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Heights and scale variation in their inherent reference frames

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Introduction

Realization of a vertical reference system (VRS) from geodetic data → 2 degrees-of-freedom!

VRS "Origin" *(zero-height reference level)*

VRS "Scale" *(spatial reference scale for heights)*

Specification of VRS datum parameters (& their temporal evolution) is a key issue for modern cm-level height systems in geodesy and Earth sciences, e.g. GGOS, IVRS, EVRS

(*) also important for the synergy of VRS with other spatial TRFs and/or space-geodetic techniques

Introduction (cont'd)

Realization of a vertical reference system (VRS) from geodetic data → 2 degrees-of-freedom!

VRS "Origin" *(zero-height reference level)*

VRS "Scale" *(spatial reference scale for heights)*

- ☑ Most research work has been performed on the definition and realization of the VRS "Origin"
- ☑ Extensive research work has also been performed for the connection of VRSs with different "origins"
- ☐ **The "spatial scale" issue has <u>not</u> been really considered within modern VRS theory and practice!**

What is a VRS ?

- In mathematical terms, a VRS corresponds to an 1D coordinate system (with possible time-dependency)
- In geodetic terms, a VRS is a framework for “height determination” that is embedded in the 3D Euclidean space and is associated with various 2D reference surfaces
- **A modern VRS should be “tied” to both a physical (geopotential) and a geometrical Earth representation**

What is a VRS ?

Primary VRS 'vertical coordinates'

$c(P) = W_o - W(P)$ ------→ *Equivalent physical heights*

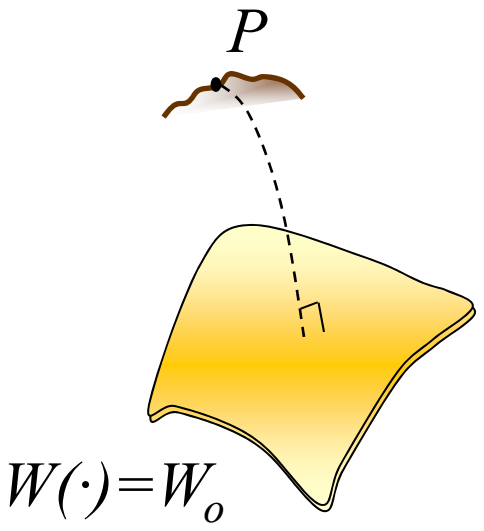
that are complemented by 3D spatial position

X(P), Y(P), Z(P)

and a gravity field "representation model"

e.g. C_{nm}, S_{nm}, W(·) or T(·)

*Not currently available in an **integrated way**…*



VRS “Scale”

- Realized directly through all input data
- Affected also by modeling choices and other approximations (e.g. computation of mean [g] or mean [γ], choice of tidal system)
- ***Remains a vague concept*** – No “standard method” for transforming VRS/VRF vertical positions and their velocities under a spatial scale change
- **No special scale-considerations currently exist in VRS realization/usage with multi-source data!**
  (compare with IERS approach for ITRS realization)

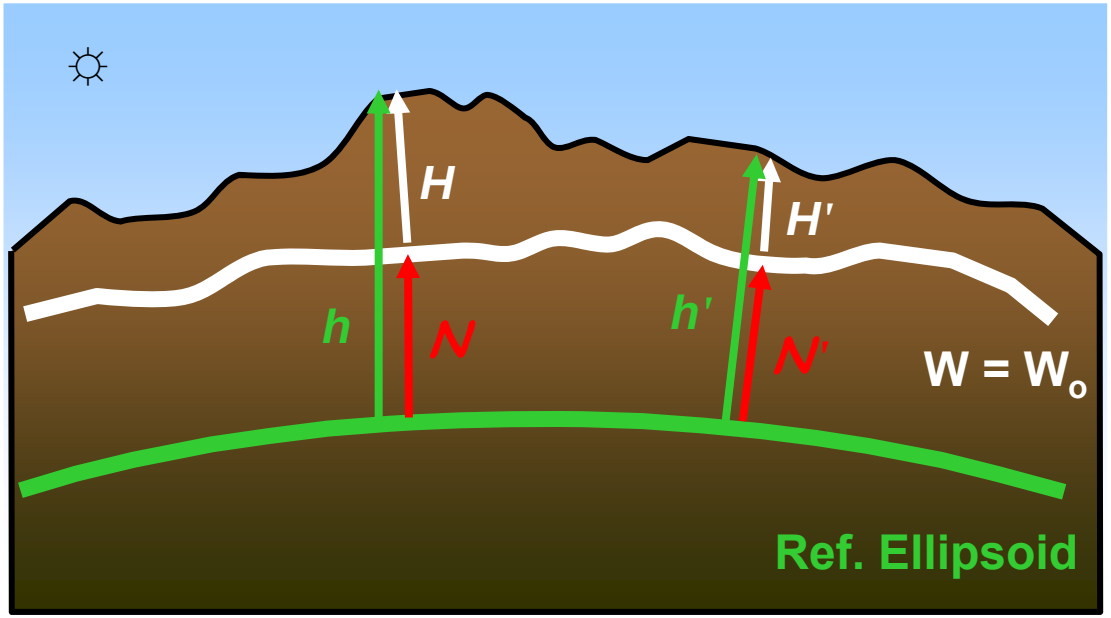
A simple example

$h - H - N = 0$

$\Delta h - \Delta H - \Delta N = 0$

Theoretical VRS constraints

the underlying height types (should) refer to an ideal and uniform spatial scale!



H, ΔH

Spirit leveling, gravimetry & other modeling assumptions

h, Δh

Space-geodetic techniques & appropriate TRFs

N, ΔN

GGMs, local gravity, DEMs, etc.

How critical are the inherent differences in spatial scale realization from the individual (absolute and/or relative) "height components" of a modern VRS ?

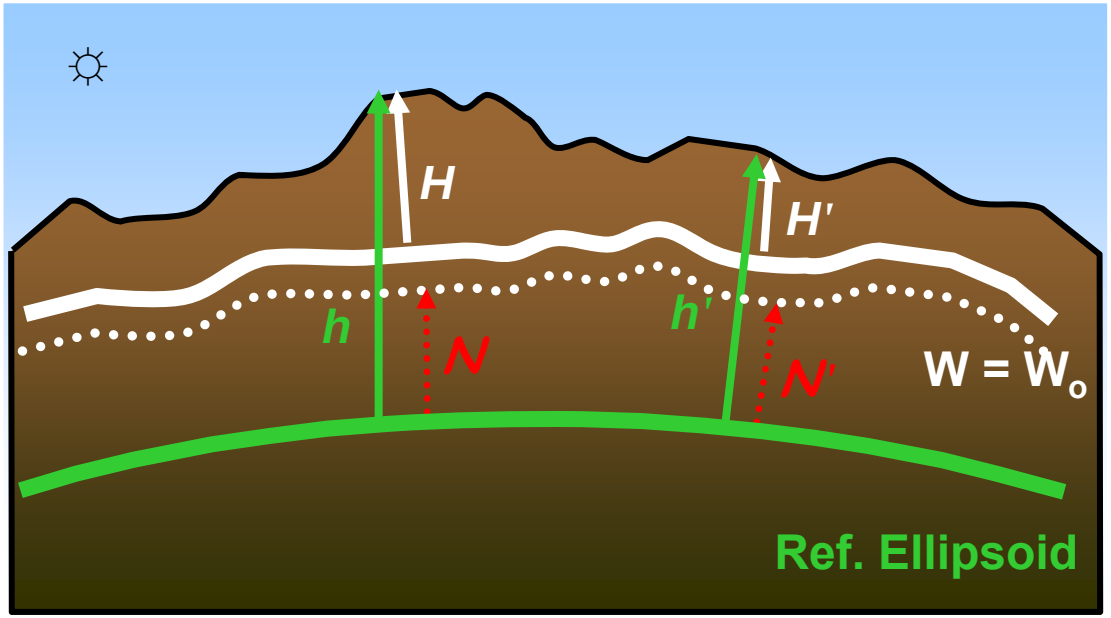
A simple example (cont'd)

$h - H - N = 0$

$\Delta h - \Delta H - \Delta N = 0$

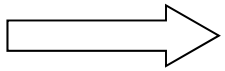
Theoretical VRS constraints

The underlying height types (should) refer to an ideal and uniform spatial scale!



In practice, *scale differences among the height types are handled through a constant correction-bias that is estimated over a test network of GPS BMs*

Such an approach is not uniquely "interpretable" and we do not really know where this scale-correction should be applied to



Bias between what ?

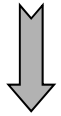
"N" geoid vs. "h-H" geoid

"h" earth vs. "H+N" earth

"H" earth vs. "h-N" earth

Levels of VRS “Scale”

- Spatial scale realized by (adjusted) geopotential numbers and their equivalent physical heights
- Spatial scale realized by VRS-linked geometric heights with respect to an underlying TRF + level ellipsoid
- Spatial scale realized by a VRS-linked (or ‘external’) Earth geopotential representation $W(\cdot) \rightarrow$ *geoid model*



Explore ways to validate the consistency
among the previous scale levels
e.g. over a network of GPS/lev BMs

Remarks…

- For a complete and rigorous study, it is required to model the effect in the VRS vertical coordinates (i.e. geopotential numbers) due to a joint perturbation of the VRS datum parameters “origin” + **“scale”**

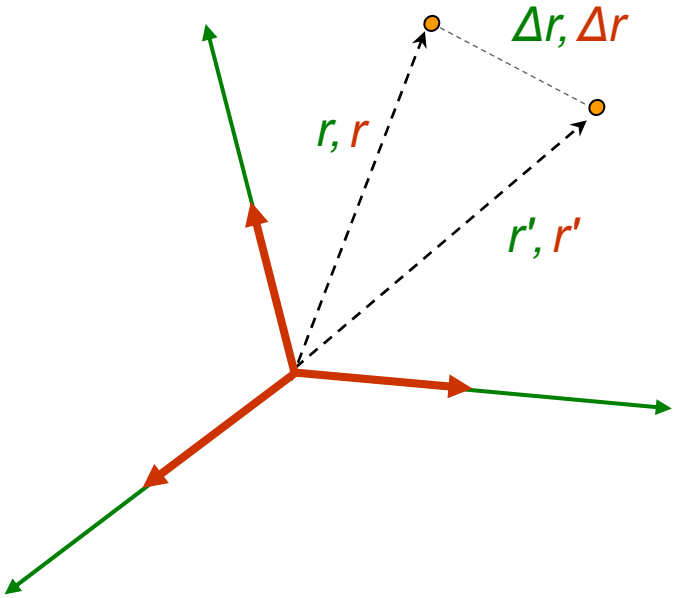
  Unsolved and rather complicated task…

- Instead, we look into the TRF spatial-scale variation and its corresponding effect on geometric heights and VRS physical heights

  (*) Relevant since the geometric heights (and their underlying TRF) play a crucial role in VRS realization and the monitoring of temporal height changes within a vertical reference system

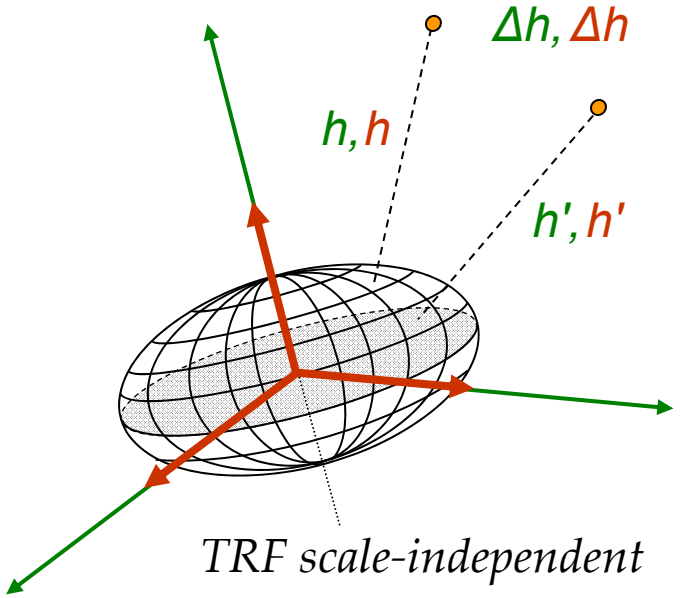
Note that…

Spatial distances



$$1+\delta s = \frac{\Delta r}{\Delta r} = \frac{r}{r} = \frac{r'}{r'}$$

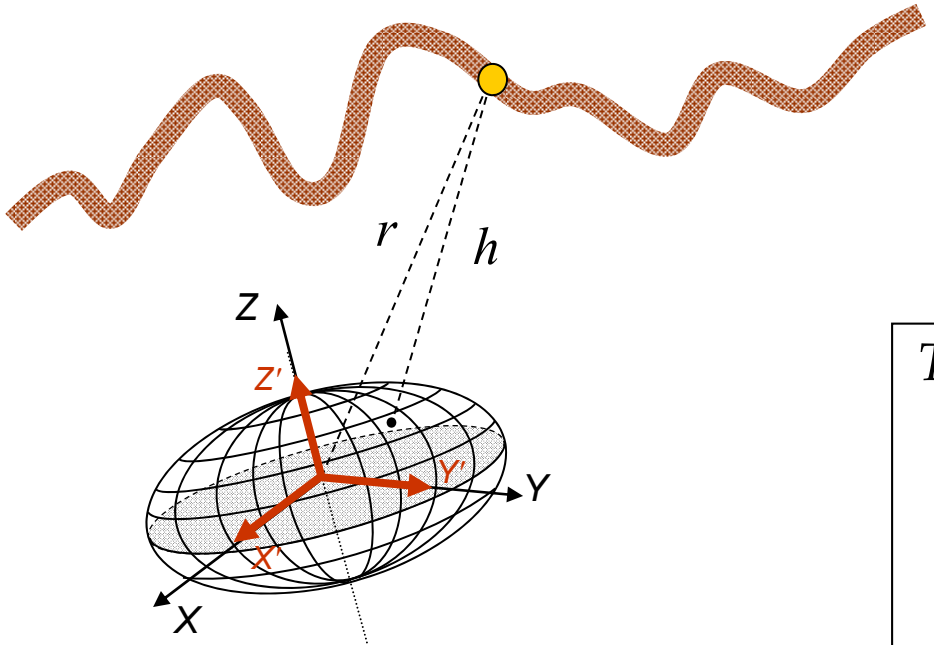
Geometric Heights



TRF scale-independent reference ellipsoid (IERS 2003)

$$1+\delta s \approx \frac{\Delta h}{\Delta h} \qquad 1+\delta s \neq \frac{h}{h} \qquad 1+\delta s \neq \frac{h'}{h'}$$

Geometric heights & TRF scale



TRF spatial-scale change

$$\boldsymbol{r}' = (1+\delta s)\boldsymbol{r}$$

TRF scale-dependent height variation

$$h' = (1+\delta s)h + a w \delta s$$

$$h' \approx h + a w \delta s$$

where: $w = (1 - e^2 \sin^2 \varphi)^{1/2}$

Existence of apparent vertical offsets due to the **implicit invariance** of the reference ellipsoid under TRF scale change

δs	$a w \delta s$
10^{-6}	~ 6.4 m
10^{-7}	~ 64 cm
10^{-8}	~ 6.4 cm
10^{-9}	~ 6.4 mm

VRS & spatial-TRF synergy

- *A TRF scale variation of 1-2 ppb causes a latitude-dependent offset of ~ 1 cm in geometric heights*

In view of the "physical height realization" approach:

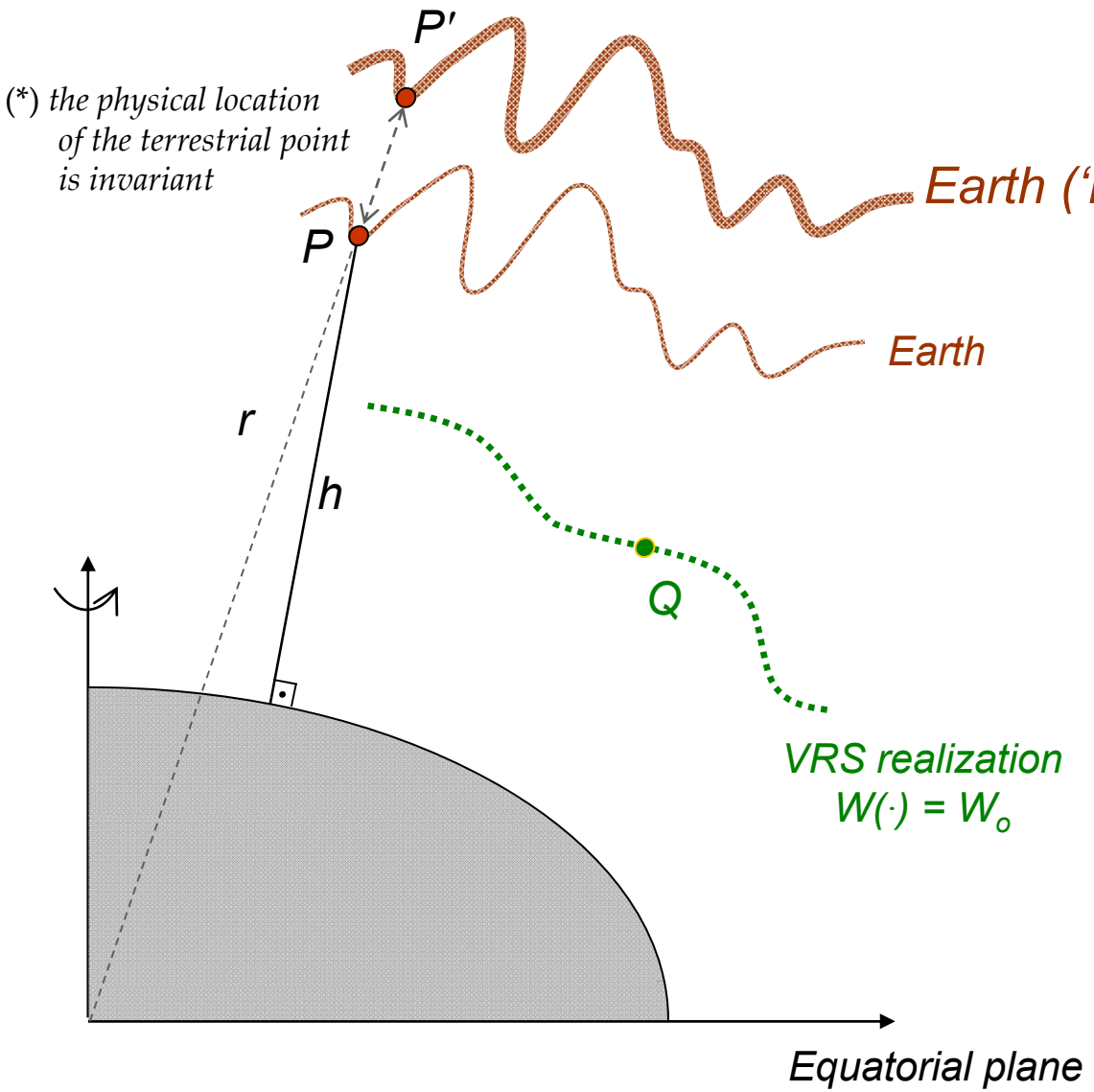
$$H = h - N \qquad \Delta H = \Delta h - \Delta N$$

how should we treat the VRS components *H* & *N* under a spatial-scale change **of the underlying TRF** ?

(*) It should be taken into account that a TRF scale change from an "old" frame to a "new" frame corresponds to a more precise realization of the Earth's **terrestrial-only** geometry

What happens under a TRF scale change ?

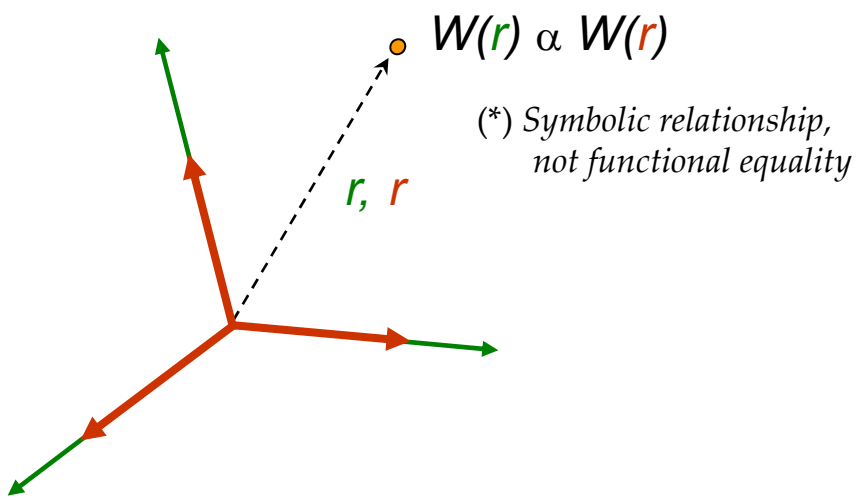
(static case – “same epoch”)



Should VRS-related quantities be affected by a TRF scale change ?

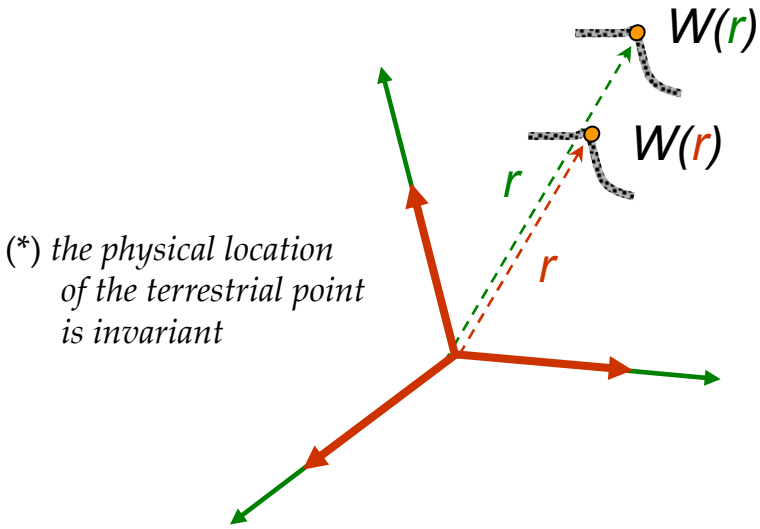
Different viewpoints…

'Math Physicist'



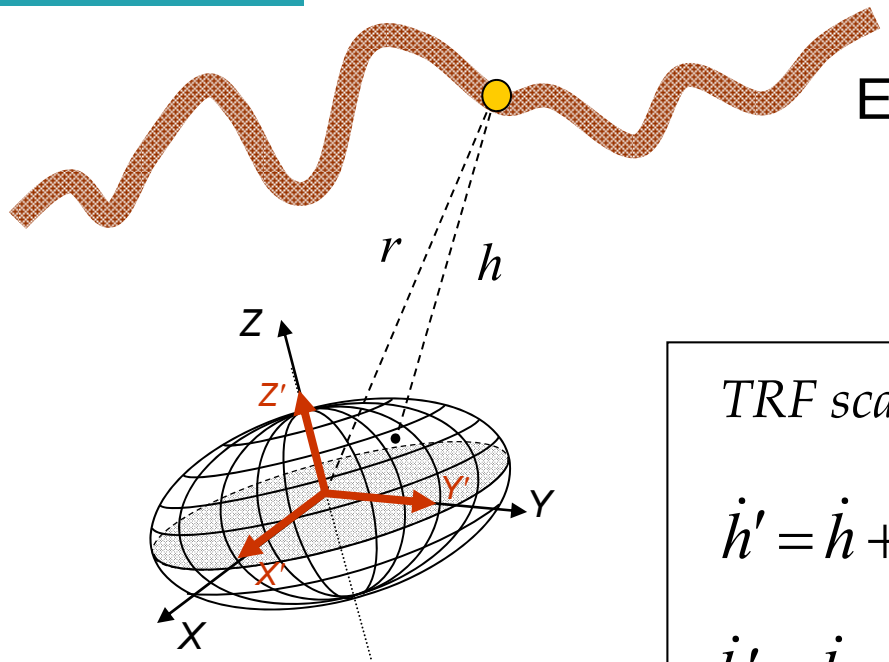
- Unequivocal position of abstract points
- Representation of *W(·)* should be 'adapted' to the particular scale of the spatial coordinate system
  (e.g. Kleusberg's formulae, GM re-scaling)
- **Need for a reference metric scale**

'Geodesist'



- Inexact position of terrestrial points
  (TRF scale change reflects a more precise estimate of Earth's geometry!)
- Knowledge of the Earth's gravity field representation *W(·)* is not perfect
- $r = (1+\delta s)\, r \rightarrow W(r) \stackrel{?}{\simeq} W(r)$

Geometric heights & TRF scale



TRF scale-dependent height variation

$$h' = (1+\delta s)h + a w \delta s$$

TRF scale-dependent height velocity variation

$$\dot{h}' = \dot{h} + (\dot{h} - \frac{e^2 \sin 2\varphi}{2w} a\dot{\varphi})\,\delta s + (h + aw)\dot{\delta s}$$

$$\dot{h}' \approx \dot{h} + a w \dot{\delta s}$$

where: $w = (1 - e^2 \sin^2 \varphi)^{1/2}$

Systematic differences (~ of a few mm/yr) in geometric height velocities between TRFs with time-dependent spatial scale variation

VRS & spatial-TRF synergy

- *A TRF scale variation with rate 0.2 ppb/yr causes a latitude-dependent change of ~ 1-2 mm/yr in the geometric height velocities*

In view of the “physical height monitoring” approach:

$$\dot{H} = \dot{h}$$

how should we treat the physical height velocities in a VRS under a spatial scale change **of its underlying TRF** ?

Summary

- Open problems exist in VRS theory and practice regarding the treatment of spatial scale issues !
- A cm-level VRS requires appropriate transformations & reductions to account for inherent differences in scale realization from its individual “height components”
- Relationship between VRS & TRS (conventions, parameters, realization, models) needs to be explored – Not completed in the frame of IAG/ICP 1.2
- A modern VRS implementation in alignment with IERS/ITRS methodology and conventions requires a clear view for scale issues…

Thanks for your attention !

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