The Abruzzo earthquake: temporal and spatial analysis of the first geodetic results

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Outline

Introduction: the Abruzzo earthquake

The network and the processing strategies

Time modelling of daily results:
displacements estimation at earthquake epoch

Spatial interpretation of the
horizontal displacements

The vertical displacements

Future works

Abruzzo earthquake

Main event: 6th april, 1:33 UTC
Location: 42.33° N, 13.33° E,
Depth: 8.8 km
Magnitude: 5.8 Richter

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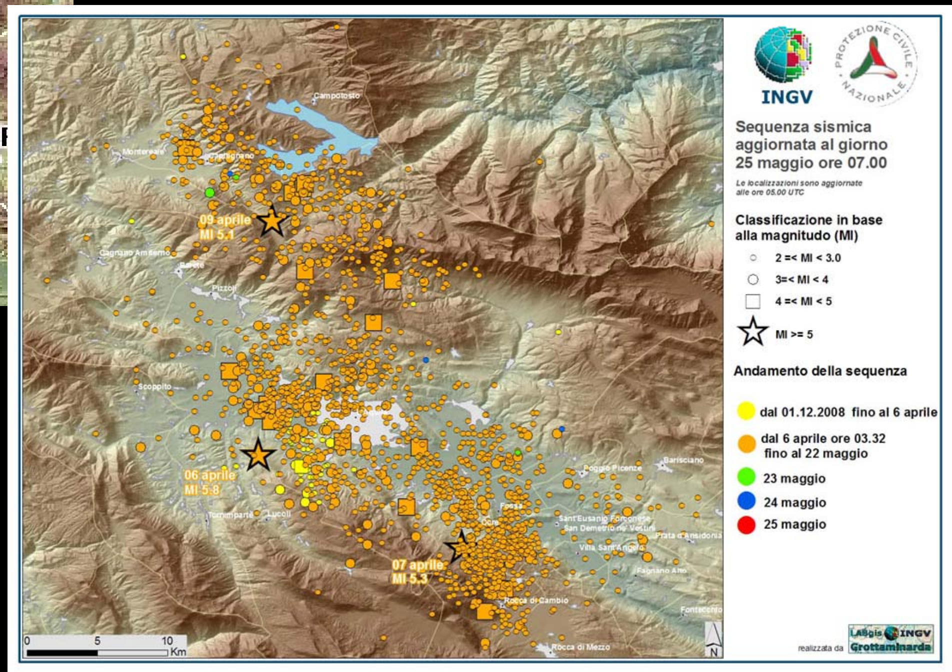
Before and after the main event:
many other
pre seismic and after seismic events



The geodetic network

3 Italian IGS stations

32 stations in Abruzzo region

17 other stations within a distance of ~ 50 km from Abruzzo boundaries

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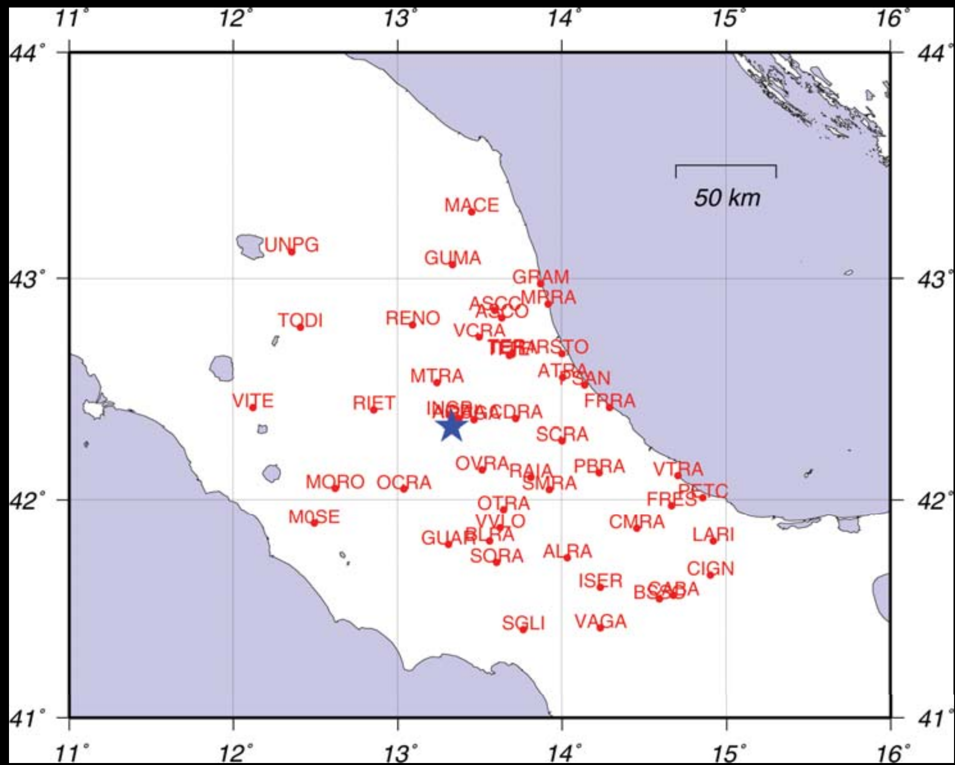
Data from
1st February (DOY 32) to 2nd, May (DOY 122)
have been adjusted up to now
from 32 to 95 (64 days): before earthquake
from 96 to 122 (27 days): after earthquake

The geodetic network

ASI-Geodaf, INGV-RING, Leica-ItalPos, TopCon-Geotop, GPSAbruzzo, GPSUmbria, ResNap



The processing strategies 1/2

IGS stations stochastically constrained:

coordinates:

interpolation of last 52 IGS05 weekly solutions,

constraints:

2 mm horizontally, 4 mm in height

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2 mm horizontally, 4 mm in height

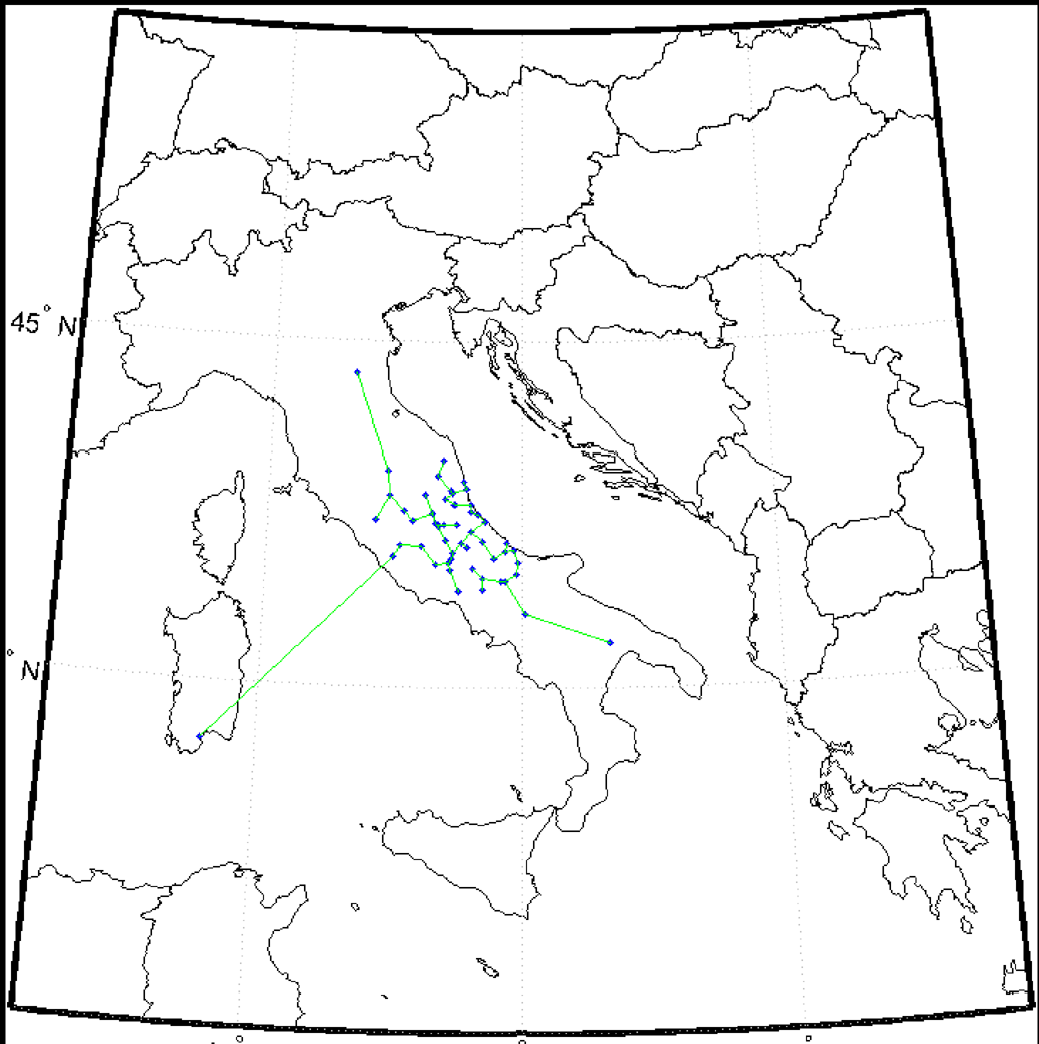
Final IGS EOP, EPH and PCV's

Adoption of the international standards

in the raw data processing

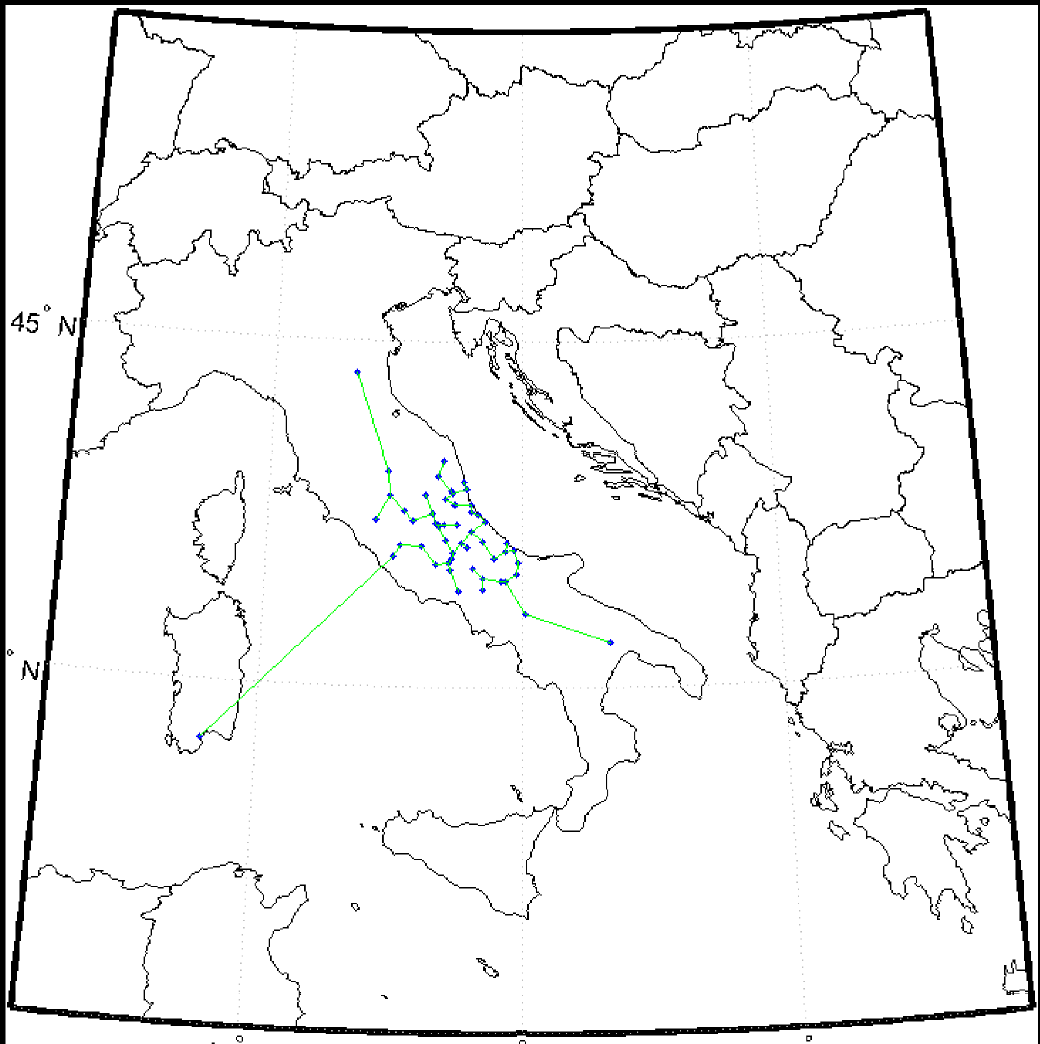
by BSW 5.0 software

The processing strategies 2 / 2



Outlier rejection

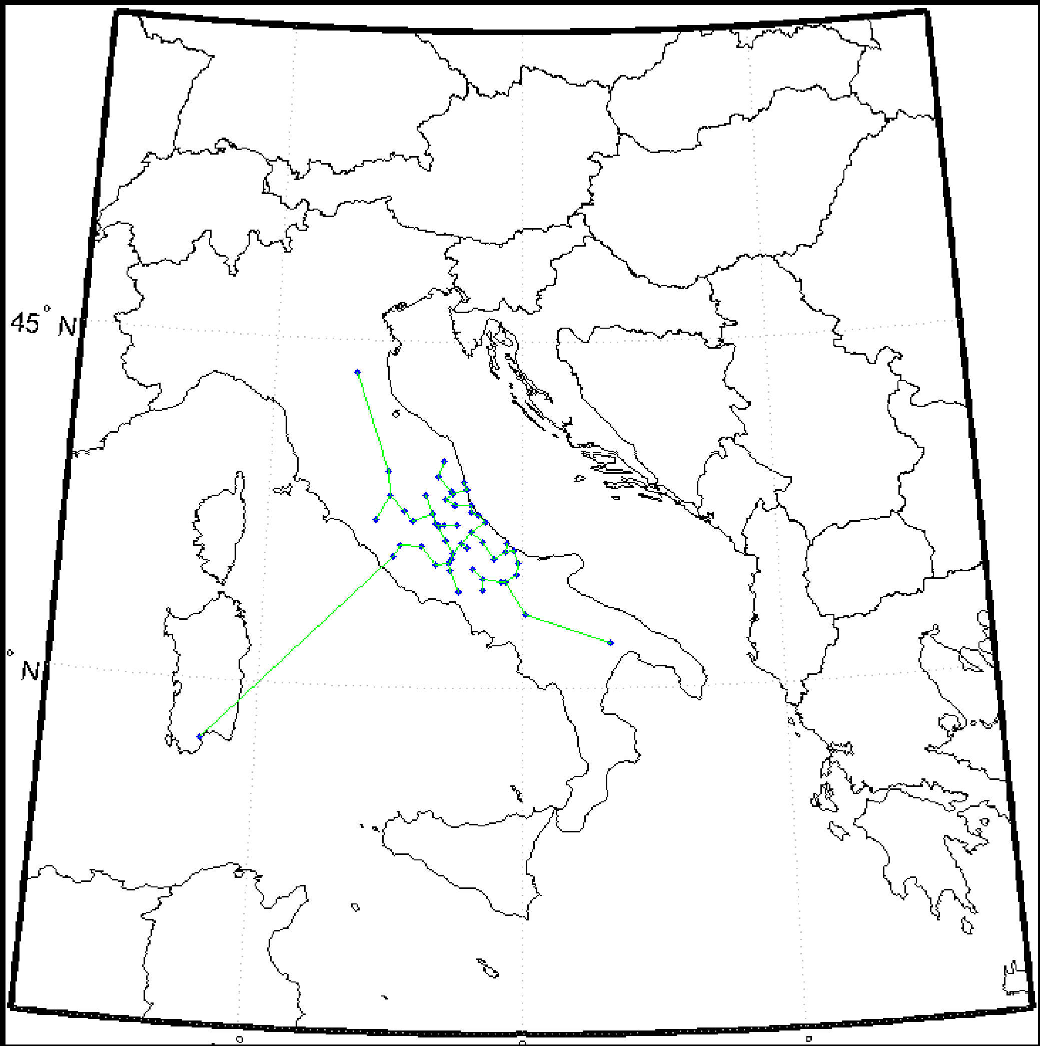
The processing strategies 2/2



Outlier rejection

Modelling the time series to estimate discontinuities

The processing strategies 2 / 2

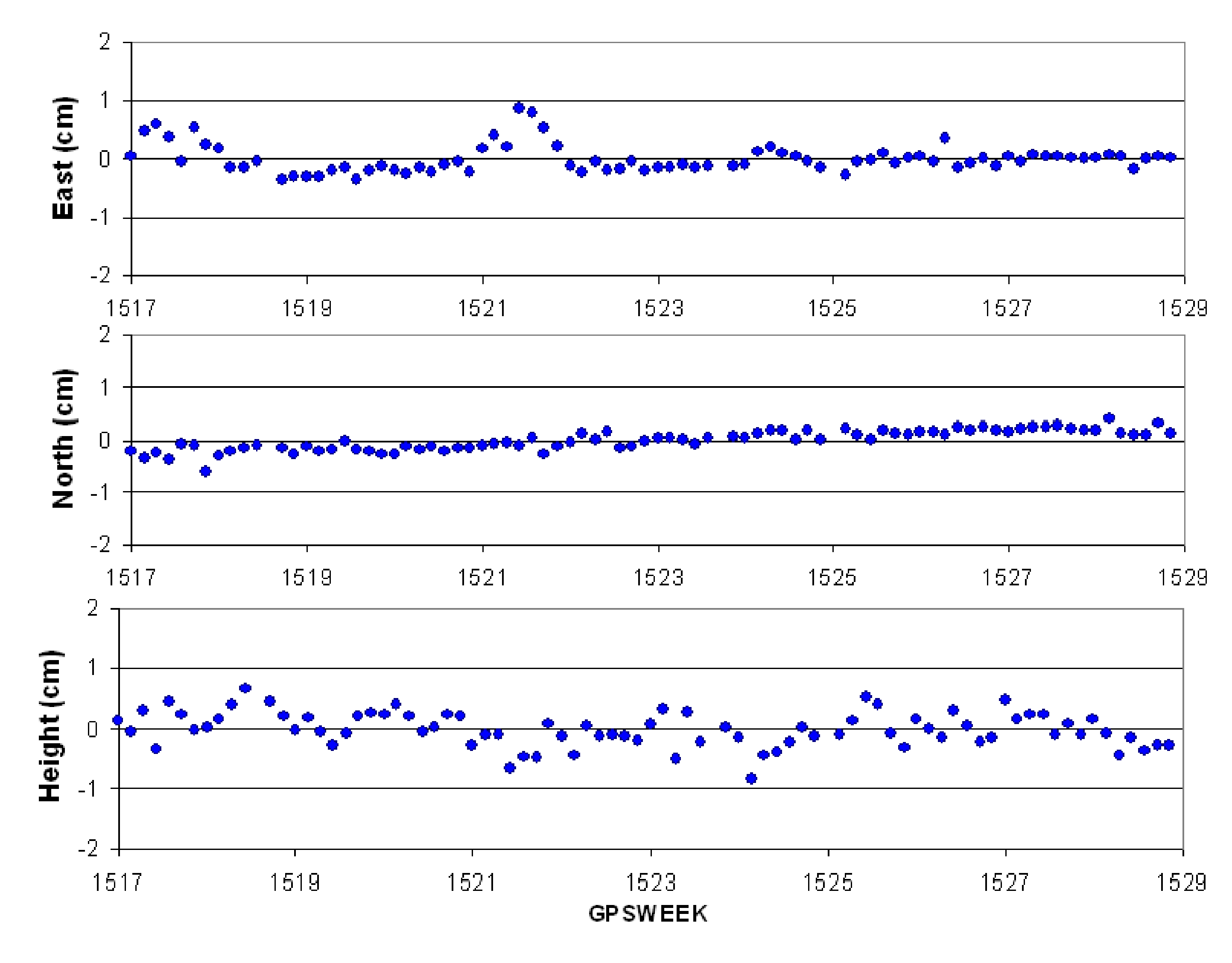


Outlier rejection

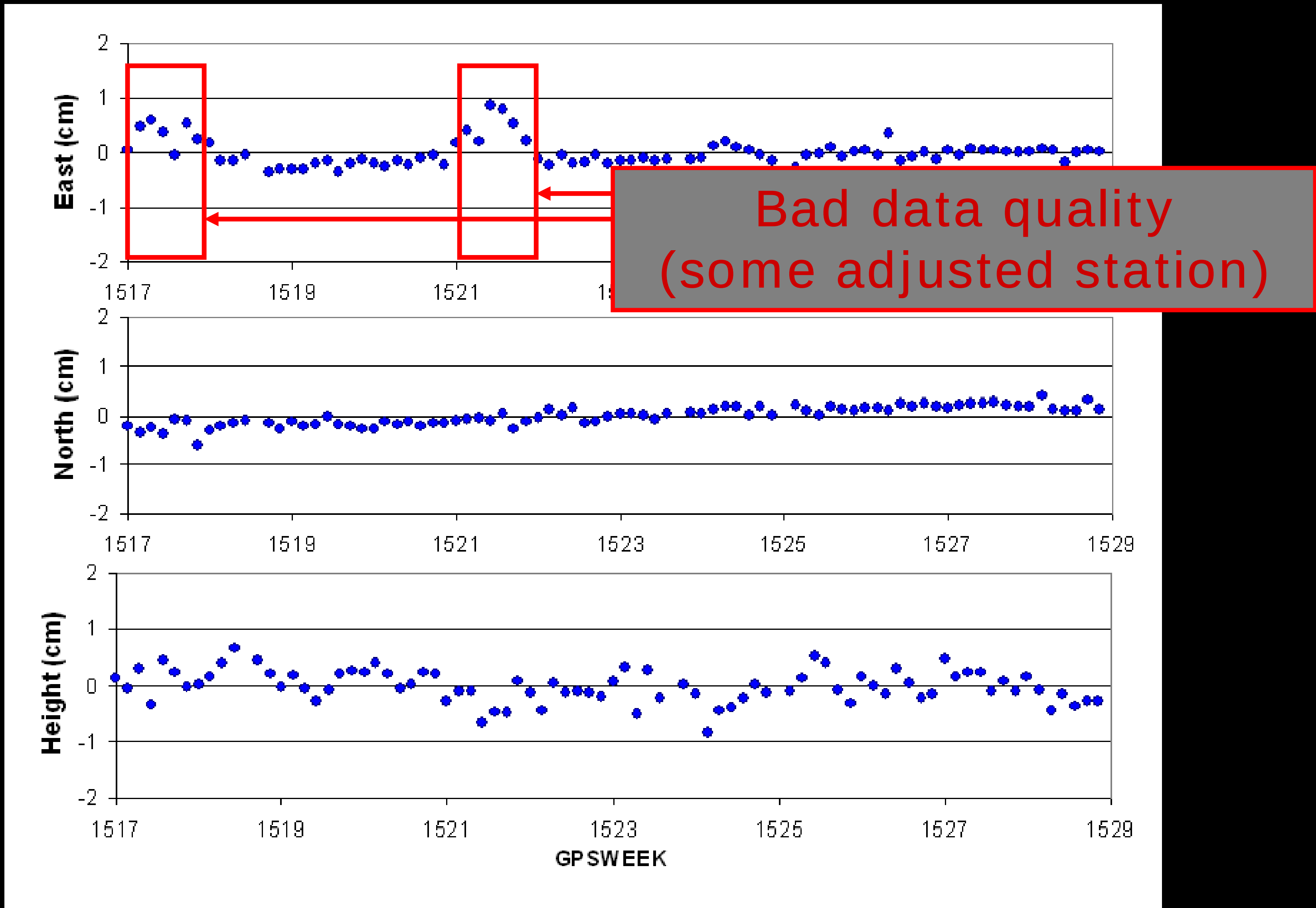
Modelling the time series to estimate discontinuities

Spatial analysis of the discontinuities and clustering in subregions

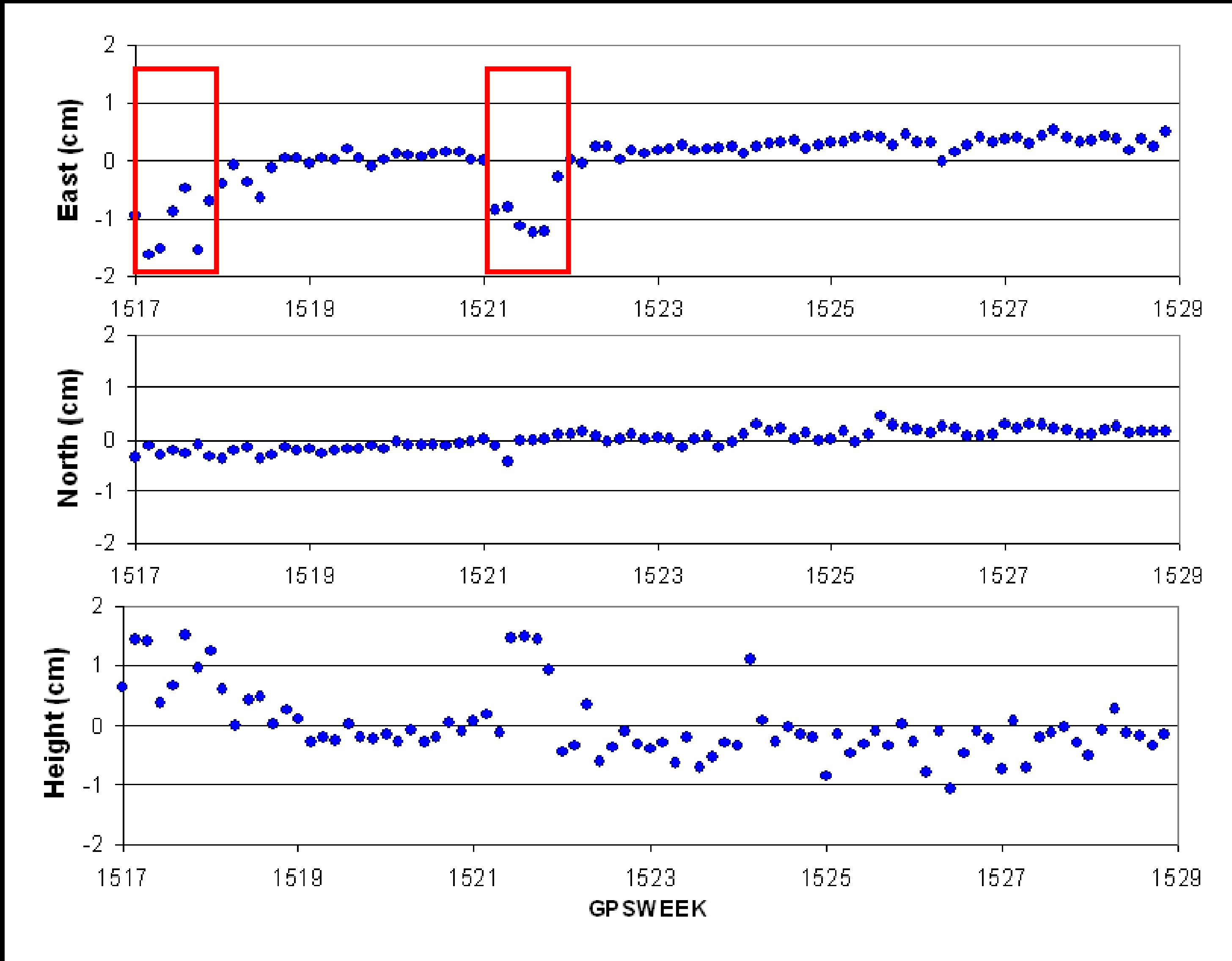
Examples of time series: MATE



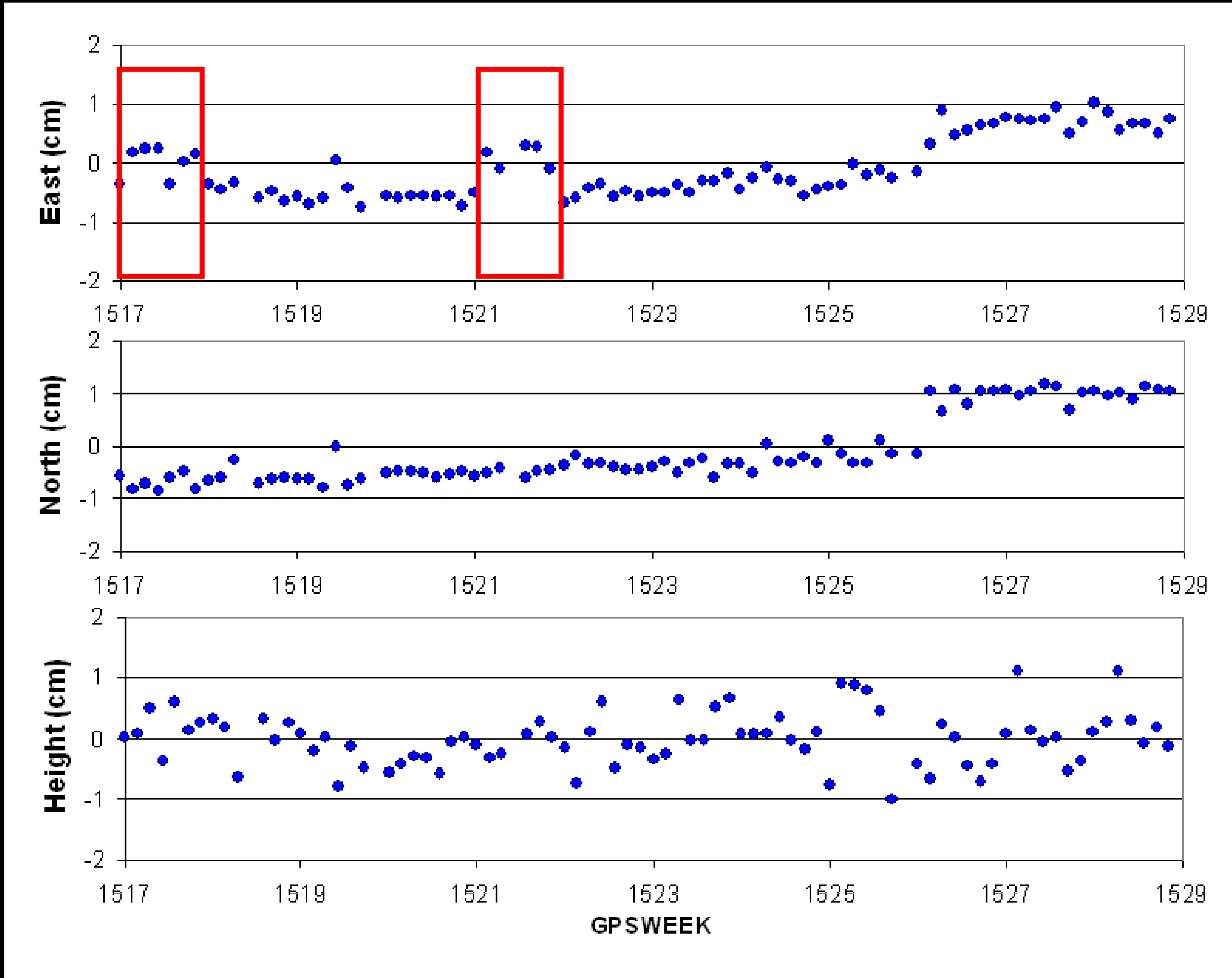
Examples of time series: MATE



Examples of time series: MEDI



Examples of time series: TERA



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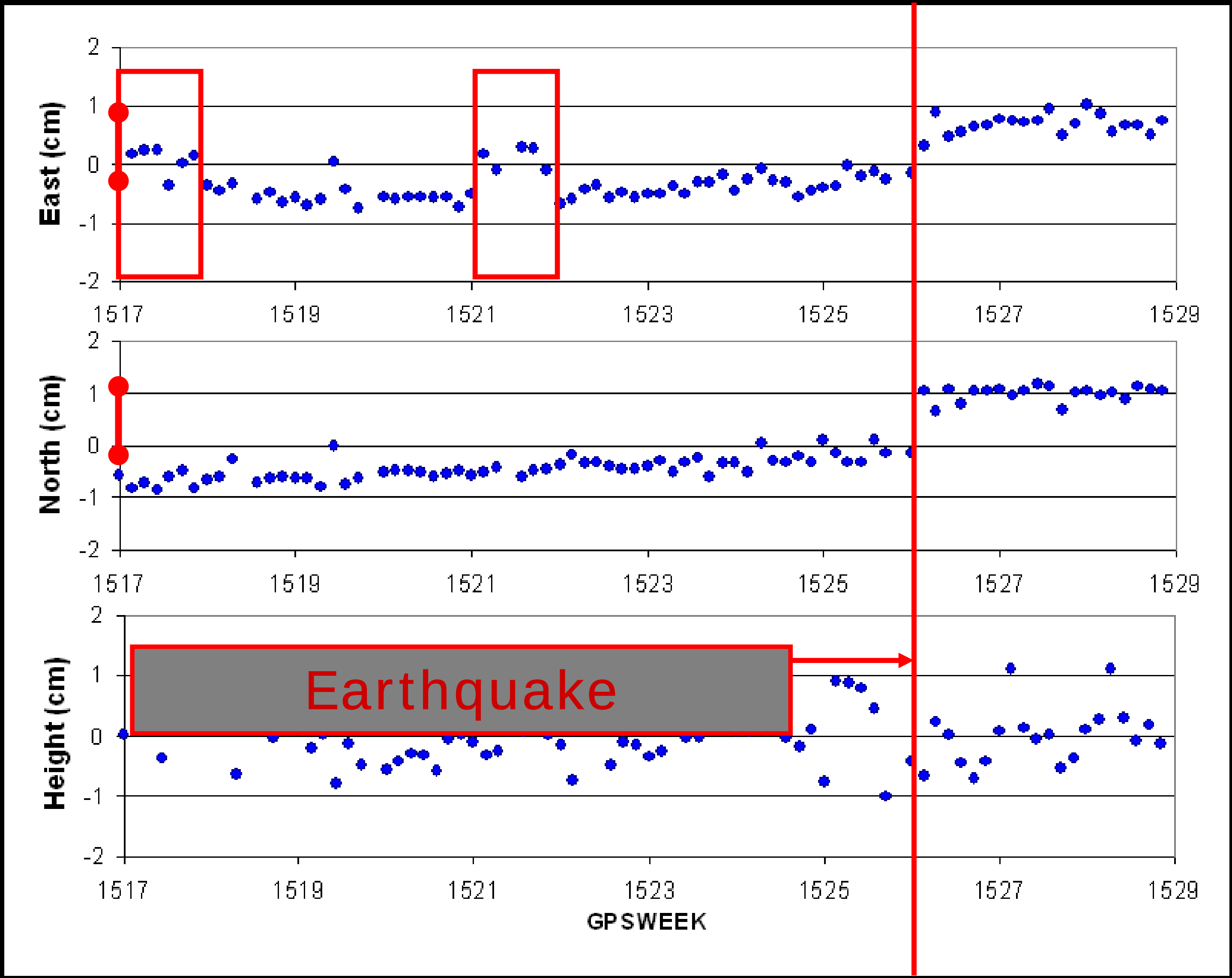
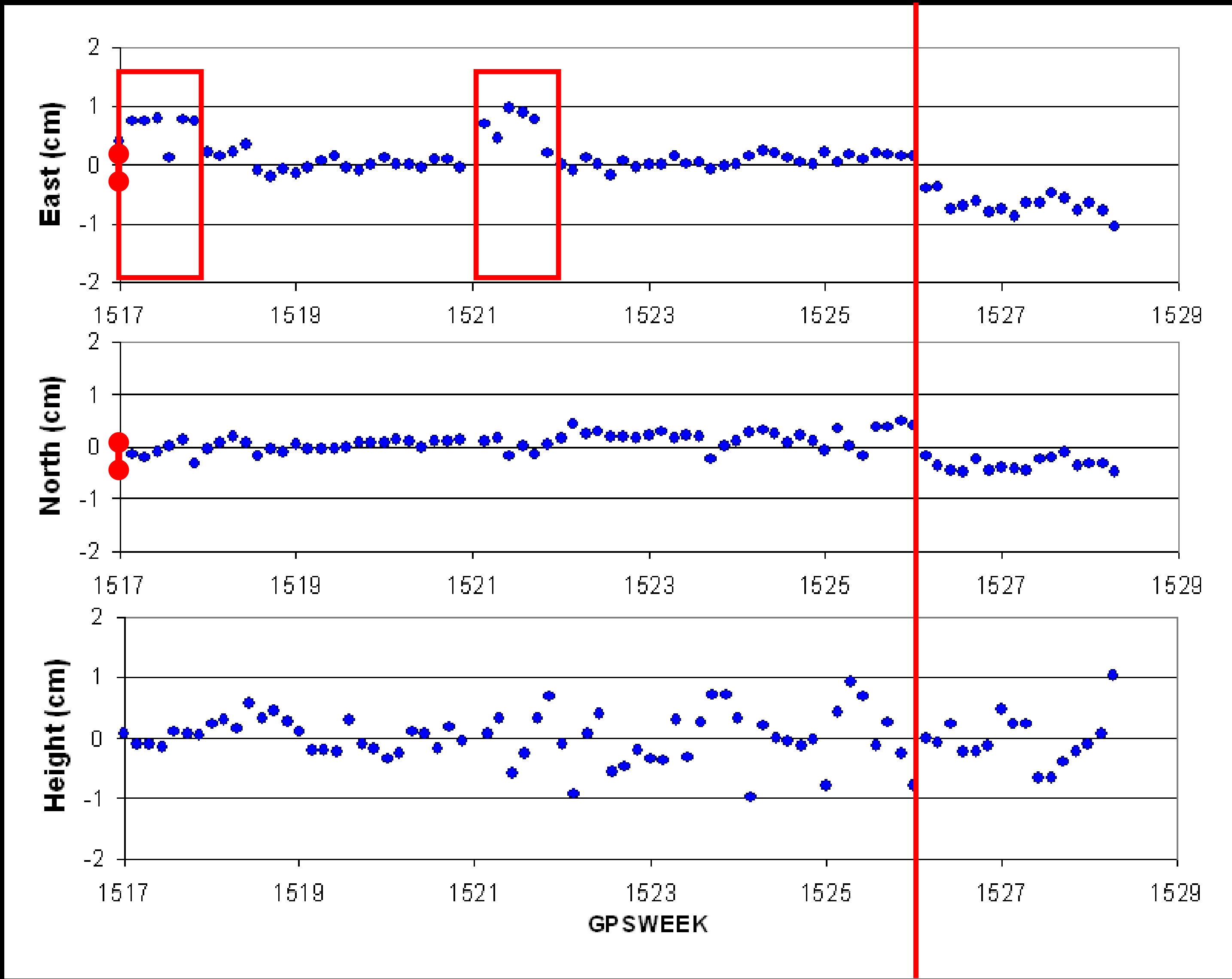
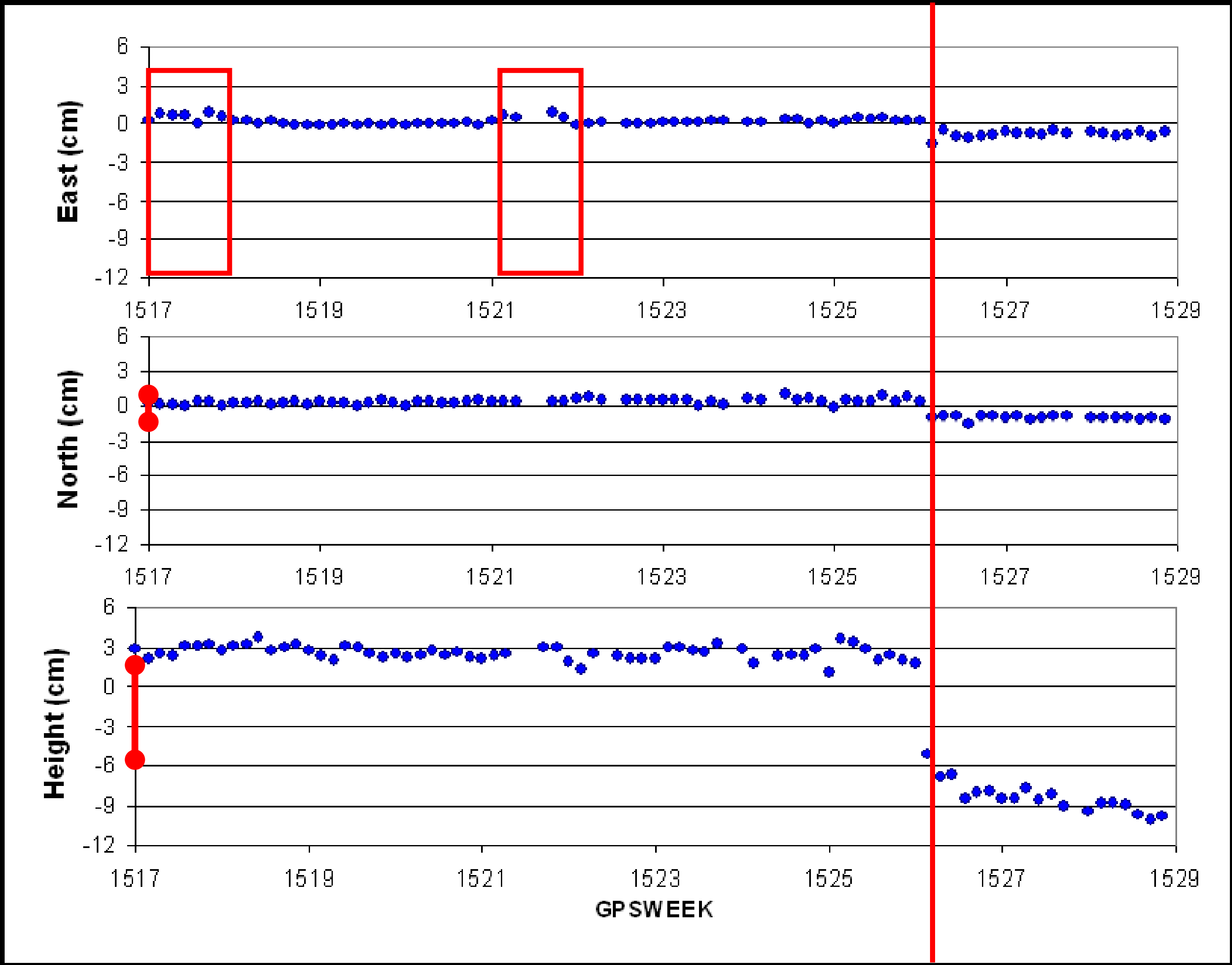


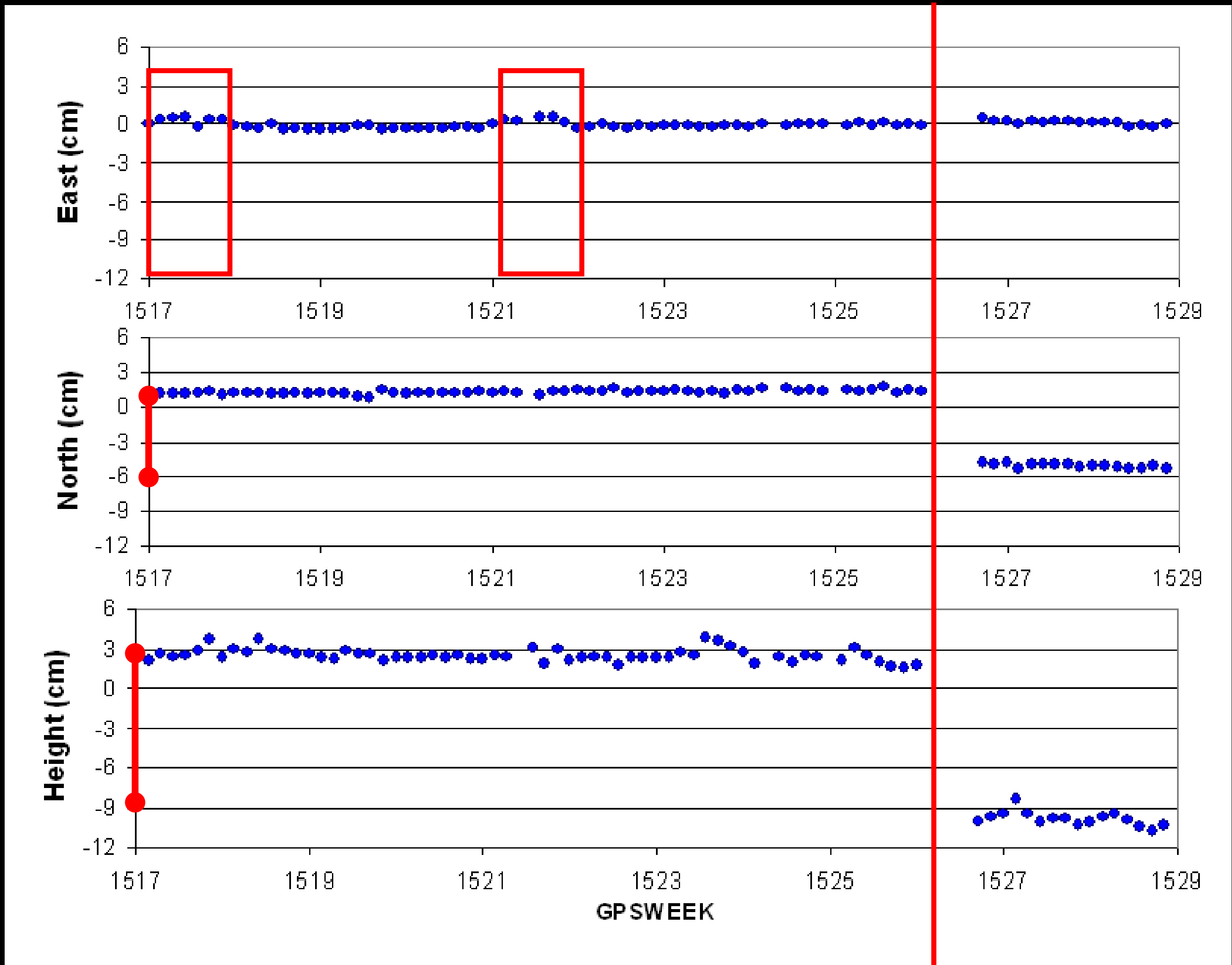
Examples of time series: OCRA



Examples of time series: PAGA



Examples of time series: AQRA



Outliers rejection

Permanent networks are intrinsically redundant

$\Downarrow$

to improve coordinates repeatabilities
a severe automated outliers rejection is useful

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a severe automated outliers rejection is useful

This is a particular case:
few data, manual analysis,

$\Downarrow$

conservative approach in outlier rejection
just bad quality sessions before earthquake removed

The results of IGS stations

3 stochastically constrained stations:
CAGL, MATE, MEDI

Residuals of daily results wrt apriori coordinates			
(mm)	East	North	Height
Mean	0.6	0.4	0.5
σ	2.7	1.0	4.1
Min	-4.0	-2.6	-10.8
Max	6.4	6.6	17.4

Time series interpretation (1/2)

Short time series in the geodetic analysis

$\Downarrow$

constant model to avoid propagation of

seasonal effects and

localized in time variations

into meaningless estimated velocities

Time series interpretation (2/2)

Before earthquake:

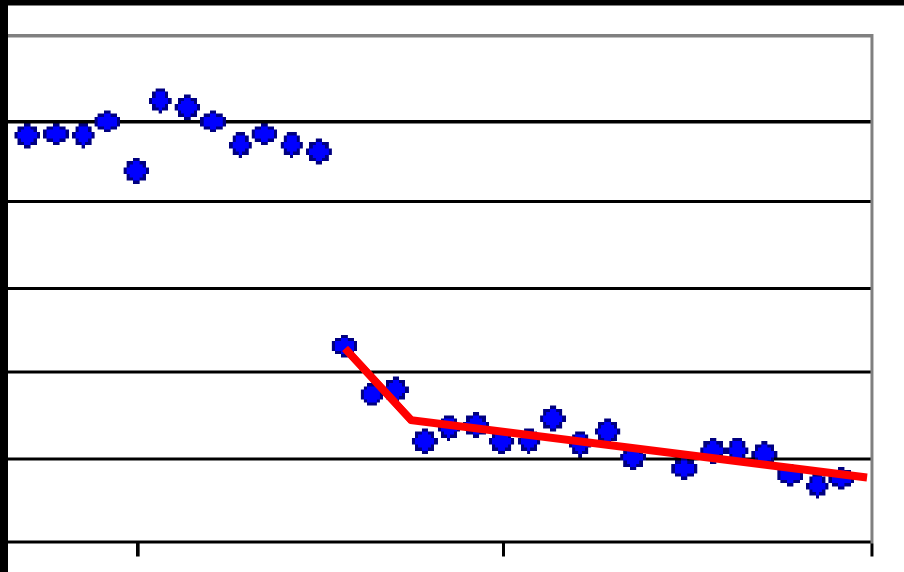
not a clear presence of pre seismic signal, just linear trend

$\Downarrow$

linear trend estimation and removal

not to estimate velocities but to better model daily solutions

Time series interpretation (2/2)



After earthquake:

a postseismic signal is often clear,

but few days are available

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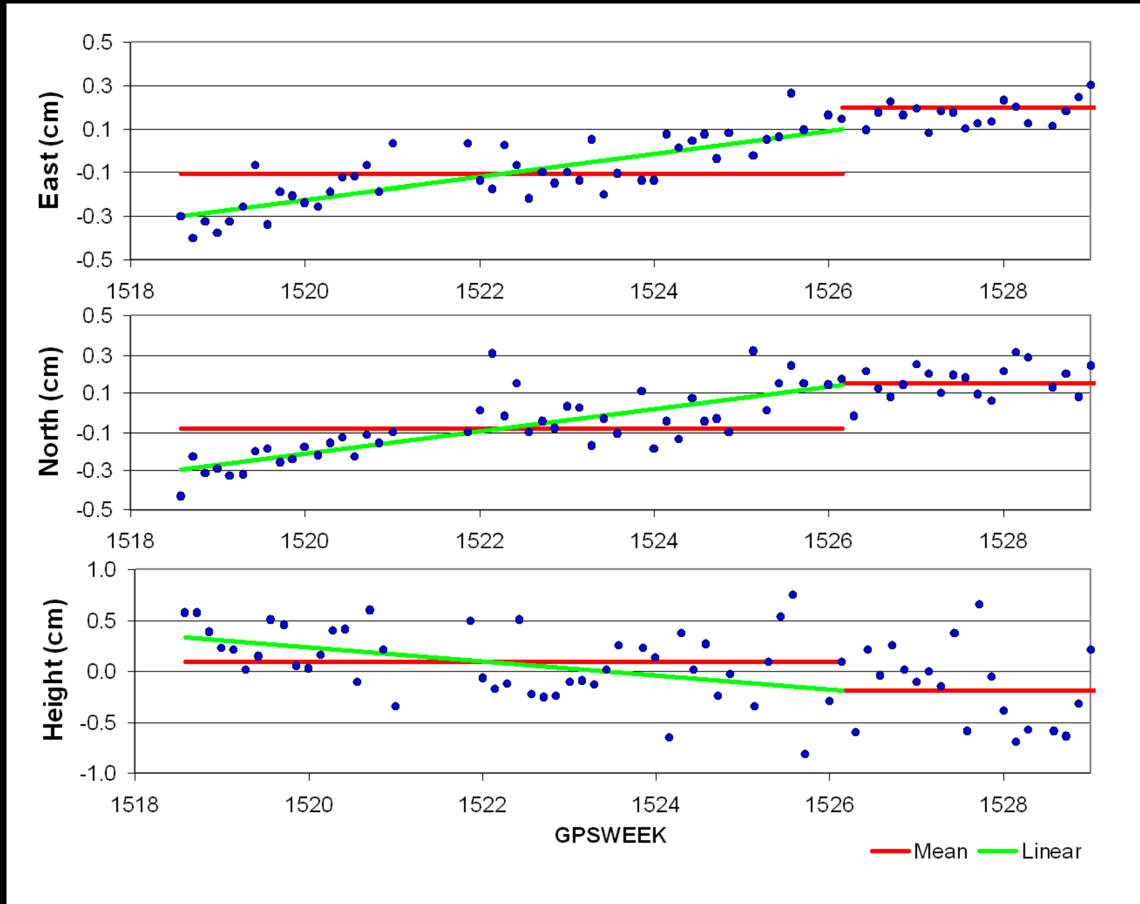
at the present, simple costant model applied,

with more data: linear and 2nd order polynomial

One example



(mm)	DE	DN	Dh
Constant	3.0	2.3	-2.9
Linear	0.9	0.1	-0.1

Residuals statistics of daily solutions

Before			
(mm)	E	N	h
Mean	0.0	0.0	0.0
σ	1.5	1.3	3.6
Min	-6.1	-6.3	-11.1
Max	7.1	5.9	11.4

After			
(mm)	E	N	h
Mean	0.0	0.0	0.0
σ	1.5	1.4	4.9
Min	-7.2	-6.5	-23.6
Max	8.0	8.5	34.7

Worse height results after earthquake:
post seismic assessment of 4 stations near L'Aquila

Parameters and covariances estimation

Daily coordinates models in time

$\Downarrow$

Model parameters estimated by LS

Formal daily covariances typically underestimated
and final covariances too much optimistic

$\Downarrow$

Empirical covariances estimation needed

Parameters and covariances estimation

Few observations

$\Downarrow$

Simplified hypotheses on
time series models and covariances

Parameters and covariances estimation

Few observations

$\Downarrow$

Simplified hypotheses on
time series models and covariances

Joint estimation of parameters and covariances

$\Downarrow$

Typically an iterative process up to final results

$$\mathbf{y}_0, \tilde{\mathbf{C}}_{yy} \Rightarrow \hat{\mathbf{x}}_I, \hat{\mathbf{C}}_{yy_I} \Rightarrow \mathbf{y}_0 \hat{\mathbf{C}}_{yy_I} \Rightarrow \hat{\mathbf{x}}_{II}, \hat{\mathbf{C}}_{yy_{II}} \Rightarrow \ldots \Rightarrow \hat{\mathbf{x}}_F, \hat{\mathbf{C}}_{yy_F}$$

Hypotheses on network covariances

1. daily network covariance constant in time
2. no correlations between consecutive days

$$\mathbf{C}(t_k) = \mathbf{C} \quad \forall k = 1,\ldots,T, \quad \underset{3P\times 3P}{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \cdots & \mathbf{C}_{1P} \\ \mathbf{C}_{21} & \mathbf{C}_{22} & \cdots & \mathbf{C}_{2P} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{C}_{P1} & \mathbf{C}_{P2} & \cdots & \mathbf{C}_{PP} \end{bmatrix},$$

$$\underset{3\times 3}{\mathbf{C}_{ij}} = \begin{bmatrix} c_{x_{1P_i}x_{1P_j}} & c_{x_{1P_i}x_{2P_j}} & c_{x_{1P_i}x_{3P_j}} \\ c_{x_{2P_i}x_{1P_j}} & c_{x_{2P_i}x_{2P_j}} & c_{x_{2P_i}x_{3P_j}} \\ c_{x_{3P_i}x_{1P_j}} & c_{x_{3P_i}x_{2P_j}} & c_{x_{3P_i}x_{3P_j}} \end{bmatrix} = \left\{ c_{ij_{lm}} \right\}_{l,m=1,2,3}$$

Estimation of the model parameters

Constant or linear model

$$\mathbf{x}_{P_i}(t) = \begin{cases} \mathbf{x}_{P_i}(\overline{t}) \\ \mathbf{x}_{P_i}(\overline{t}) + \dot{\mathbf{x}}_{P_i} \cdot (t - \overline{t}) \end{cases}$$

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1. For each point i=1,…,P, each component l=1,2,3
an independent regression is estimated by Least Squares

$$\mathbf{y}_0 = \begin{bmatrix} x_{l_i 0}(t_1) \\ x_{l_i 0}(t_2) \\ \dots \\ x_{l_i 0}(t_T) \end{bmatrix}, \tilde{\mathbf{C}}_{yy} = \sigma_0^2 \mathbf{I} \Rightarrow \boxed{\mathbf{LS}} \Rightarrow \hat{\mathbf{x}}_I = \begin{bmatrix} \hat{x}_{l_i}(\bar{t}) \\ \hat{\dot{x}}_{l_i} \end{bmatrix}_I$$

Empirical estimation of the covariances

Estimated vector of the residuals

$$\hat{\mathbf{r}}_{l_i\,I} = \begin{bmatrix} x_{l_i\,0}(t_1) - [\hat{x}_{l_i\,I}(\bar{t}) + \hat{\dot{x}}_{l_i\,I} \cdot (t_1 - \bar{t})] \\ x_{l_i\,0}(t_2) - [\hat{x}_{l_i\,I}(\bar{t}) + \hat{\dot{x}}_{l_i\,I} \cdot (t_2 - \bar{t})] \\ \dots \\ x_{l_i\,0}(t_T) - [\hat{x}_{l_i\,I}(\bar{t}) + \hat{\dot{x}}_{l_i\,I} \cdot (t_T - \bar{t})] \end{bmatrix}$$

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Estimated covariances and correlations

$$\hat{c}_{ij_{lm}\,I} = \frac{1}{T-N}\,\hat{\mathbf{r}}^T_{l_i\,I}\,\hat{\mathbf{r}}_{m_j\,I}$$

(N=1/2 for the constant/ linear model)

Final results

With the above hypotheses, no need of iterations

Final parameters

$$\hat{\mathbf{x}}_F = \left\{ \begin{array}{l} \left\{ \hat{x}_{l_i}(\bar{t}), \hat{\dot{x}}_{l_i} \right\}_{l=1,2,3;\ i=1,\dots,P} \\ \left\{ \hat{x}_{l_i}(\bar{t}) \right\}_{l=1,2,3;\ i=1,\dots,P} \end{array} \right\} = \hat{\mathbf{x}}_I$$

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Related covariances

$$\hat{\sigma}^2_{x_{l_i} x_{m_j}} = \frac{1}{T} \hat{c}^2_{x_{l_i} x_{m_j}}, \quad \hat{\sigma}_{x_{l_i} \dot{x}_{m_j}} = 0$$

$$\hat{\sigma}^2_{\dot{x}_{l_i} \dot{x}_{m_j}} = \frac{1}{m_t^2 T} \hat{c}^2_{x_{l_i} x_{m_j}}$$

$$m_t^2 = \frac{1}{N} \sum_i (t_i - \bar{t})^2$$

Propagation of coordinates and covariances

Displacement at earthquake epoch

$$\hat{\mathbf{x}}_{iB}(t_E) = \hat{\mathbf{x}}_i(\overline{t_B}) + \hat{\dot{\mathbf{x}}}_i(t_E - \overline{t_B})$$

$$\hat{\mathbf{x}}_{iA}(t_E) = \hat{\mathbf{x}}_{iA}(\overline{t_A})$$

$$\delta\hat{\mathbf{x}}_i(t_E) = \hat{\mathbf{x}}_{iA}(t_E) - \hat{\mathbf{x}}_{iB}(t_E)$$

Propagation of coordinates and covariances

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$$\delta\hat{\mathbf{x}}_i(t_E) = \hat{\mathbf{x}}_{iA}(t_E) - \hat{\mathbf{x}}_{iB}(t_E)$$

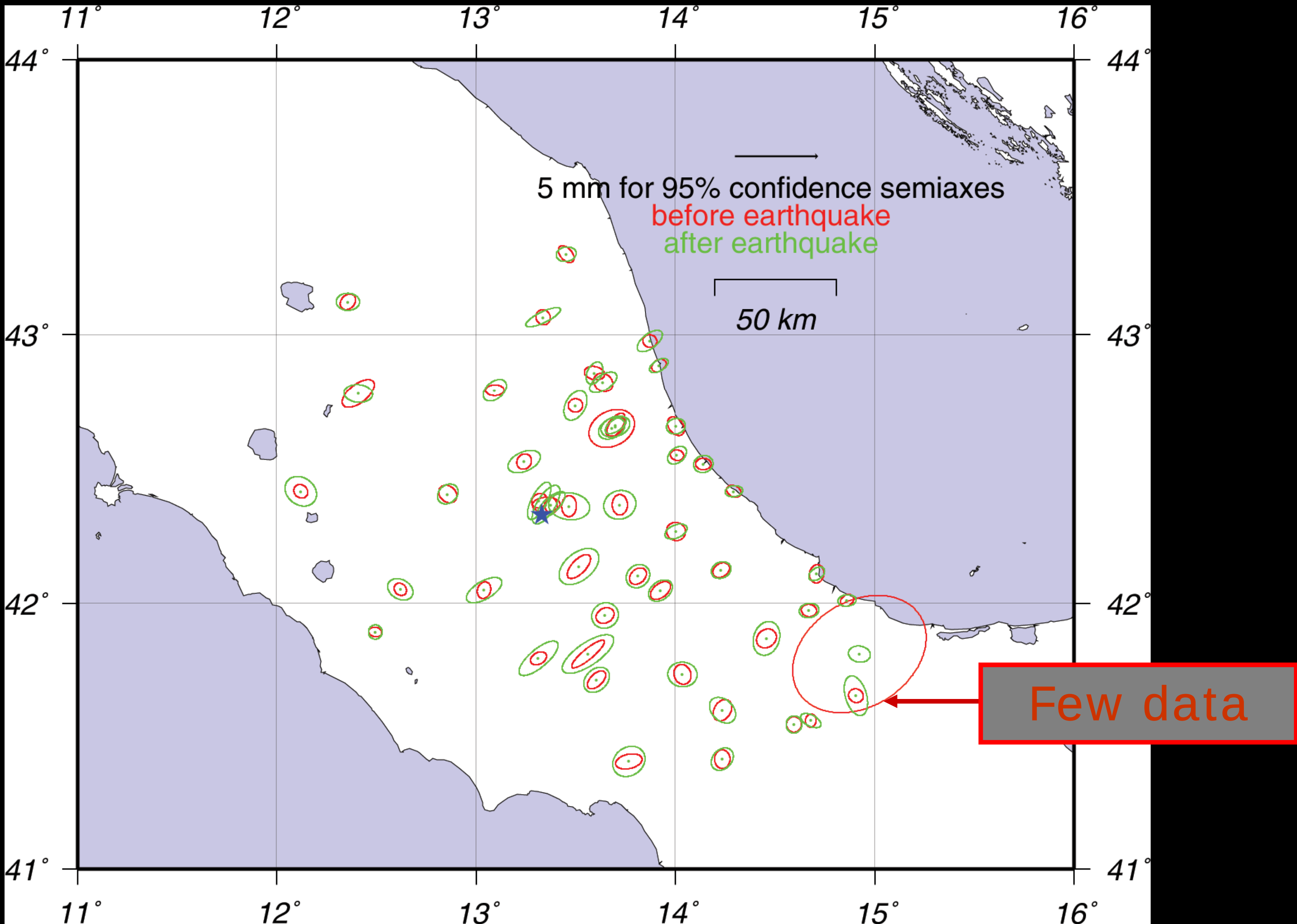
Covariance of the displacement

$$\mathbf{C}_{iB}(t_E) = \mathbf{C}_{\overline{xx}iB} + \mathbf{C}_{\dot{x}\dot{x}i}(t_E - \overline{t}_B)^2$$

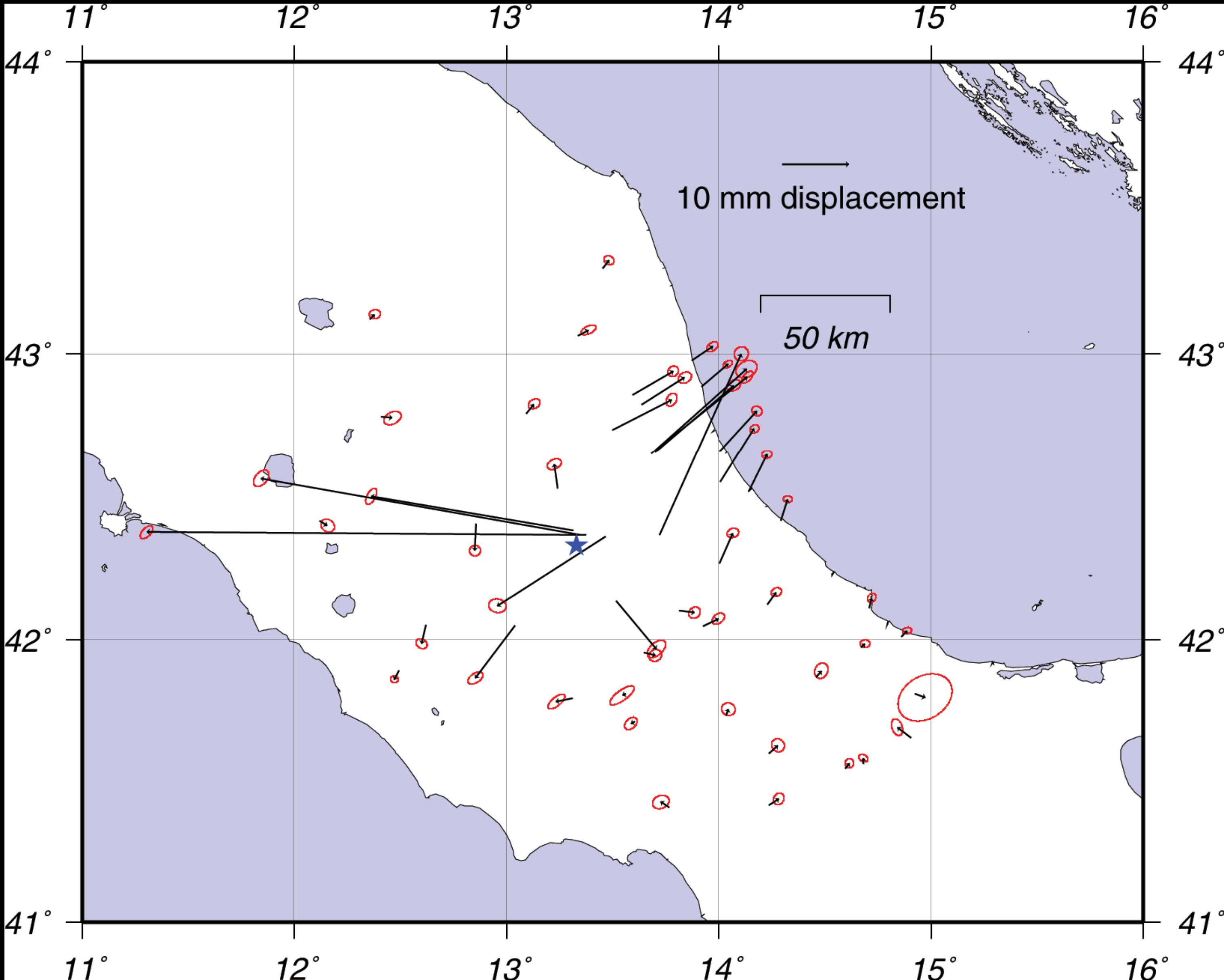
$$\mathbf{C}_{iA}(t_E) = \mathbf{C}_{\overline{xx}iA}$$

$$\mathbf{C}_{i\delta\delta}(t_E) = \mathbf{C}_{iA}(t_E) + \mathbf{C}_{iB}(t_E)$$

Covariances of the two propagations



Horizontal displacements map



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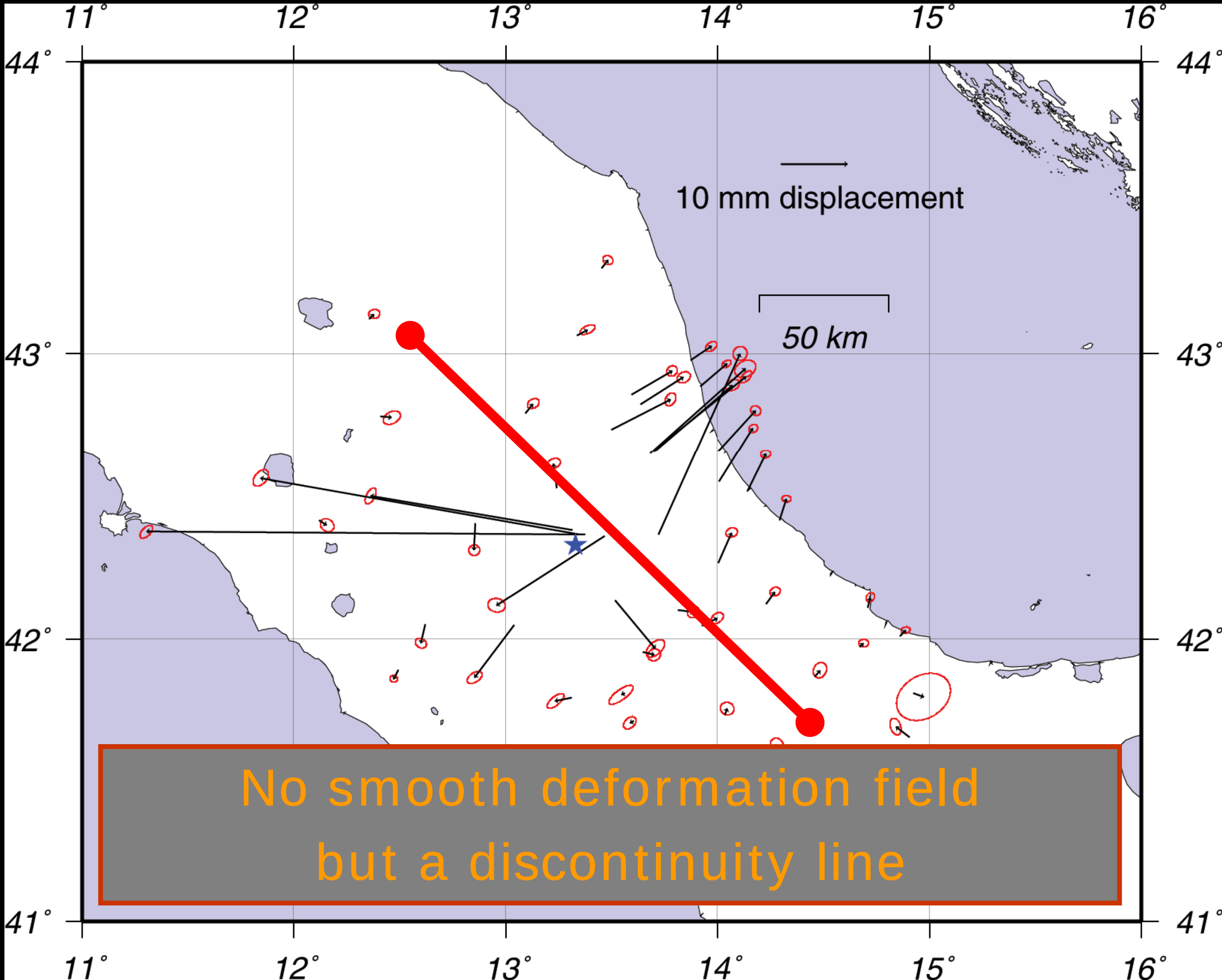
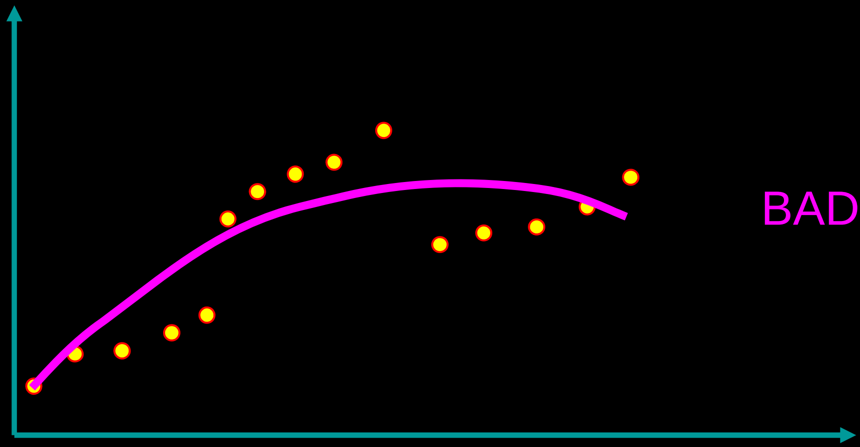


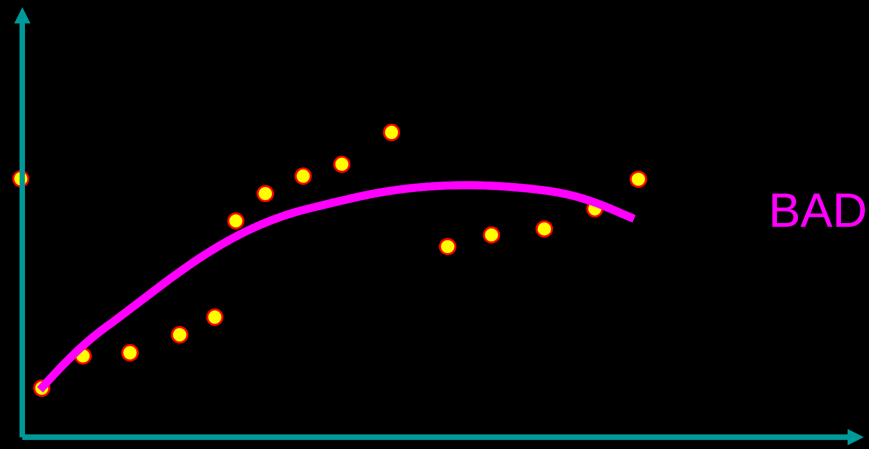
No smooth deformation field
but a discontinuity line

Separation of rigid motion from deformation



BAD SPATIAL INTERPOLATION

Separation of rigid motion from deformation



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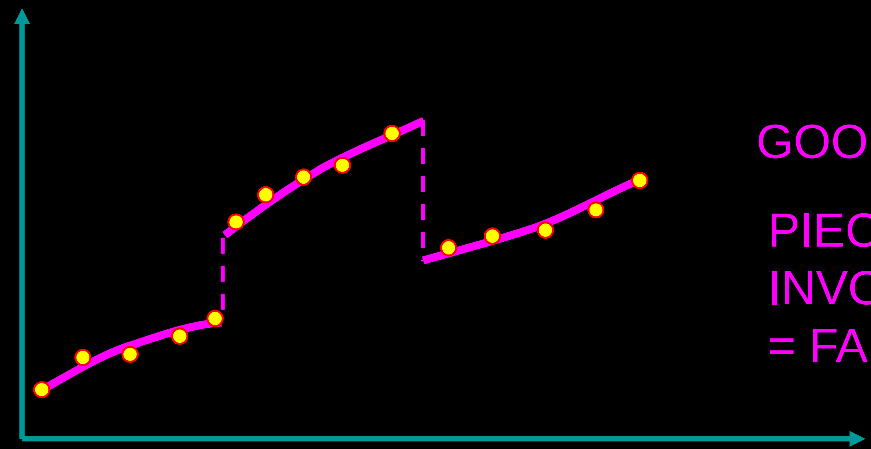
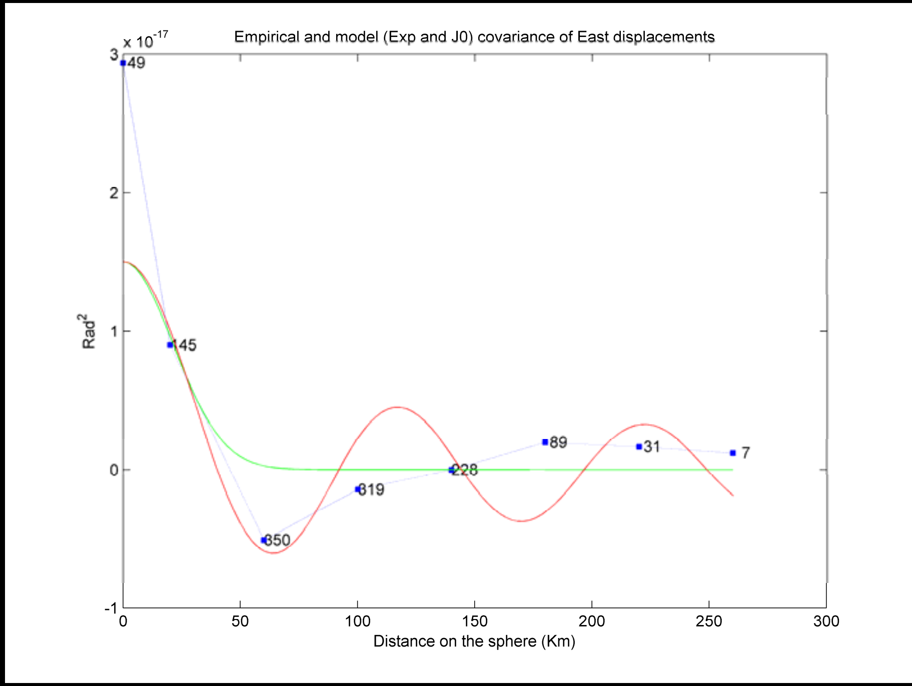
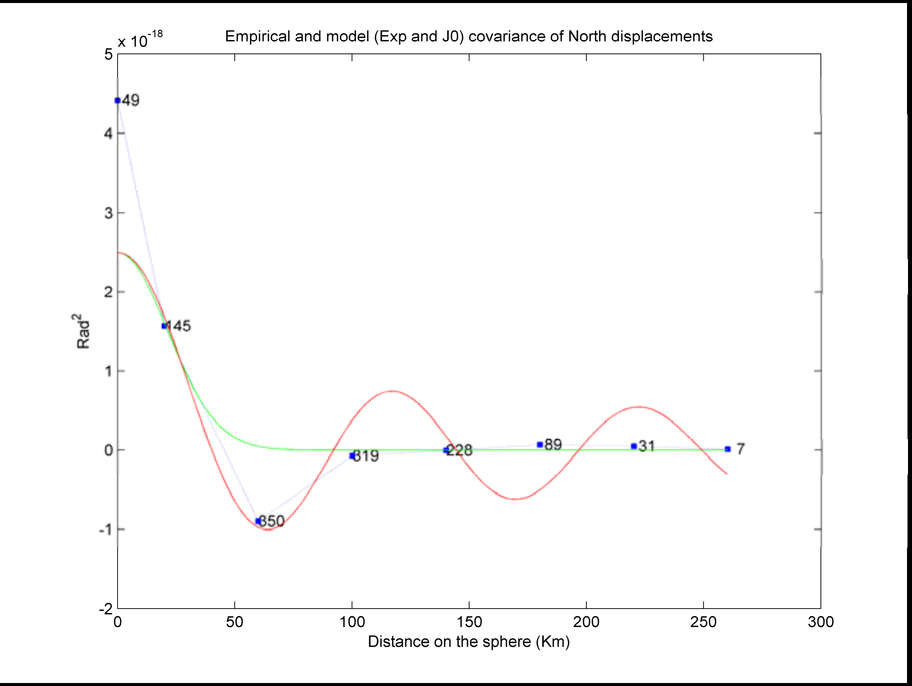


GOOD SPATIAL INTERPOLATION

PIECEWISE INTERPOLATION
INVOLVES DISCONTINUITIES
= FAULTS !

Spatial covariances and interpolation





A signal could be isolated, but quite arbitrarily

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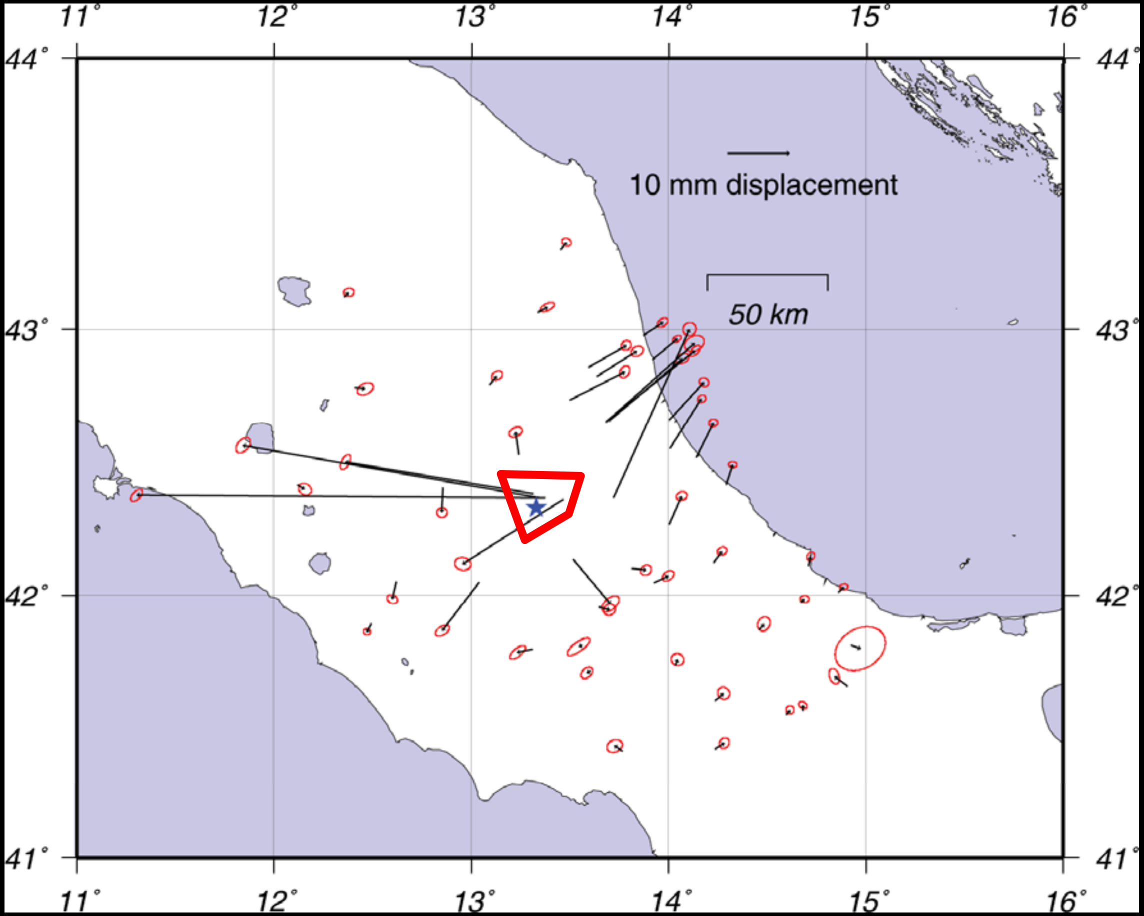
a preliminary clustering of homogeneous areas needed

Spatial clustering

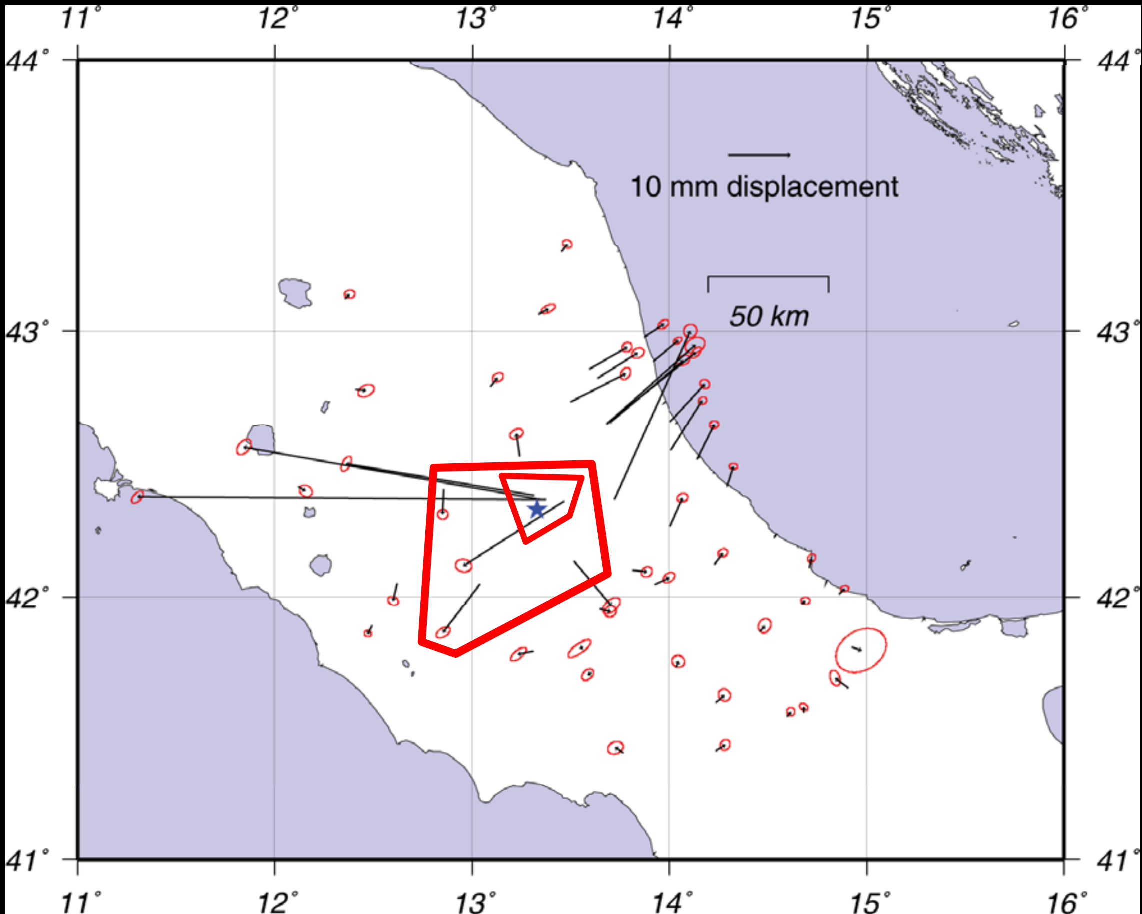
1. L'Aquila sites:
20-70 mm W displacements



Spatial clustering

1. L'Aquila sites: 20-70 mm W displacements

2. Around them: smaller S-W displacements



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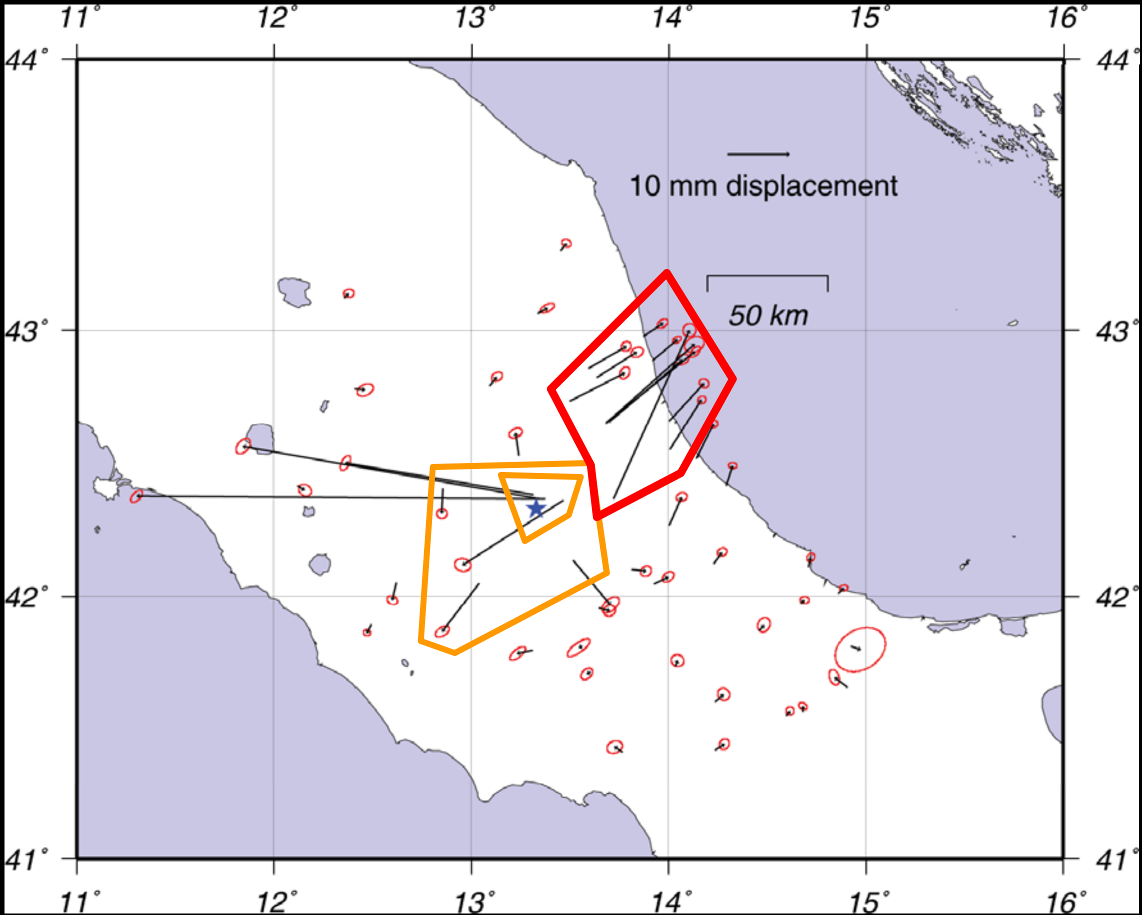

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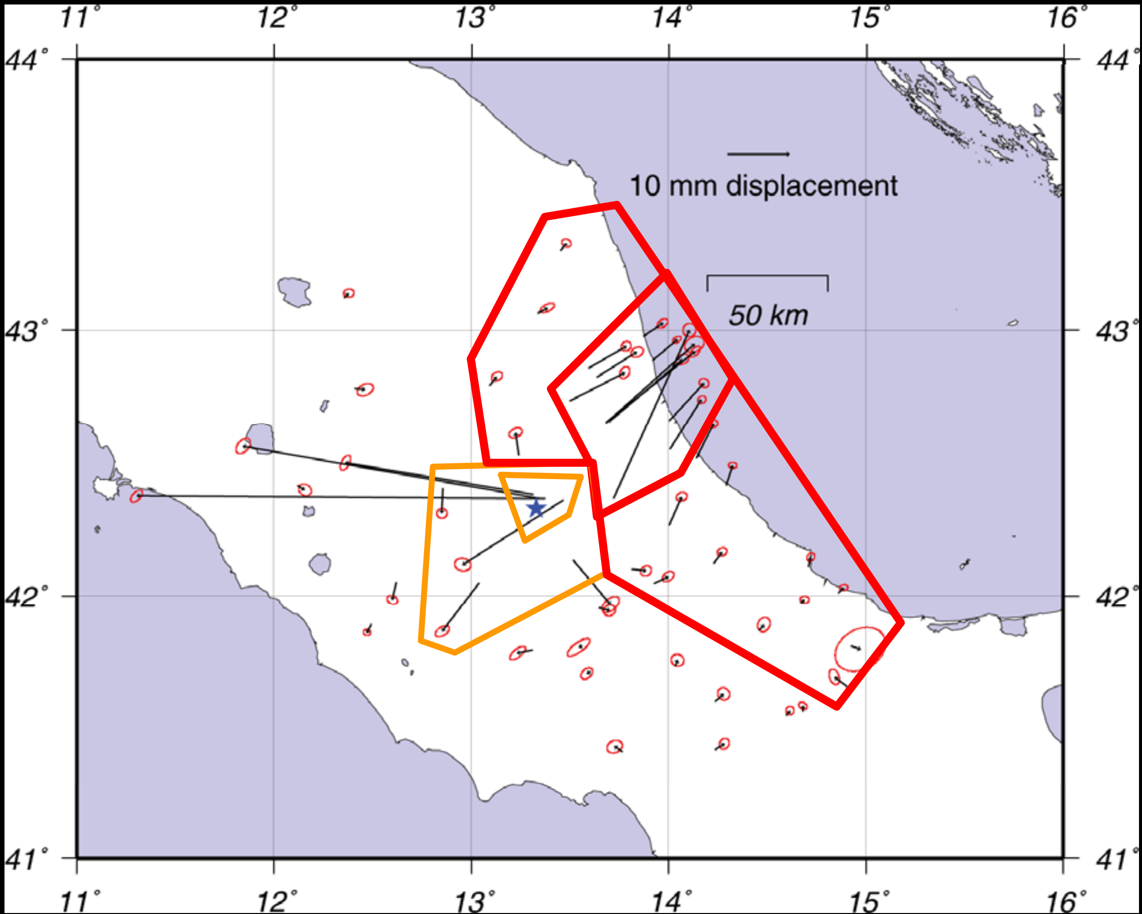
3. East region: 2-30 mm NE displacements

Spatial clustering



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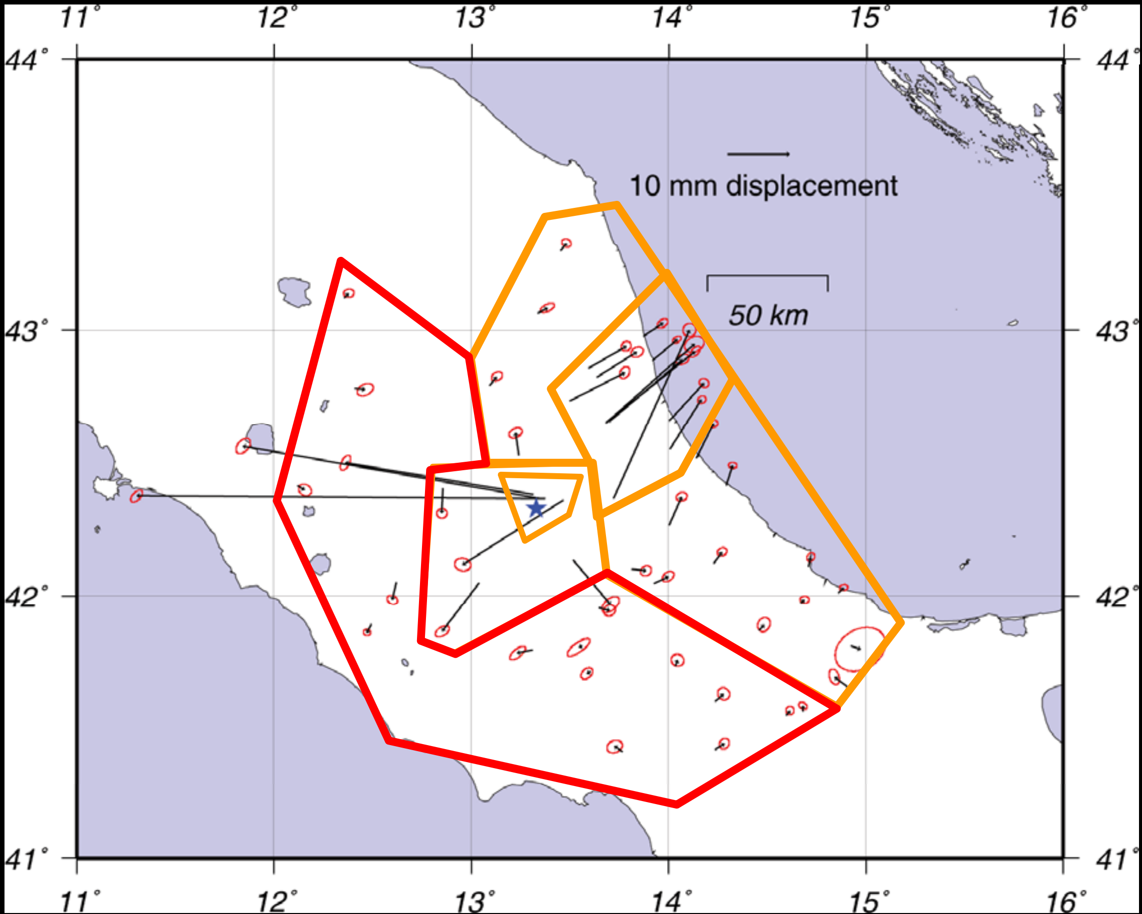
4. Around it: no significant displacements, but consistent directions.

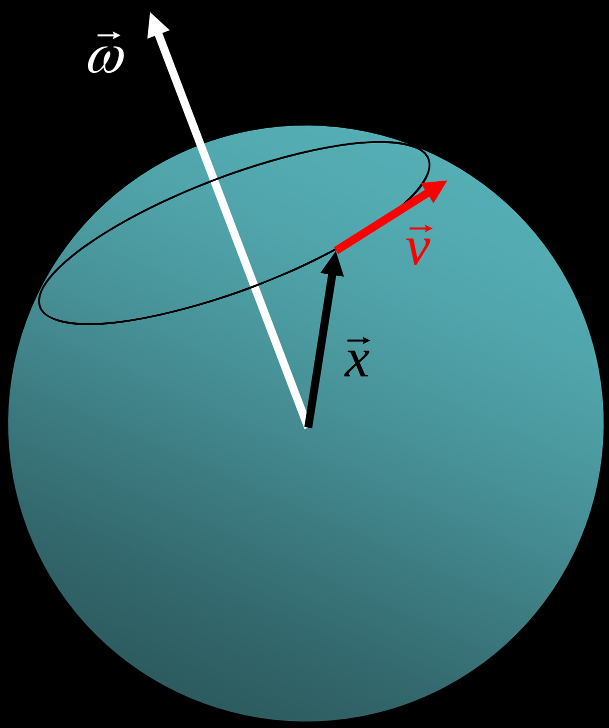
Spatial clustering



1. L'Aquila sites : 20-70 mm W displacements
2. Around them: smaller S-W displacements
3. East region: 2-30 mm NE displacements
4. Around it: no significant displacements, but consistent directions.
5. No other significant displacements

Separation of rigid motion from deformation

Horizontal motion of a network on earth surface:
rotation of all the points
around an axis with angular velocity ω



$$\mathbf{v}_i = [\boldsymbol{\omega}\times]\mathbf{x}_i$$

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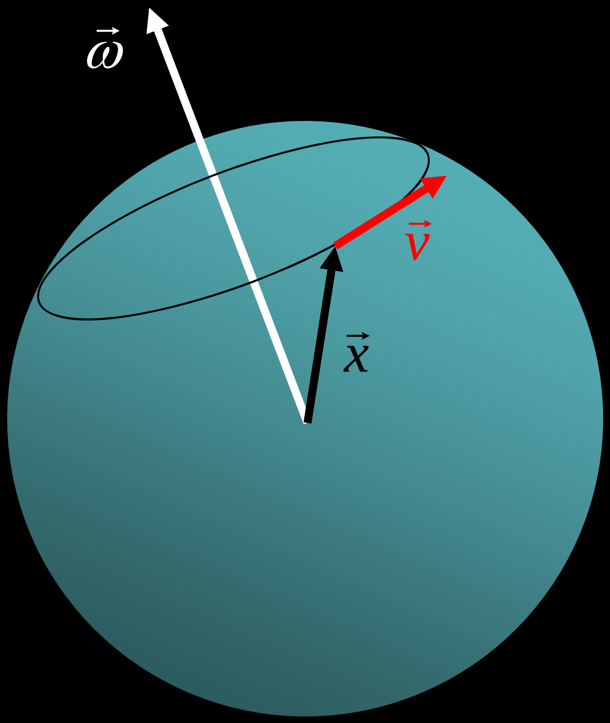


$$\mathbf{v}_i = [\boldsymbol{\omega}\times]\mathbf{x}_i$$

ω can be estimated
by minimization of relative kinetic
energy of the network

$$T_{\mathrm{ap}} = \sum_{i=1,\dots,P} \mathbf{v}_i^{\mathrm{T}}\mathbf{v}_i = \min$$

Realization of a
Discrete Tisserand reference system

Horizontal analysis in separate regions

Probably no significant rotation of networks
but differential displacements

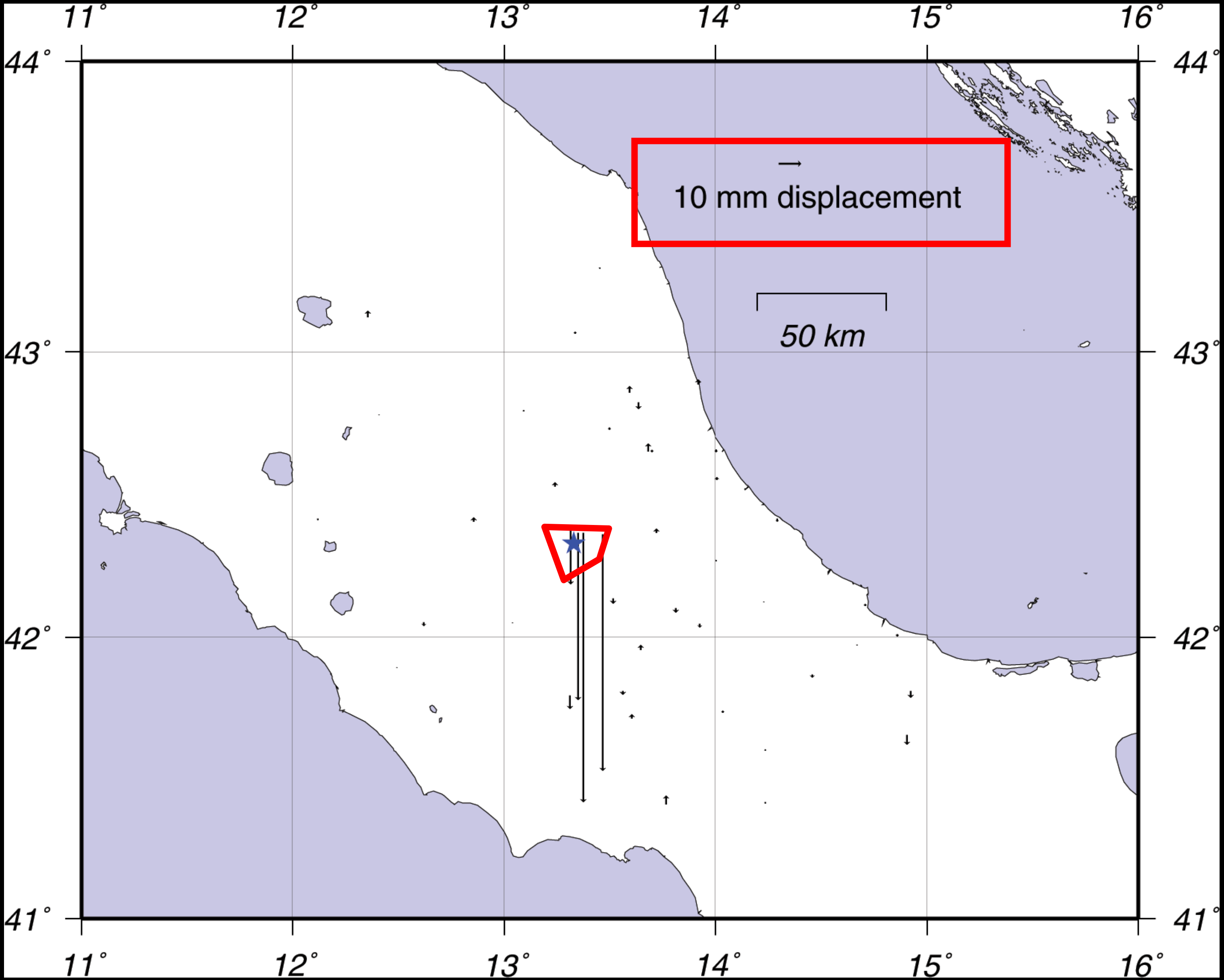
...

up to now no Tisserand analysis but
statistics on displacements for the two main regions

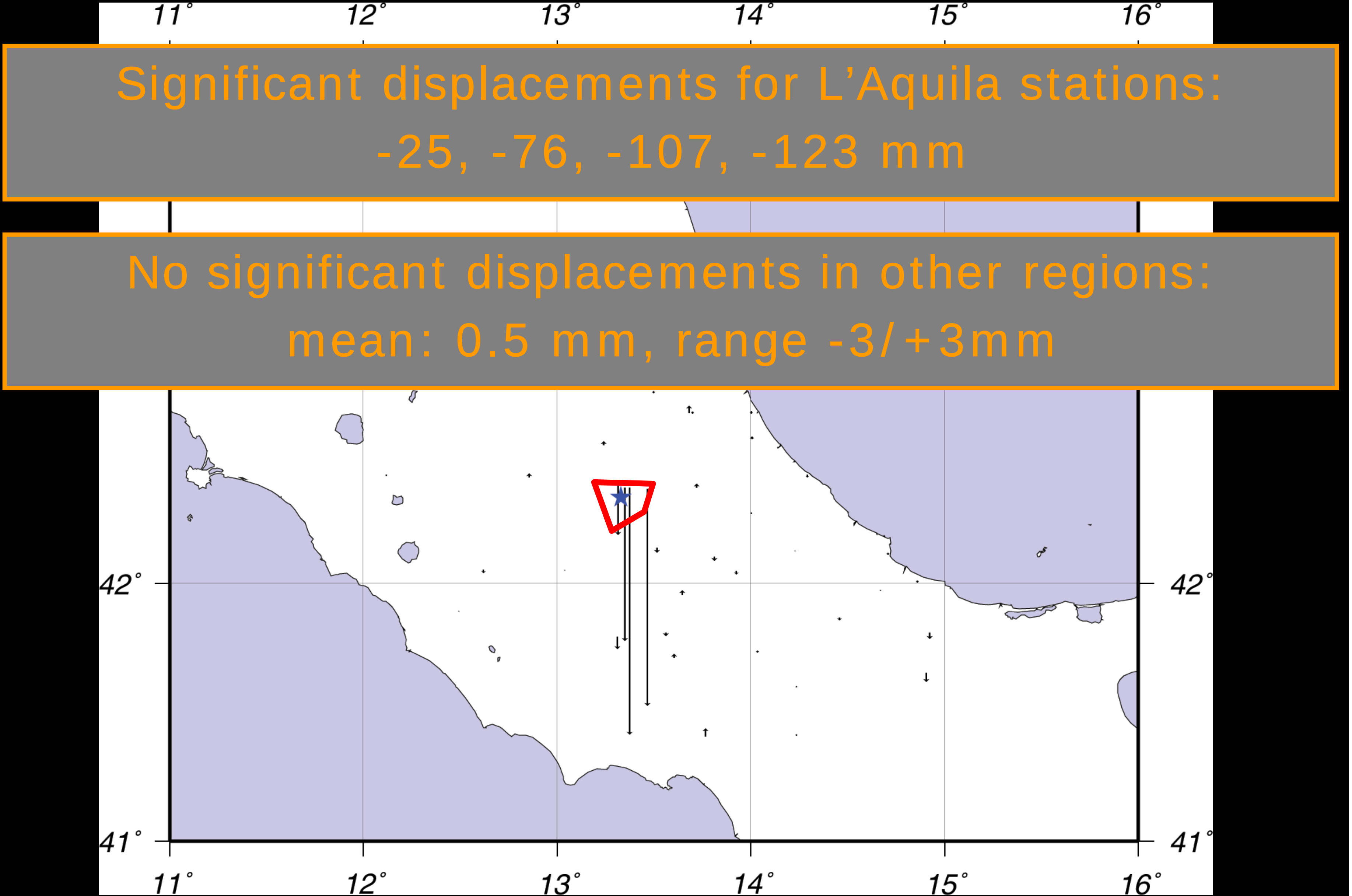
East (14 stations)			
(mm)	E	N	2D
Mean	7.0	7.6	10.7
σ	4.5	6.6	7.5
Min	1.0	2.1	3.4
Max	14.4	27.5	30.1

L'Aquila (4 stations)			
(mm)	E	N	2D
Mean	-40	1	41
σ	22	8	21
Min	-66	-11	19
Max	-16	8.5	66

Vertical displacements



Vertical displacements



Conclusions

6th April earthquake in L'Aquila
has been accompanied by
an extension along an axis oriented NW-SE:

L'Aquila area and an Eastern Adriatic area interested
by significant opposite horizontal displacements

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L’Aquila area and an Eastern Adriatic area interested
by significant opposite horizontal displacements

Significant gradients in the horizontal displacements
of the Eastern Adriatic area

L’Aquila sites interested by
vertical displacements of about 10 cm

Future analyses

Longer time series, to:

increase the populations after the earthquake,

analyze the post seismic time series

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More rigorous clustering in separate regions,
rigorous Tisserand analysis,
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Cross comparison in L’Aquila

with SAR interferograms