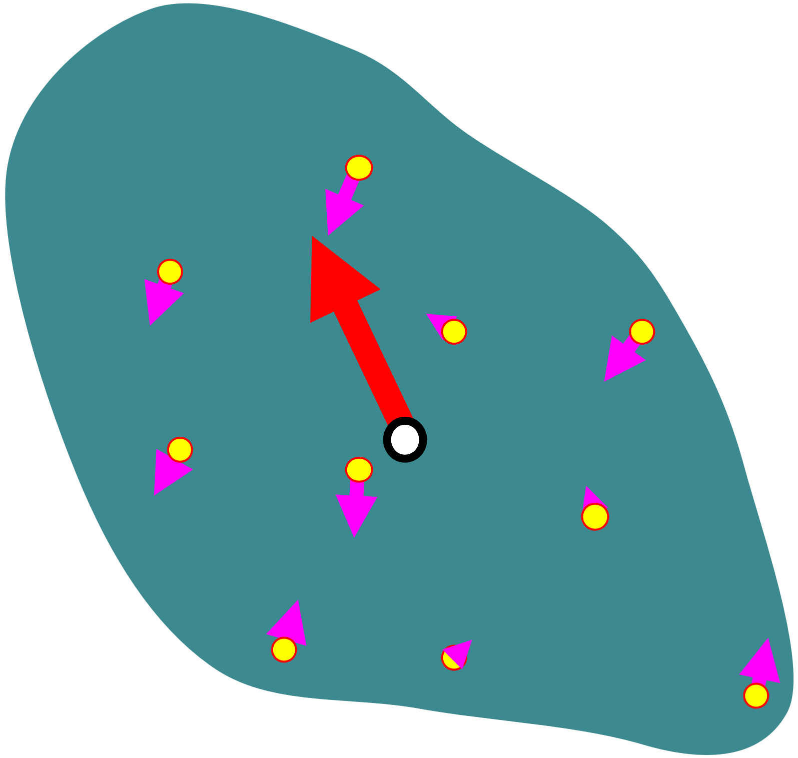


Apart from internal deformation regions are in relative motion

Separation of rigid motion from deformation



Original RS



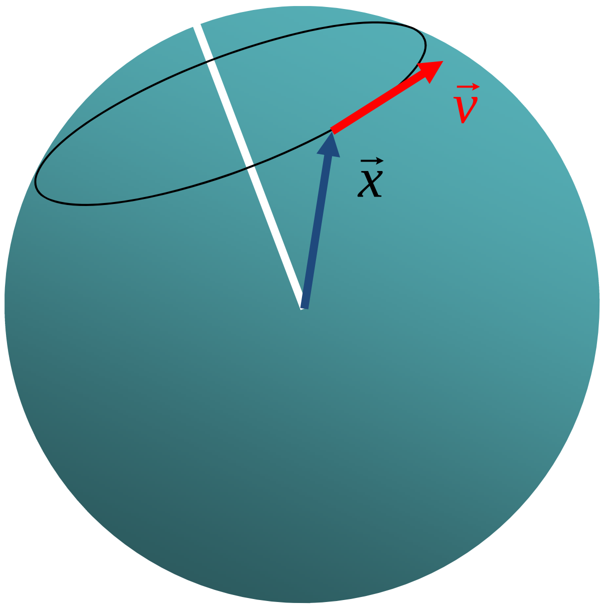
Optimal RS

How to represent the motion of a deforming region as a whole?
By the motion of a regional optimal reference system.
Optimal: such that the corresponding displacements are as small as possible.

Separation of rigid motion from deformation

Horizontal motion on earth ellipsoid (≈ sphere):
Rotation around an axis with angular velocity ω

$$\vec{v}_{\mathrm{ap}} = \vec{\omega} \times \vec{x} \qquad \mathbf{v}_{\mathrm{ap}} = [\boldsymbol{\omega} \times]\mathbf{x}$$



Definition of optimal reference frame:
Minimization of relative kinetic energy of regional network

$$T_{\mathrm{ap}} = \sum_i \mathbf{v}_{i,\mathrm{ap}}^{\mathrm{T}} \mathbf{v}_{i,\mathrm{ap}} = \min$$

Discrete Tisserand reference system

Solution: $$\boldsymbol{\omega}(t) = \mathbf{C}(t)^{-1}\mathbf{h}(t)$$

$$\mathbf{C} = -\sum_i [\mathbf{x}_i \times]^2$$ = inertia matrix

$$\mathbf{h} = \sum_i [\mathbf{x}_i \times] \frac{d\mathbf{x}_i}{dt}$$ = relative angular momentum

Deformation analysis within one network

Estimation of the inner deformations between two network states at two epochs t and t'.

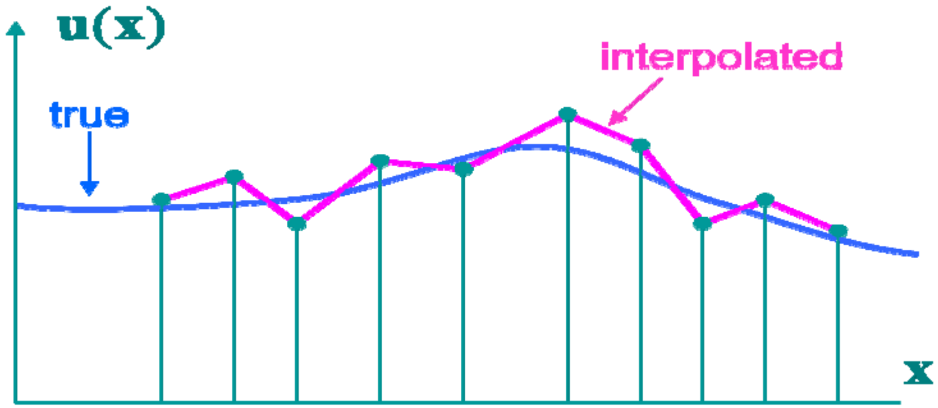
Deformation function:

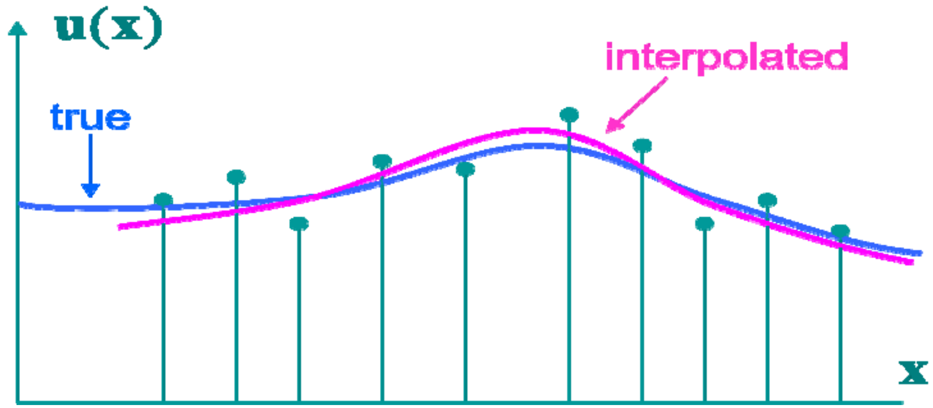
$$\mathbf{x}(t') = \mathbf{f}((t',t),\mathbf{x}(t))$$

Deformation gradient in space domain:

$$\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x})$$

Interpolation of $\mathbf{F}$ by finite elements:
Biagi and Dermanis, IAG Symposia Volumes 131





Finite elements

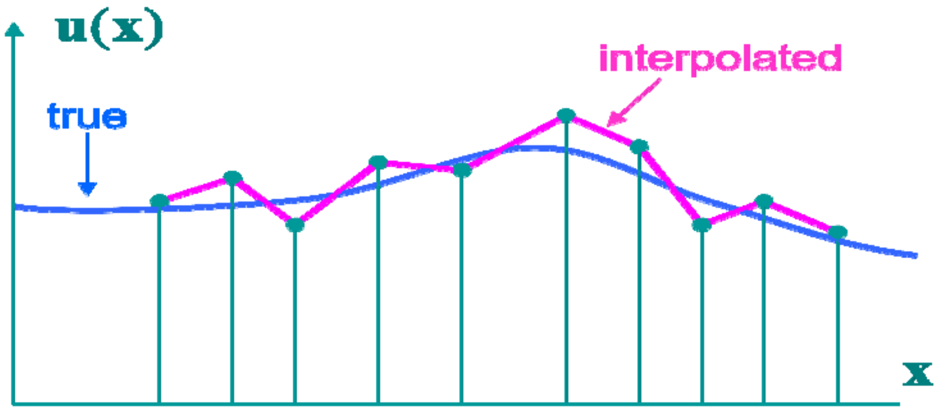
Deterministic interpolation

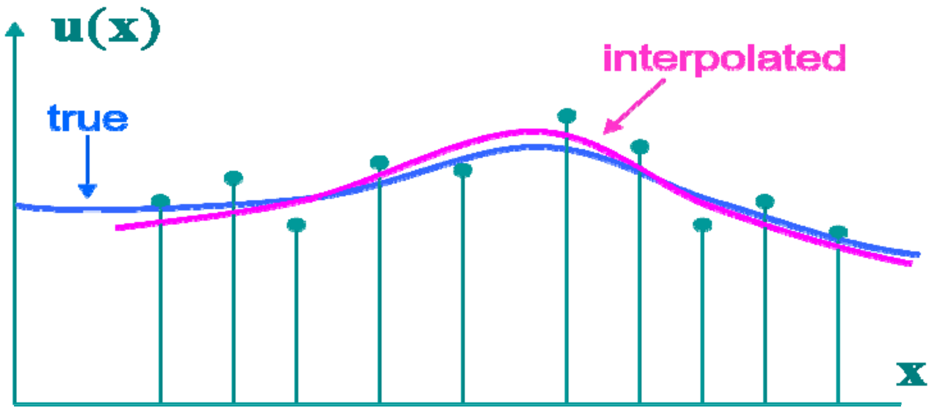
Piecewise displacements and deformations.

Collocation

Stochastic prediction

Continuous displacements and deformation field.





Observations errors

completely absorbed in estimated deformations prediction.

can be filtered by the prediction.

Accuracies estimates

only functions of observations uncertainties.

reflect both observations and signals uncertainties.

Prediction of $\mathbf{F}$ by collocation

Estimated displacements $u_{x_i}(t) = v_{x_i} t$, $u_{y_i}(t) = v_{y_i} t$ at the network points P_i are used to predict displacements and/or displacement gradient elements at any point P:

$$\mathbf{J} = \mathbf{F} - \mathbf{I},\ J_{11} = \frac{\partial u_x}{\partial x},\ J_{12} = \frac{\partial u_x}{\partial y},\ J_{21} = \frac{\partial u_y}{\partial x},\ J_{22} = \frac{\partial u_y}{\partial y}$$

2. A rotation is applied to the network to obtain

$$C_{u_x u_y}(P,Q) = 0$$

>>Remove:

>>empirical decorrelation of displacements on the sphere.

Choice of the model covariance function for the displacements:

Exp: $C_{u_x}(P,Q) = C_{0_x} e^{-\alpha \rho_{PQ}^2}, \; C_{u_y}(P,Q) = C_{0_y} e^{-\alpha \rho_{PQ}^2}, \; C_{0_i} > 0$

>>Legendre Polinomials:

$$C_{u_x}(P,Q) = \sum_{n=0}^{N} c_{n\,x} P_n(\cos\psi_{PQ}), \; C_{u_y}(P,Q) = \sum_{n=0}^{N} c_{n\,y} P_n(\cos\psi_{PQ}), \; c_{ij} > 0$$

3. Empirical estimation of the covariance function parameters.

>>In case of Legendre polynomials:
>>application of Not Negative Least Squares to
>>find and estimate not zero coefficients.

Construction of observation vector/covariance matrix

$$\mathbf{s} = \begin{bmatrix} \mathbf{u}_x \\ \mathbf{u}_y \end{bmatrix}, \; u_{x_i} = u_x(P_i), \; u_{y_i} = u_y(P_i), \; \mathbf{C}_{ss} = \begin{bmatrix} \mathbf{C}_{ux} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{uy} \end{bmatrix}$$

5. Prediction of a signal $\mathbf{t}$ at any point P: two approaches

A. Exact interpolation: $\hat{\mathbf{t}} = \mathbf{C}_{ts}\mathbf{C}_{ss}^{-1}\mathbf{s}$

B. Smoothing: $\hat{\mathbf{t}} = \mathbf{C}_{ts}[\mathbf{C}_{ss} + \mathbf{C}_{\varpi\varpi}]^{-1}\mathbf{s}$

where $C_{\varpi\varpi}$ is the estimated covariance matrix of the displacements errors.

Computation of the crosscovariance matrix $\mathbf{C}_{ts}$

If

$$\mathbf{t} = \mathbf{u}(P)) = [u_x(P) \;\; u_y(P)]^T$$

it follows

$$\mathbf{C}_{ts} = [\mathbf{C}_{tu_x} \;\; \mathbf{C}_{tu_y}]$$

Id est

$$\{\mathbf{C}_{tu_x}\}_{1k} = C_{ux}(x, y, x_k, y_k)$$
$$\{\mathbf{C}_{tu_x}\}_{2k} = C_{u_x u_y}(x, y, x_k, y_k) = 0 = C_{u_y u_x}(x, y, x_k, y_k) = \{\mathbf{C}_{tu_y}\}_{1k}$$
$$\{\mathbf{C}_{tu_y}\}_{2k} = C_{u_y}(x, y, x_k, y_k)$$

Deformation Jacobian

$$\mathbf{t} = Vect(\mathbf{J}(P)) = [J_{11}(P) \;\; J_{12}(P) \;\; J_{21}(P) \;\; J_{22}(P)]^T$$

$$\Rightarrow \;\; \mathbf{C}_{ts} = [\mathbf{C}_{tu_x} \;\; \mathbf{C}_{tu_y}]$$

$$\Rightarrow \{\mathbf{C}_{tu_x}\}_{1k} = C_{J_{11}u_x}(P, P_k) = C_{\frac{\partial u_x}{\partial x}u_x}(P, P_k) = \frac{\partial}{\partial x} C_{u_x}(x, y, x_k, y_k)$$

$$\Rightarrow \{\mathbf{C}_{tu_x}\}_{2k} = C_{J_{12}u_x}(P, P_k) = C_{\frac{\partial u_x}{\partial y}u_x}(P, P_k) = \frac{\partial}{\partial y} C_{u_x}(x, y, x_k, y_k)$$

$$\Rightarrow \{\mathbf{C}_{tu_x}\}_{3k} = C_{J_{21}u_x}(P, P_k) = C_{\frac{\partial u_y}{\partial x}u_x}(P, P_k) = \frac{\partial}{\partial x} C_{u_x u_y}(x, y, x_k, y_k) = 0$$

$$\Rightarrow \{\mathbf{C}_{tu_x}\}_{4k} = C_{J_{22}u_x}(P, P_k) = C_{\frac{\partial u_y}{\partial y}u_x}(P, P_k) = \frac{\partial}{\partial y} C_{u_x u_y}(x, y, x_k, y_k) = 0$$

The same can be applied to $\{\mathbf{C}_{tu_y}\}$

Estimation of the covariance matrix of the predicted signal

A.

Exact interpolation

Covariance matrix of the predictions: $\mathbf{C}_{\hat{t}\hat{t}} = \mathbf{C}_{ts}\mathbf{C}_{ss}^{-1}\mathbf{C}_{\varpi\varpi}\mathbf{C}_{ss}^{-1}\mathbf{C}_{ts}^{T}$

Covariance matrix of the errors: $\mathbf{C}_{\varepsilon\varepsilon} = \mathbf{C}_{\hat{t}\hat{t}}$

B.

Smoothing

Covariance matrix of the predictions: $\mathbf{C}_{\hat{t}\hat{t}} = \mathbf{C}_{ts}(\mathbf{C}_{ss} + \mathbf{C}_{\varpi\varpi})^{-1}\mathbf{C}_{ts}^{T}$

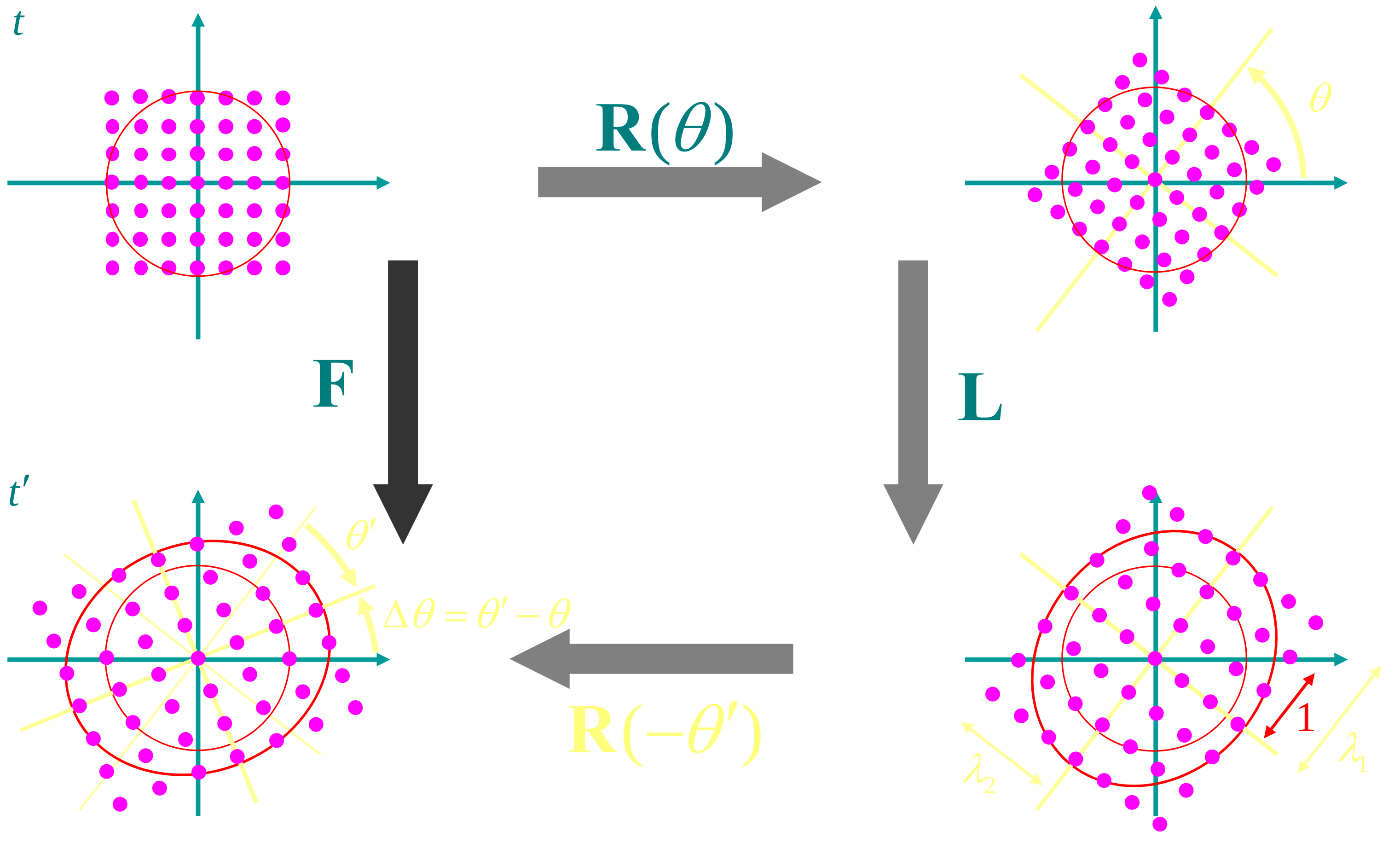
Covariance matrix of the errors: $\mathbf{C}_{\varepsilon\varepsilon} = \mathbf{C}_{tt} - \mathbf{C}_{ts}(\mathbf{C}_{ss} + \mathbf{C}_{\varpi\varpi})^{-1}\mathbf{C}_{ts}^{T}$

>>Restore of the decorrelation into the predicted signals

From $\hat{\mathbf{F}}(P) = \mathbf{I} + \hat{\mathbf{J}}(P)$ the deformation parameters are computed.

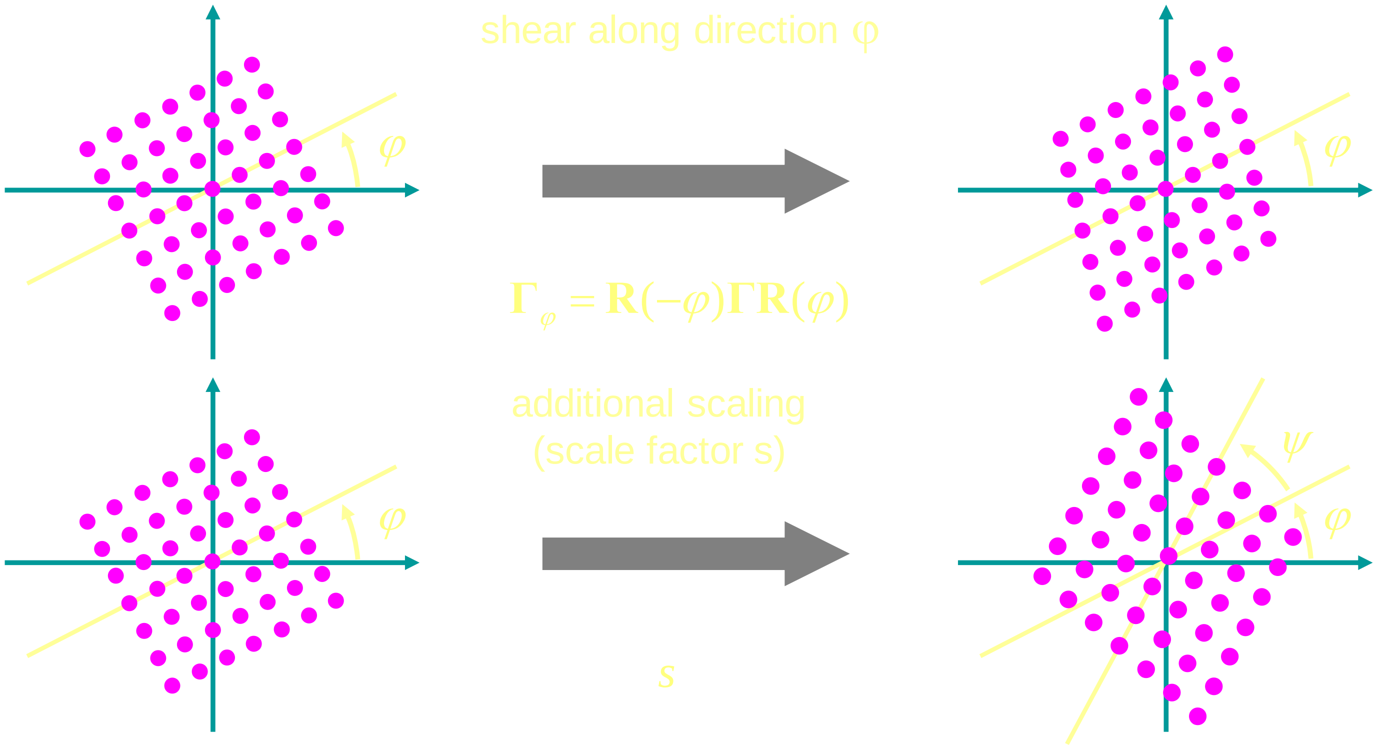
Eigenvalues and angles: $\lambda_{\max}(P), \lambda_{\min}(P), \theta_{\lambda(P)}$

$$\mathbf{F} = \mathbf{R}(-\theta')\mathbf{L}\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$



Shears $\gamma(P), \theta_{\gamma(P)}$ and scale s

$$\mathbf{F} = \mathbf{F}(s,\gamma,\varphi,\psi) = s\,\mathbf{R}(\psi)\mathbf{R}(-\varphi)\mathbf{G}\mathbf{R}(\varphi)\,,\quad \mathbf{G} = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix}$$



Covariance propagation from the deformation gradient to the deformation parameters.

For example, if

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_{\max} \\ \lambda_{\min} \\ \theta_{\lambda} \end{bmatrix} = \boldsymbol{\lambda}(Vec(\mathbf{F})),$$

under a first order approximation $\dfrac{\partial \boldsymbol{\lambda}}{\partial Vec(\hat{\mathbf{F}})} = \mathbf{L}$,

then, simply

$$\mathbf{C}_{\lambda\lambda} = \mathbf{L}\mathbf{C}_{Vec(F)}\mathbf{L}^{T}$$

The metric on the ellipsoid

In case of planar coordinates, the computation

$$[\boldsymbol{\eta}_0, \mathbf{u}] \Rightarrow \mathbf{F} \Rightarrow \mathbf{F}^T\mathbf{F} \Rightarrow [\lambda_{\max}, \lambda_{\min}, \theta_\lambda, \gamma, \theta_\gamma, \Delta]$$

is direct, either by finite elements or by collocation.

In case of curvilinear (geodetic) coordinates,
one more step is needed:

$$[\boldsymbol{\eta}_0, \mathbf{u}] \Rightarrow \mathbf{F}_{\eta\eta} \Rightarrow \mathbf{C} = \mathbf{D}^T\mathbf{F}_{\eta\eta}^T\mathbf{F}_{\eta\eta}\mathbf{D} \Rightarrow [\lambda_{\max}, \lambda_{\min}, \theta_\lambda, \gamma, \theta_\gamma, \Delta],$$

where $\mathbf{D}$ is the matrix of the metric deformation on the ellipsoid surface, here not discussed into details.

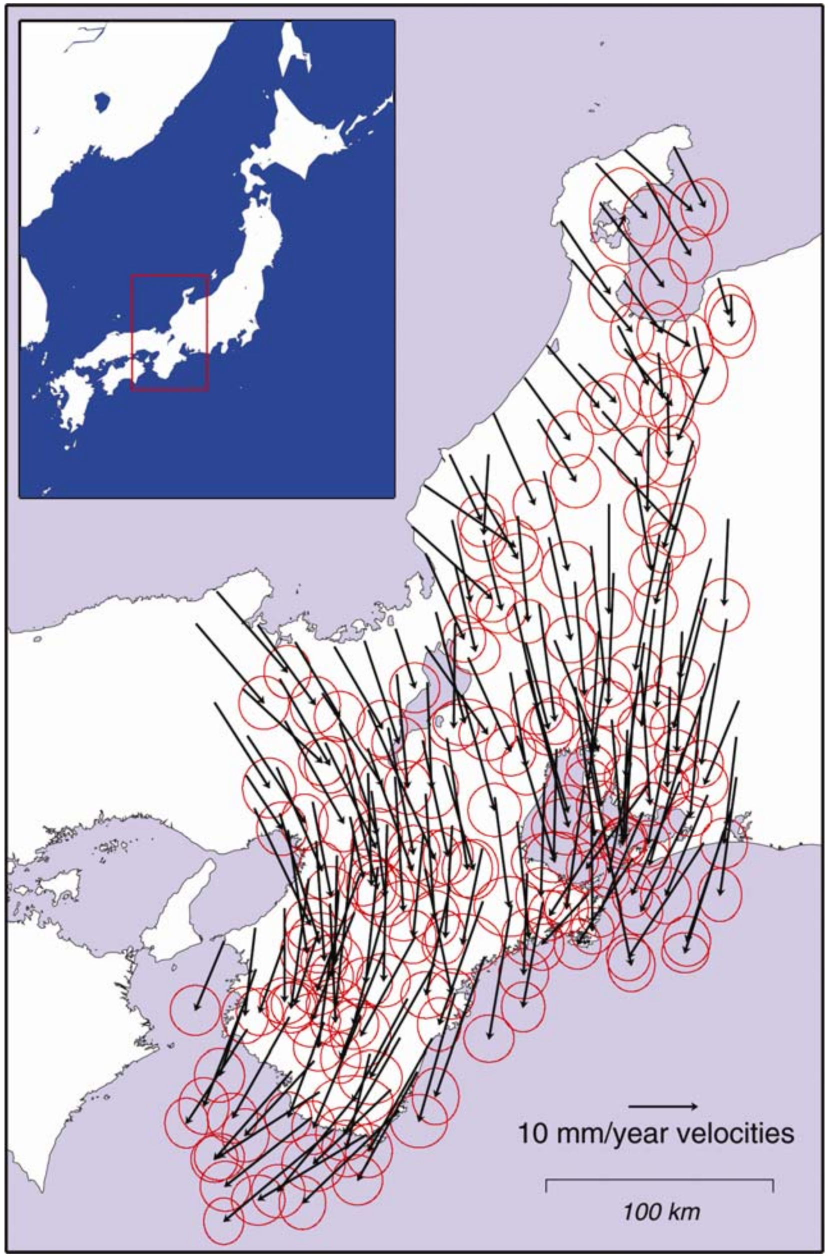
The case study: Japan network

One year (2005) of daily solutions of the
Japan PN ($\approx$ 1200 SP).

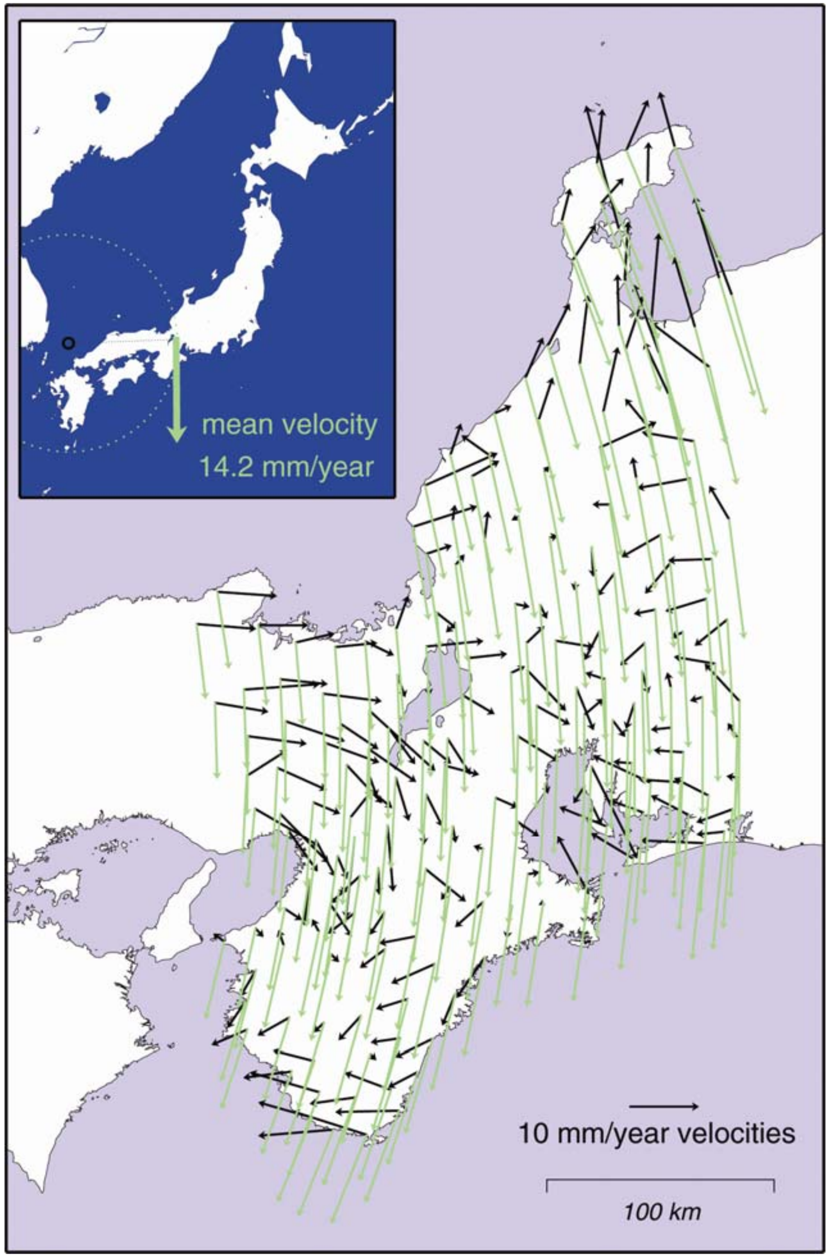
Preprocessing:
rejection of bundlers,
clustering of the network in homogeneous subregions,
selection of a particular subregion as a case study,
Delaunay triangulation.

Prediction of the deformation parameters,
by finite elements and collocation.

Comparisons.

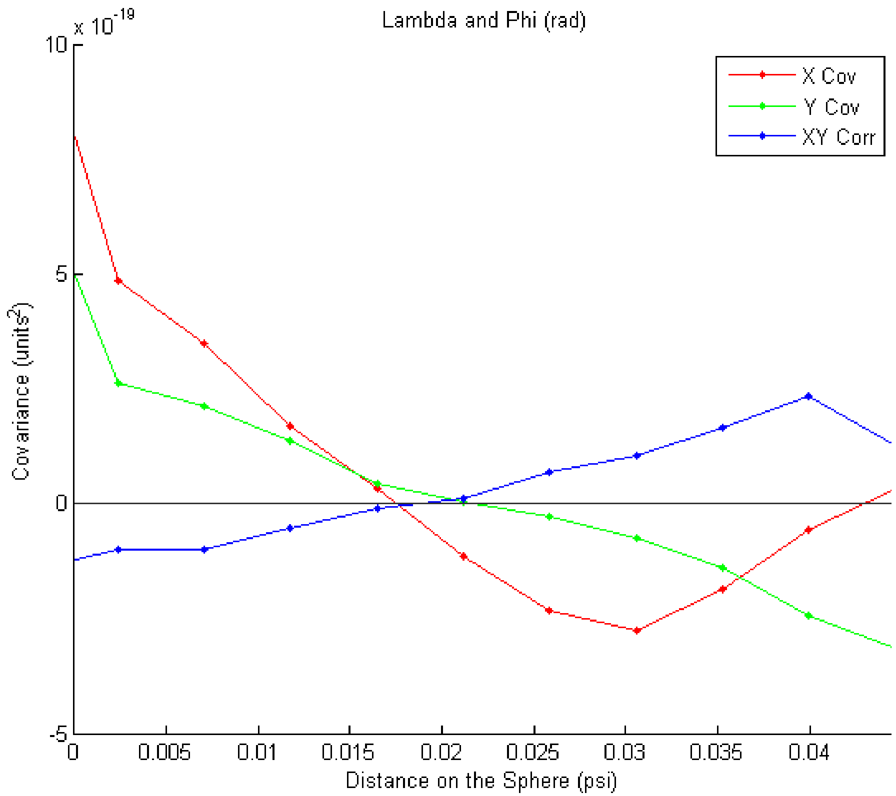


Original displacements

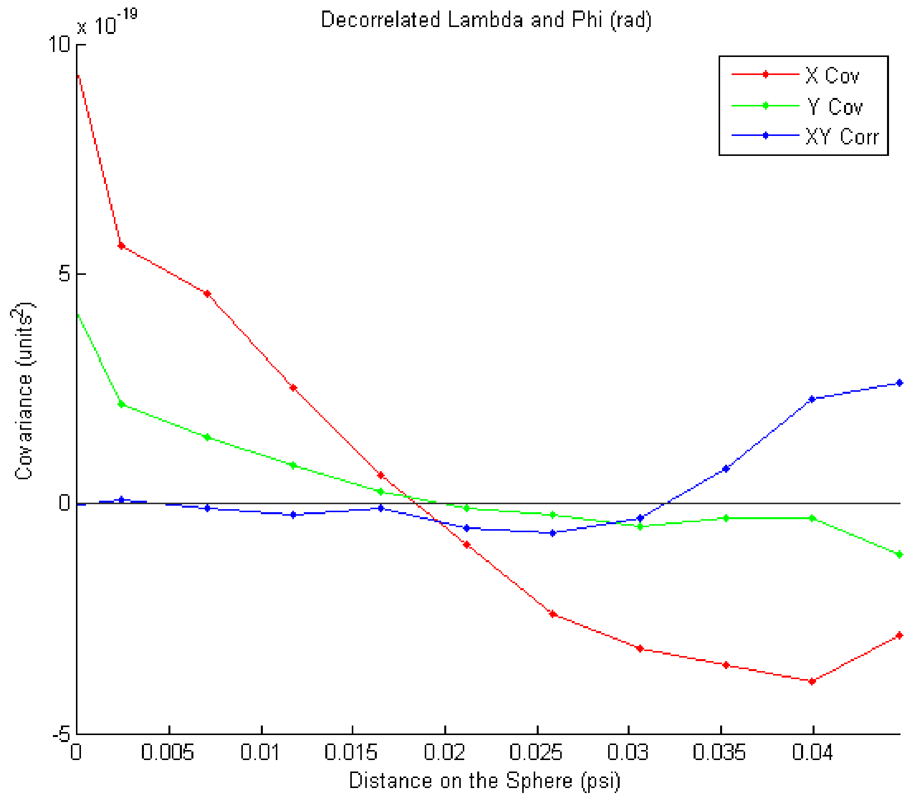


Tisserand displacements

Decorrelation of the signals in longitude and latitude

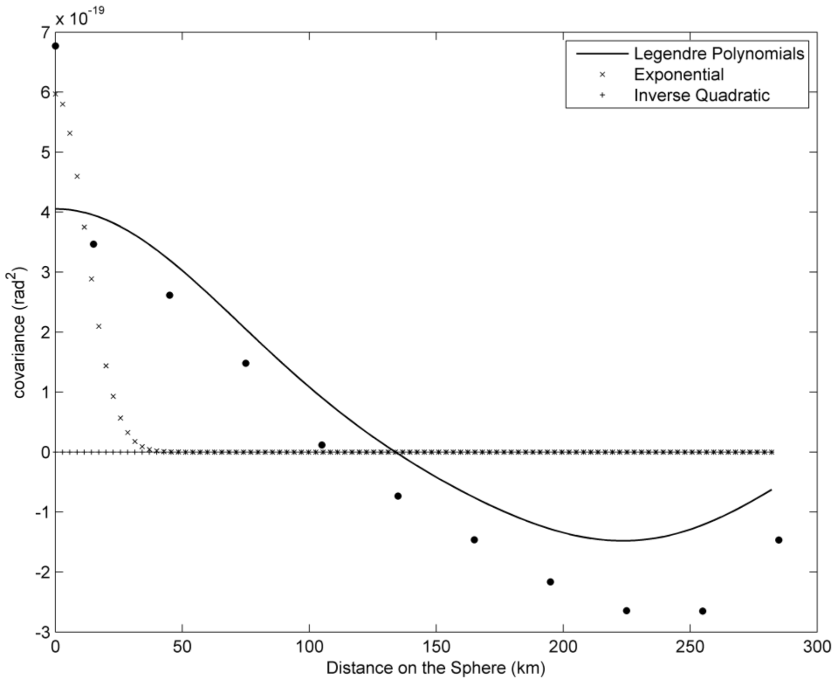


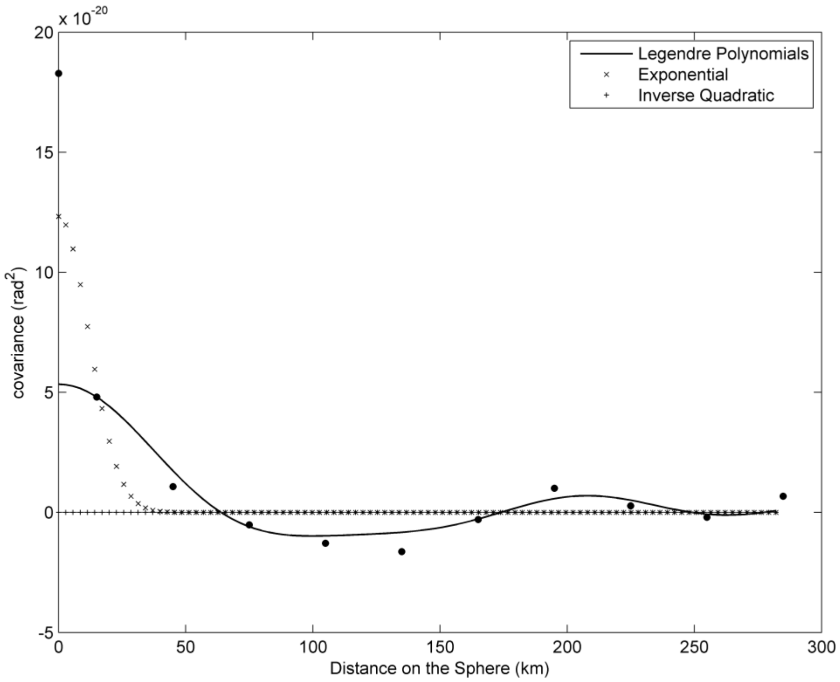
Before decorrelation



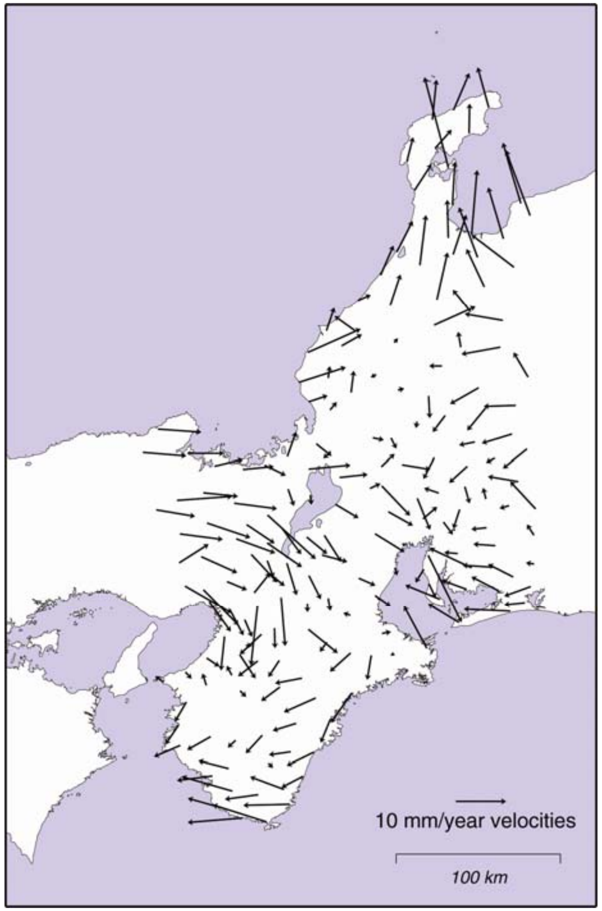
After decorrelation

Empirical estimation of the model covariances





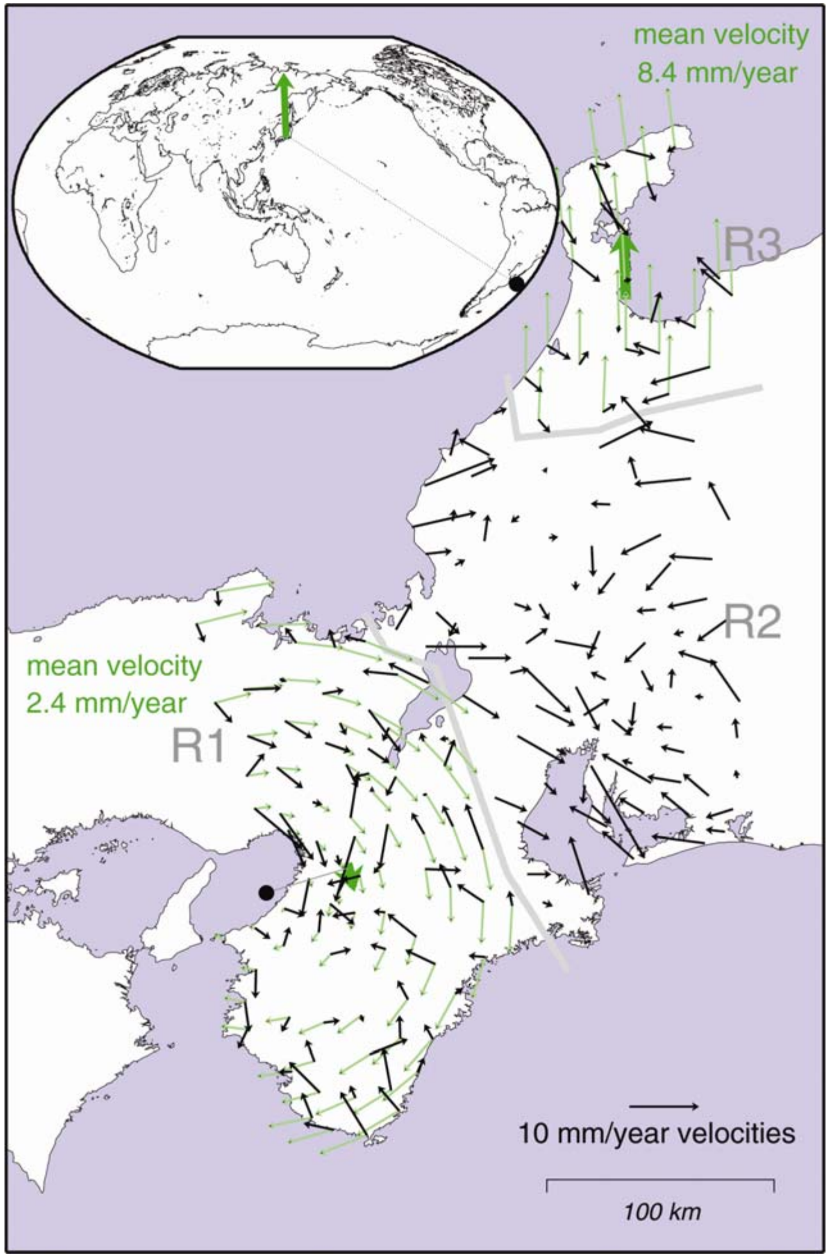
Longitude and latitude displacements:
comparisons between NNLS Legendre fitting and
a typical choice of exponential covariance



R1-SouthWest rotation

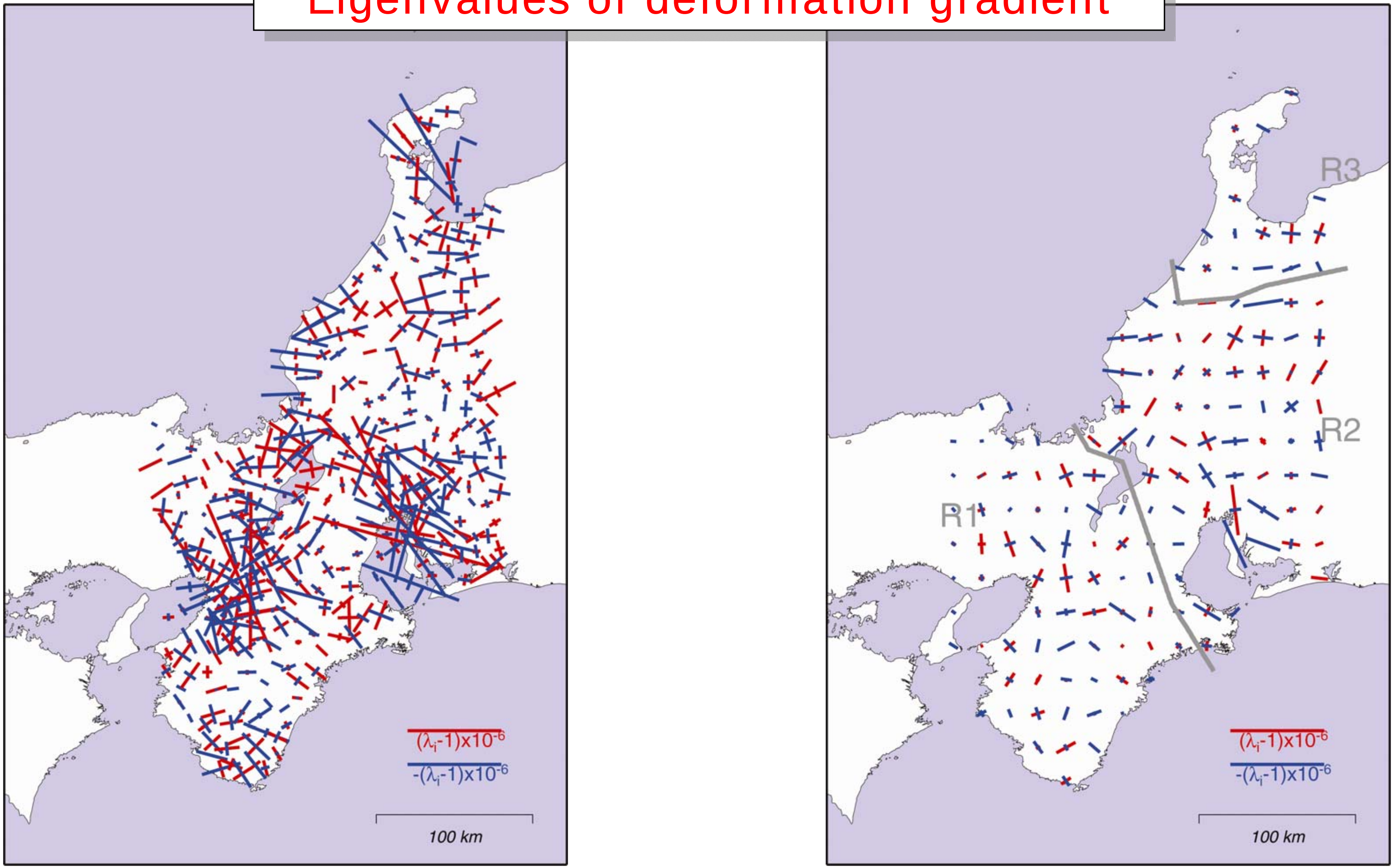
R3-North translation

R2-Center compression



Tisserand analyses of
R1 and R3 wrt R2.
Separate deformation predictions.

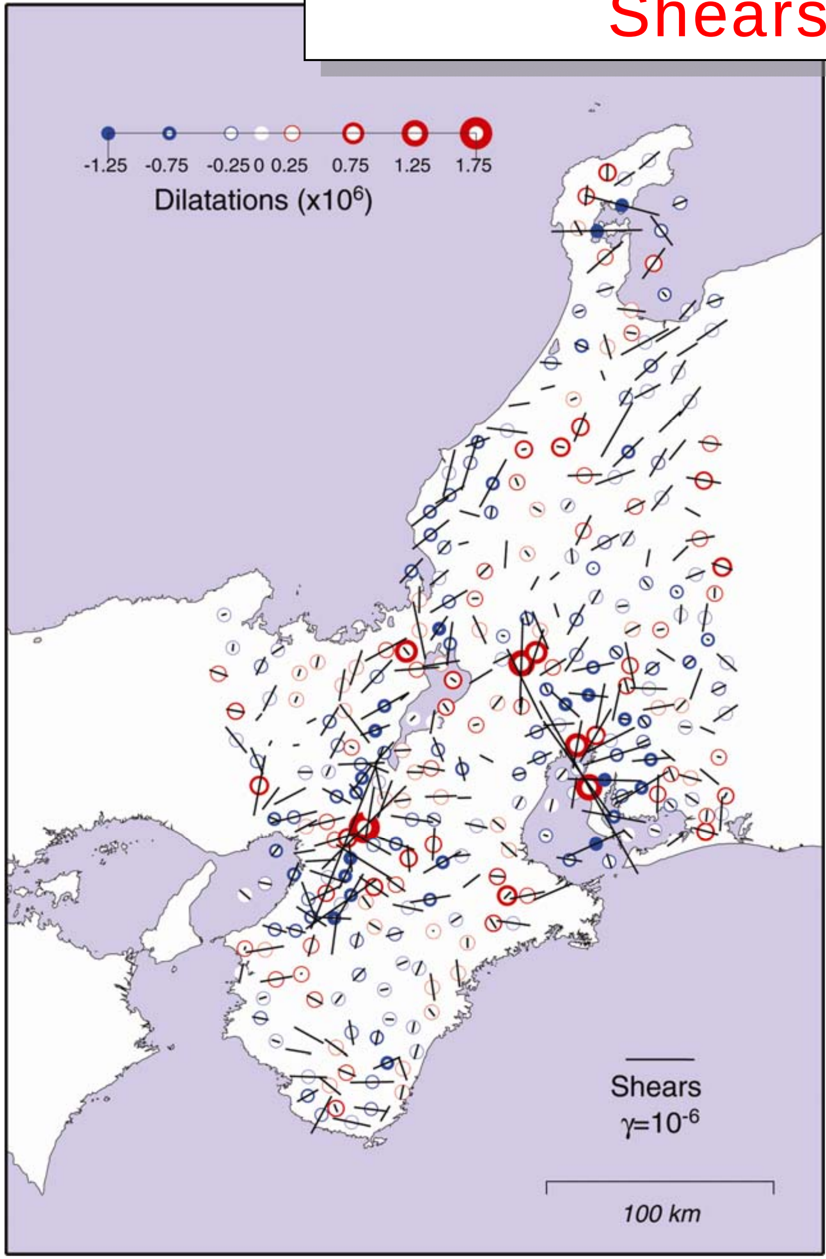
Eigenvalues of deformation gradient

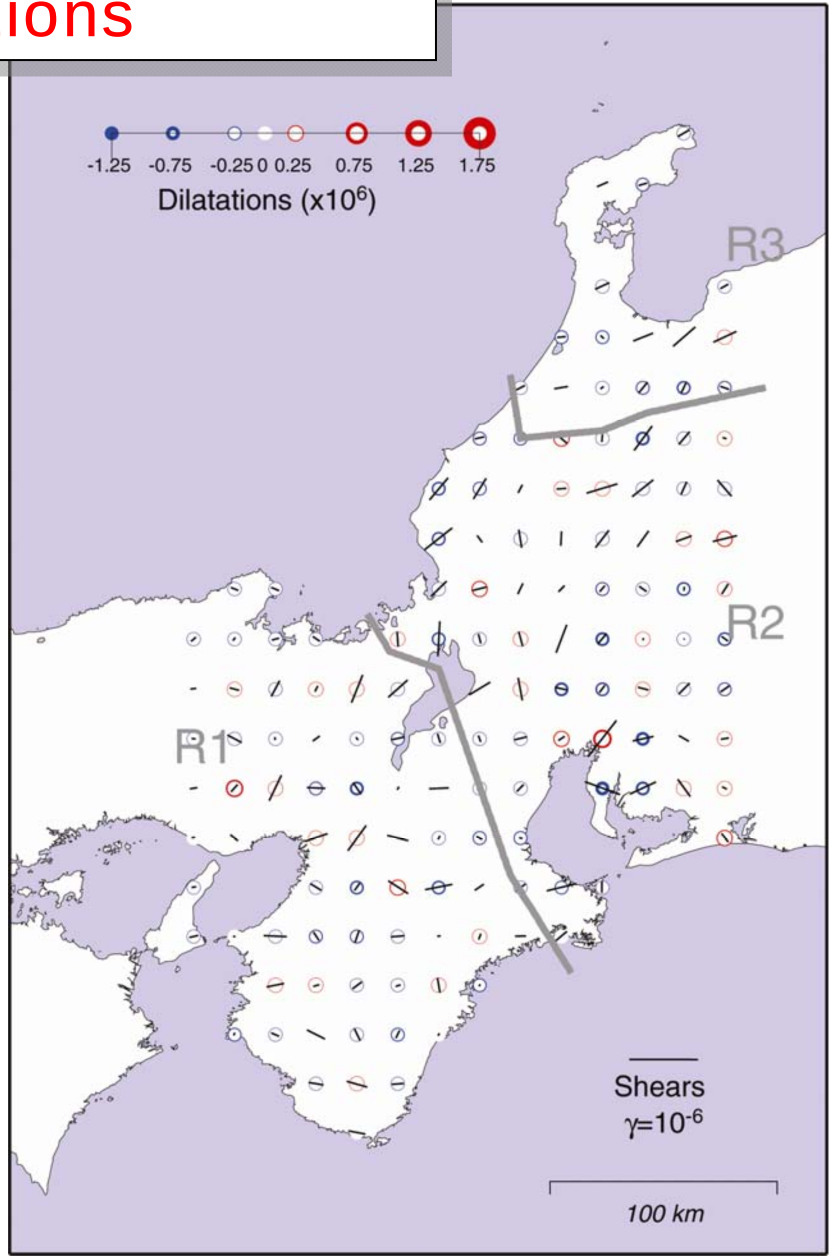


Finite elements

Collocation

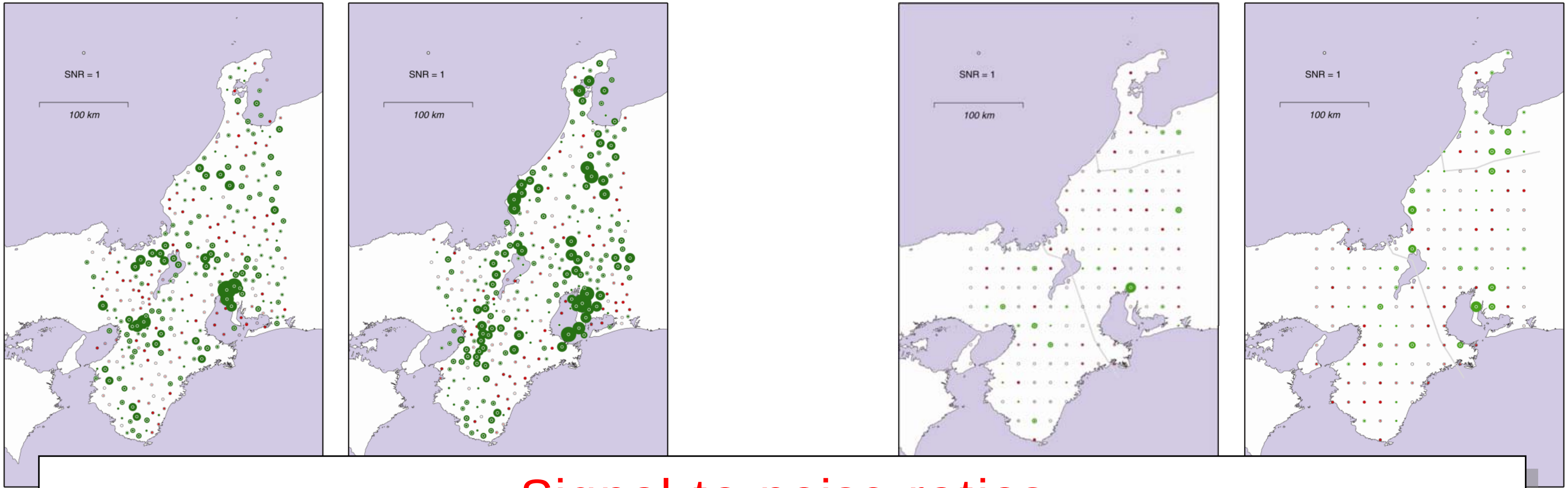
Shears and dilatations





Finite elements

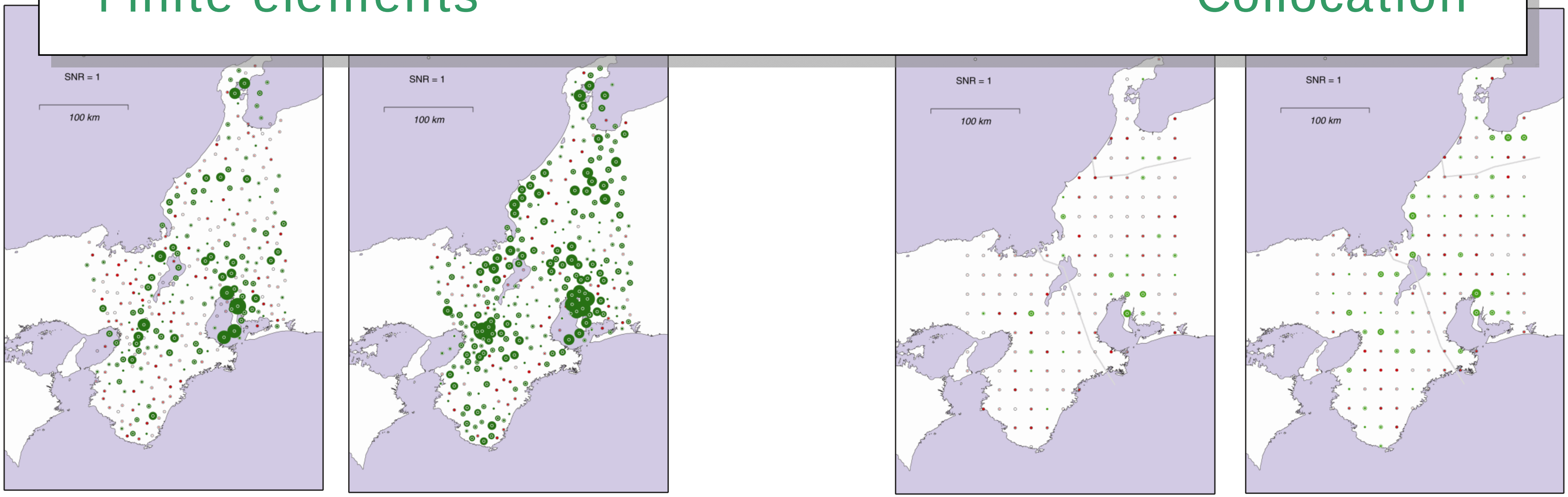
Collocation



Signal to noise ratios

Finite elements

Collocation



Methodological results and conclusions

New algorithms for curvilinear (geodetic) coordinates
have been studied and implemented for

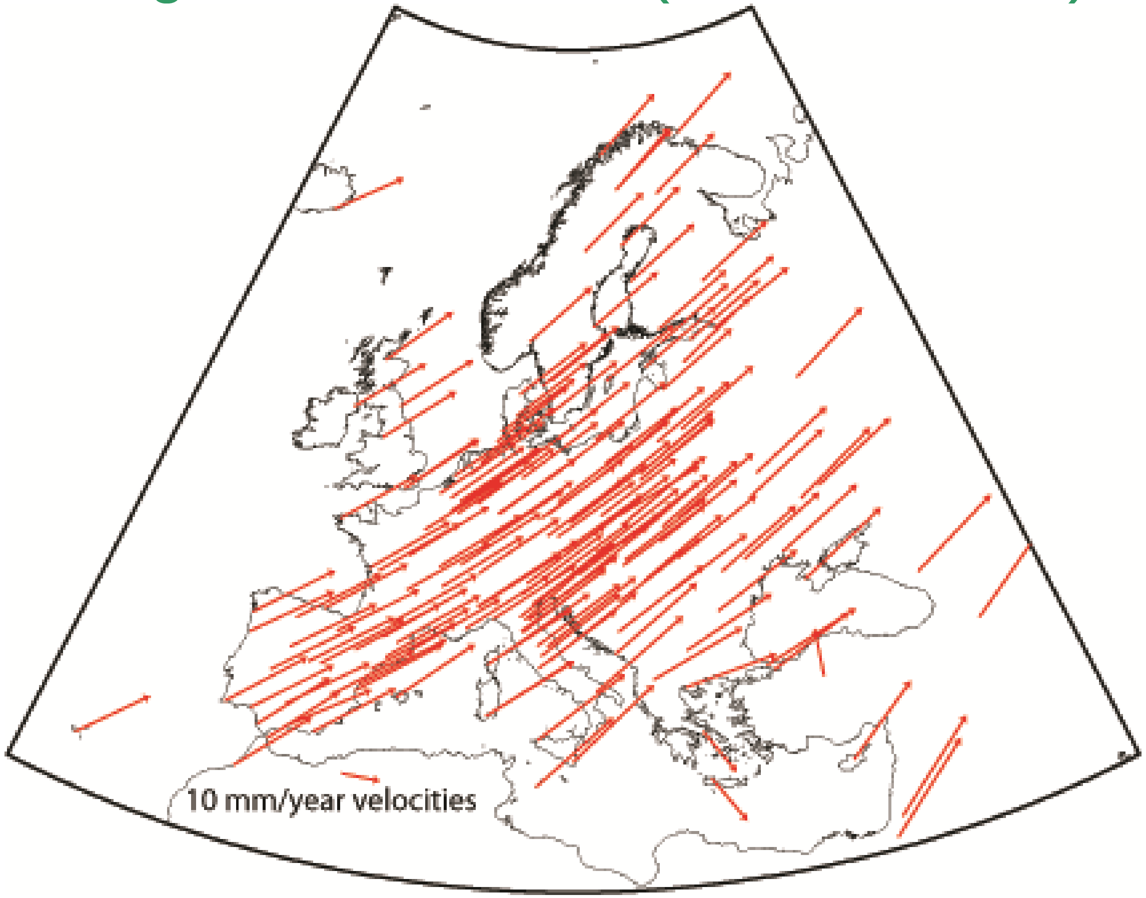
1. Tisserand analysis on networks.
2. Displacements and deformation field prediction,
   both by finite elements and by collocation.
3. LP's NNLS fitting of empirical covariance function.
4. Covariance propagation from time series to
   deformation parameters.

WRT finite elements, collocation provides

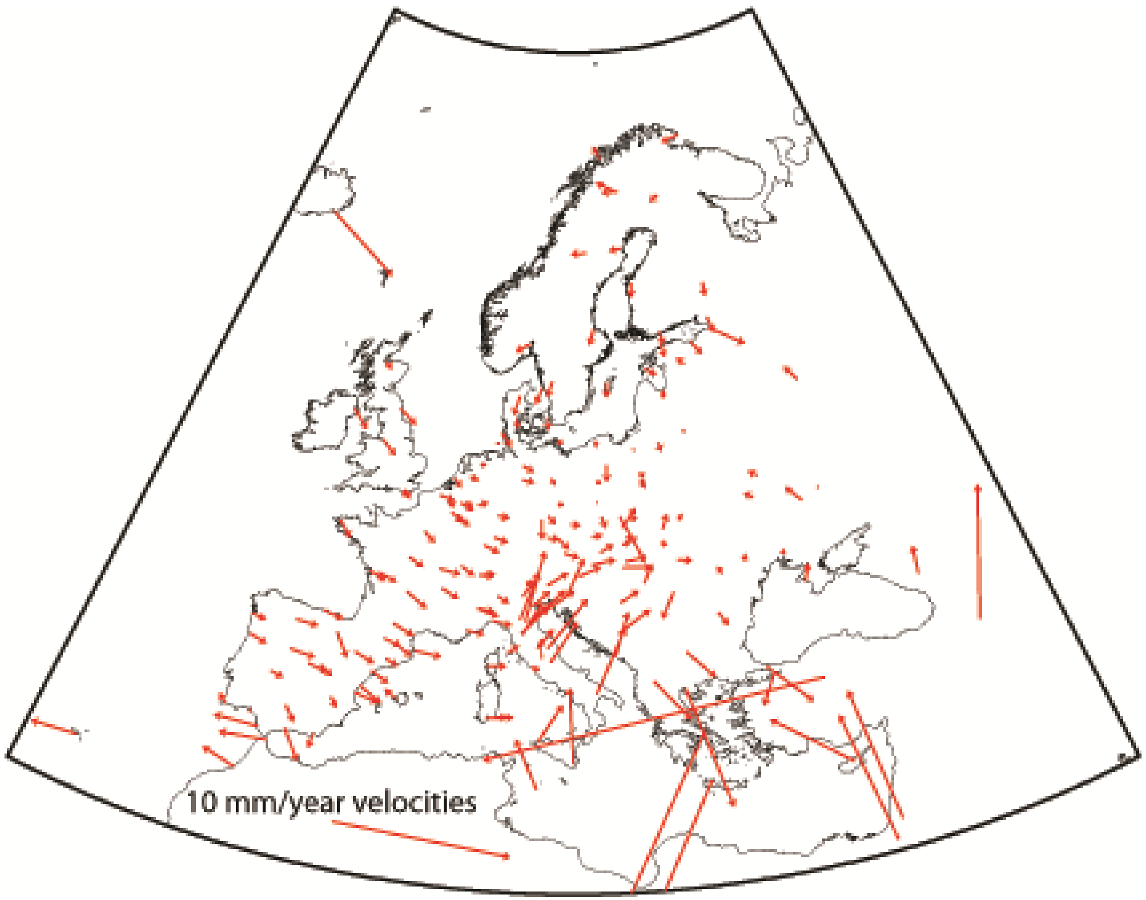
1. continuous estimates,
2. smooth results,
3. more realistic variance-covariance assessment.

First trials on EPN: class A solution of GPSW 1570

Original velocities (in ITRF2005)

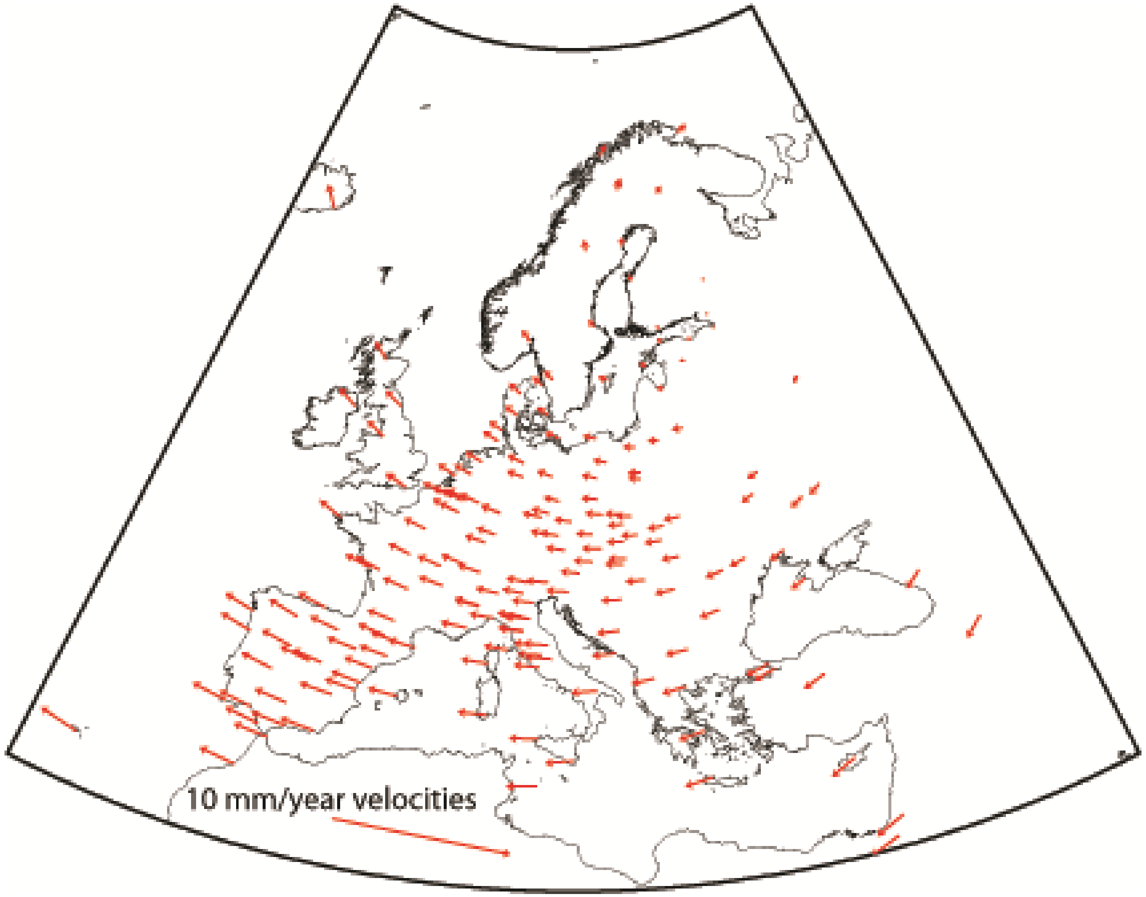


Tisserand velocities



Note: in this very first analysis:
all the EPN stations used, no subregion clustering.

Comparisons between ITRF-Tisserand and ETRF



A small rotation due to the no subregion clustering between class A EPN stations (stable part of Europe).

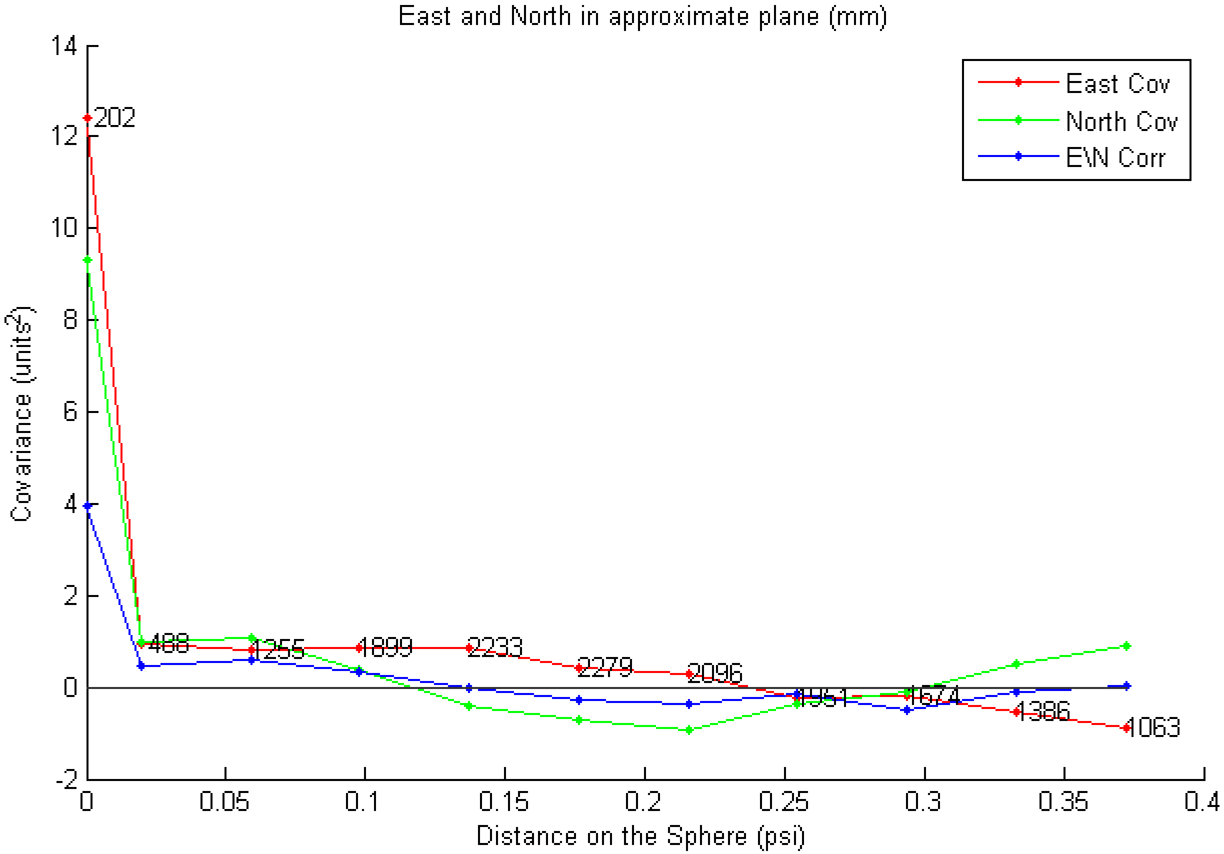
Numerical differences

	(ITRF+Tiss)-(ETRF)		(ITRF+Tiss)-(ETRF+Tiss)	
	East	North	East	North
E	-1.3	0.2	0.0	0.0
StdDev	0.6	0.4	0.2	0.2
min	-2.8	-1.0	-1.2	-0.7
max	0.8	1.4	0.5	1.0

Application of Tisserand principle to ETRF coordinates leads to almost identical results.

What about spatial covariances of Tisserand displacements?

Empirical covariances (without model estimation)



East and North displacements in pseudo-mm: no correlated signal to predict.

A macro clustering between subregions would be useful.

The relative motion of subregions could be estimated; to predict deformations in subregions, denser networks are needed.