# *A new mapping function based on GPS Radio Occultation for the ZTD estimation*

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# Content

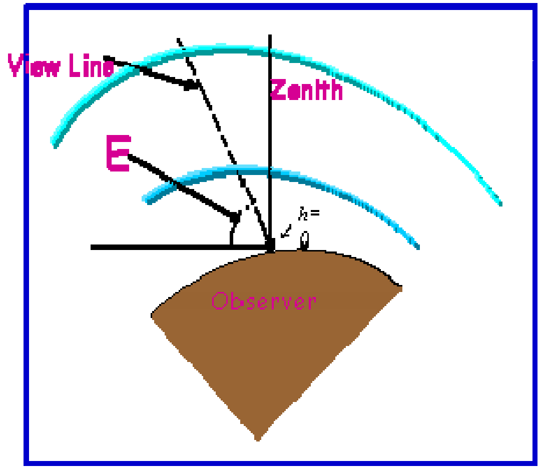
- *Historic background*
- *Niell mapping function: How it was computed*
- *MF with Radio Occultation observation*
- *Global M.F. with RO*
- *Results*

The Phase Observed by GPS: $\Phi = \rho + c(dt - dT) + \lambda A - \Delta_{ion} + \Delta_{trop} + \varepsilon$

The path of GPS signal from the satellite to the ground receiver is ruled by Fermat Principle:

$$L = \int n \, ds, \quad \Delta_{trop} = \int (n-1)\, ds = 10^{-6} \int N^{trop} \, ds$$

$$N^{trop} = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$ Refractivity as from Smith & Weintraub (1953)




| |
|---|
| $P_d$=surface pressure (mbar) |
| e=wet pressure |
| T=temperature |

$$\Delta_{trop} = ZTD \cdot M(E)$$ With:

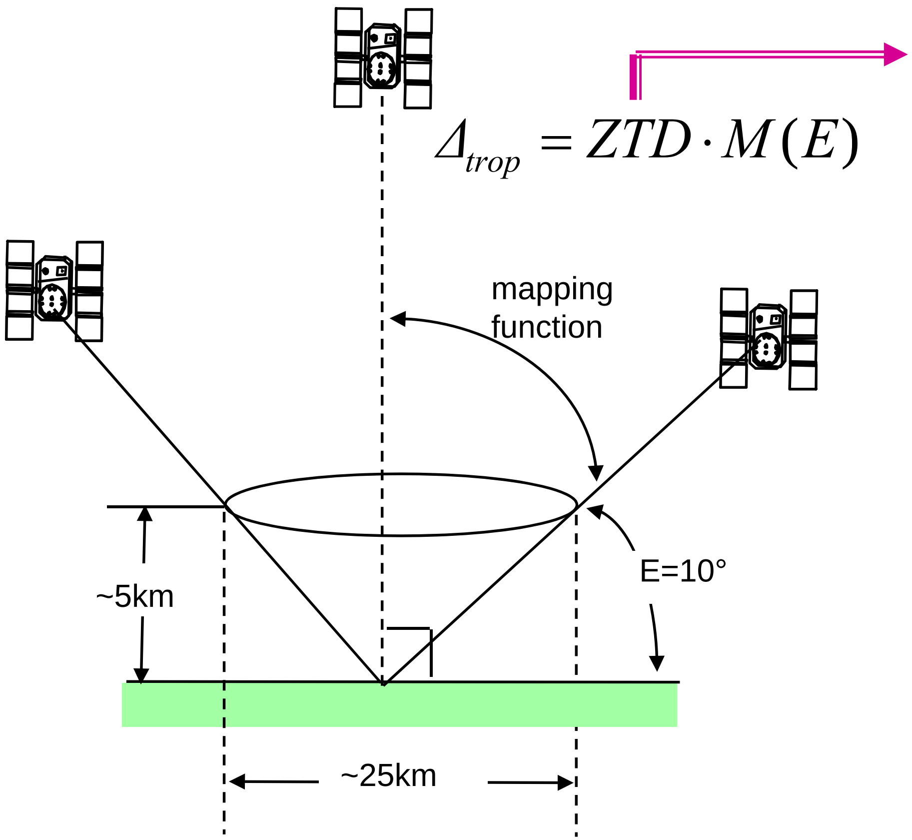
M(E)=Mapping Function

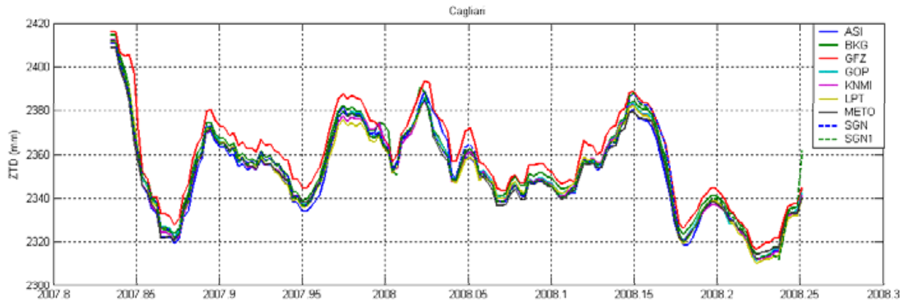
$$ZTD = 10^{-6}\left[\int k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}\, dz\right] = ZHD + ZWD$$

$$\Delta_{trop} = m_h(E) ZHD + m_w(E) ZWD + m_\Delta(E) \cot E \left[G_N \cos\phi + G_E \sin\phi\right]$$

## Ground-Based GPS Meteorology

$$\Delta_{trop} = ZTD \cdot M(E)$$




**Fundamental Measurement**

$$L_S = 10^{-6} \int N(s)\,ds$$

$$N = k_1 \cdot \left(\frac{P_d}{T}\right) + k_2 \cdot \left(\frac{e}{T}\right) + k_3 \cdot \left(\frac{e}{T^2}\right)$$

A mapping function is applied to determine how the signal delay changes with elevation angle.

The results are averaged over all the satellites to give the ZTD.

# HOPFIELD Era (1969)

$$N^{trop} = N_d^{trop} + N_w^{trop}$$

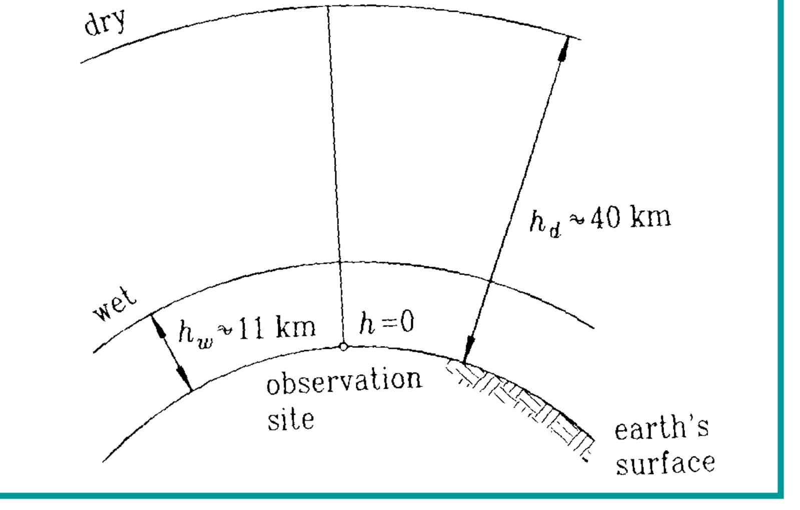
$$N_d^{trop}(h) = N_{d,0}^{trop}\left[\frac{h_d - h}{h_d}\right]^4$$

$$N_w^{trop}(h) = N_{w,0}^{trop}\left[\frac{h_w - h}{h_w}\right]^4$$

$$N_{d,0}^{trop} = 77.64\,\frac{P_0}{T_0}$$

$$N_{w,0}^{trop} = -12.96\,\frac{e}{T_0} + 3.718\cdot 10^5\,\frac{e}{T_0^{\,2}}$$

**$P_0$ e $T_0$ are ground measurement.**




$$h_d = 40136 + 148.72(T - 273.16) \quad [m]$$

$$h_w = 11000\ m$$

# Hopfield MF

$$M_d(E) = \frac{1}{Sin(\sqrt{E^2 + 6.25})}$$

$$M_w(E) = \frac{1}{Sin(\sqrt{E^2 + 2.25})}$$

# Saastamoinen Era (1973)

$$\Delta_{trop} = \frac{0.002277}{\cos(z)}\left[ p + \left( \frac{1255}{T} + 0.05 \right) e - B(h) \cdot \tan^2(z) \right] + \delta R(z,h))$$

***z= zenith angle (90-E)***
***P=Pressure***
***T=Temperature***
***e=wet pressure***
***B=tabled pressure correction***
***δR(z,h)=tabled delay corrections***

# Mapping Function
# Marini-Murray like (1972)

$$m(E) = \cfrac{1}{\sin E + \cfrac{a}{\sin E + \cfrac{b}{\sin E + \cfrac{c}{\sin E + \ldots}}}}$$

$$\Delta^{trop} = ZHD \cdot M_d(E) + ZWD \cdot M_w(E)$$

## *Mapping Function*
## *Marini-Murray like (1972)*

Herring (1992)

$$a_d = \left[1.2320 + 0.0139\cos\varphi - 0.0209h + 0.00215(T-283)\right]\cdot 10^{-3};$$
$$b_d = \left[3.1612 - 0.1600\cos\varphi - 0.0331h + 0.00206(T-283)\right]\cdot 10^{-3};$$
$$c_d = \left[71.244 - 4.293\cos\varphi - 0.149h - 0.0021(T-283)\right]\cdot 10^{-3}.$$

$$a_w = \left[0.583 - 0.011\cos\varphi - 0.052h + 0.0014(T-283)\right]\cdot 10^{-3};$$
$$b_w = \left[1.402 - 0.102\cos\varphi - 0.101h + 0.0020(T-283)\right]\cdot 10^{-3};$$
$$c_w = \left[45.85 - 1.91\cos\varphi - 0.29h + 0.015(T-283)\right]\cdot 10^{-3}.$$

Parameters involved:
latitude “eT”
Height “*h*”
Surface Temperature T

# Niell (1996)

$$m(\varepsilon)=\frac{1+\cfrac{a}{1+\cfrac{b}{1+c}}}{\sin(\varepsilon)+\cfrac{a}{\sin(\varepsilon)+\cfrac{b}{\sin(\varepsilon)+c}}}+\left(\frac{1}{\sin(\varepsilon)}-\frac{1+\cfrac{a_h}{1+\cfrac{b_h}{1+c_h}}}{\sin(\varepsilon)+\cfrac{a_h}{\sin(\varepsilon)+\cfrac{b_h}{\sin(\varepsilon)+c_h}}}\right)\cdot h$$

$$a(\lambda_i,t)=a_{avg}(\lambda_i)-a_{amp}(\lambda_i)\cos\left(2\pi\frac{t-T_0}{365.25}\right)$$

$$m_{wet}(E)=\frac{1+\cfrac{a_{wet}}{1+\cfrac{b_{wet}}{1+c_{wet}}}}{\sin(E)+\cfrac{a_{wet}}{\sin(E)+\cfrac{b_{wet}}{\sin(E)+c_{wet}}}}$$

An equivalent formulation is given for the wet MF

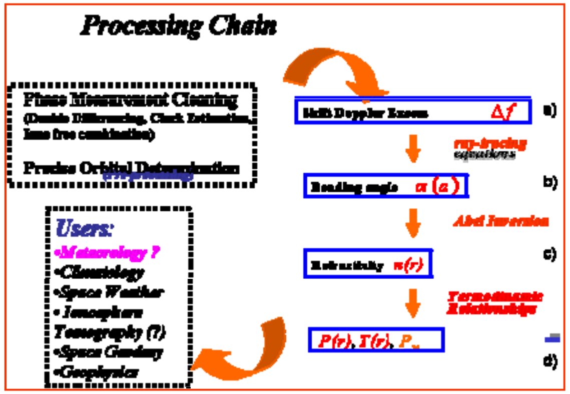
*We are using just the same formulation of Niell but*

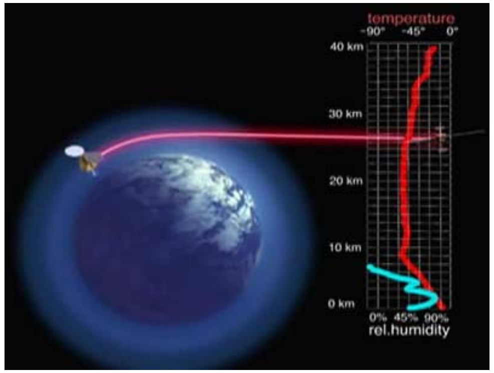
*Using Radio Occultation Data* **!!**

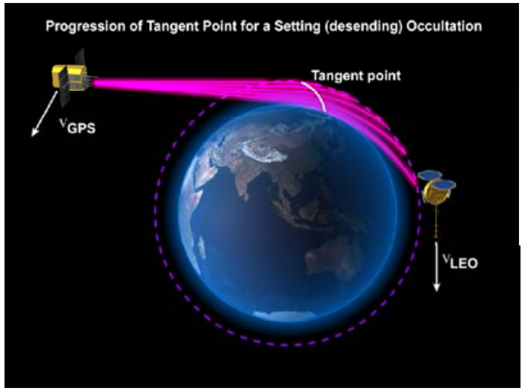
Parameters involved:
latitude “$\varphi$”
Height “*h*”
DoY “*t*”

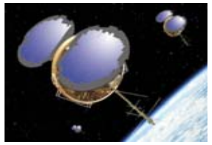
***For the computation of the coefficients RAOB data were used***

# Radio Occultation Data suitable now to build the MF

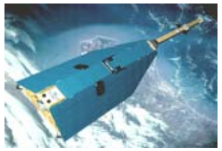


COSMIC 6 satellites in orbit since 14 April 2006 (~2000 occ/day). Still active



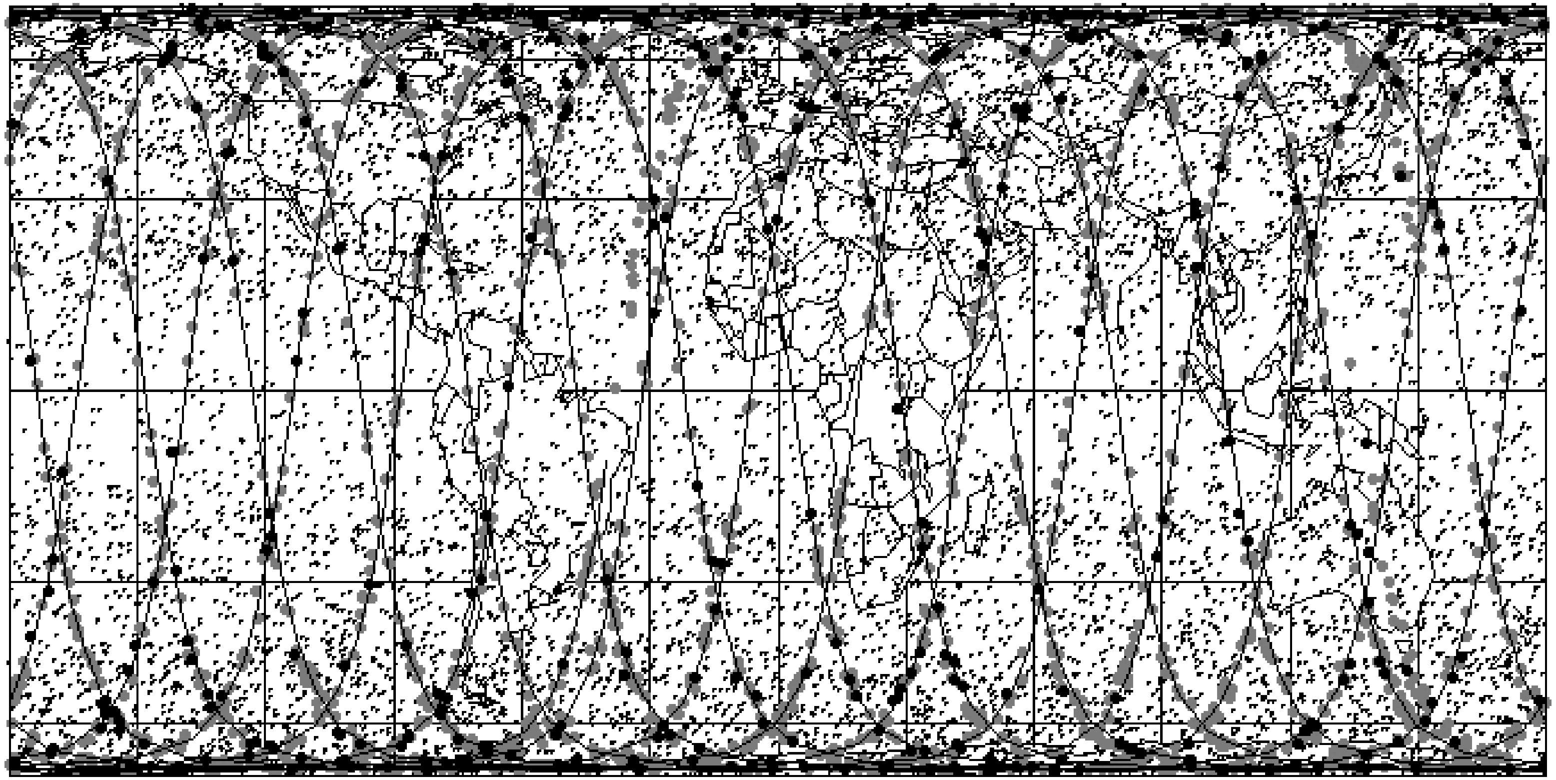
CHAMP launched on July 2000 (~200 occ/day). Still active

SAC-C launched on November 2000 (one year only of data)

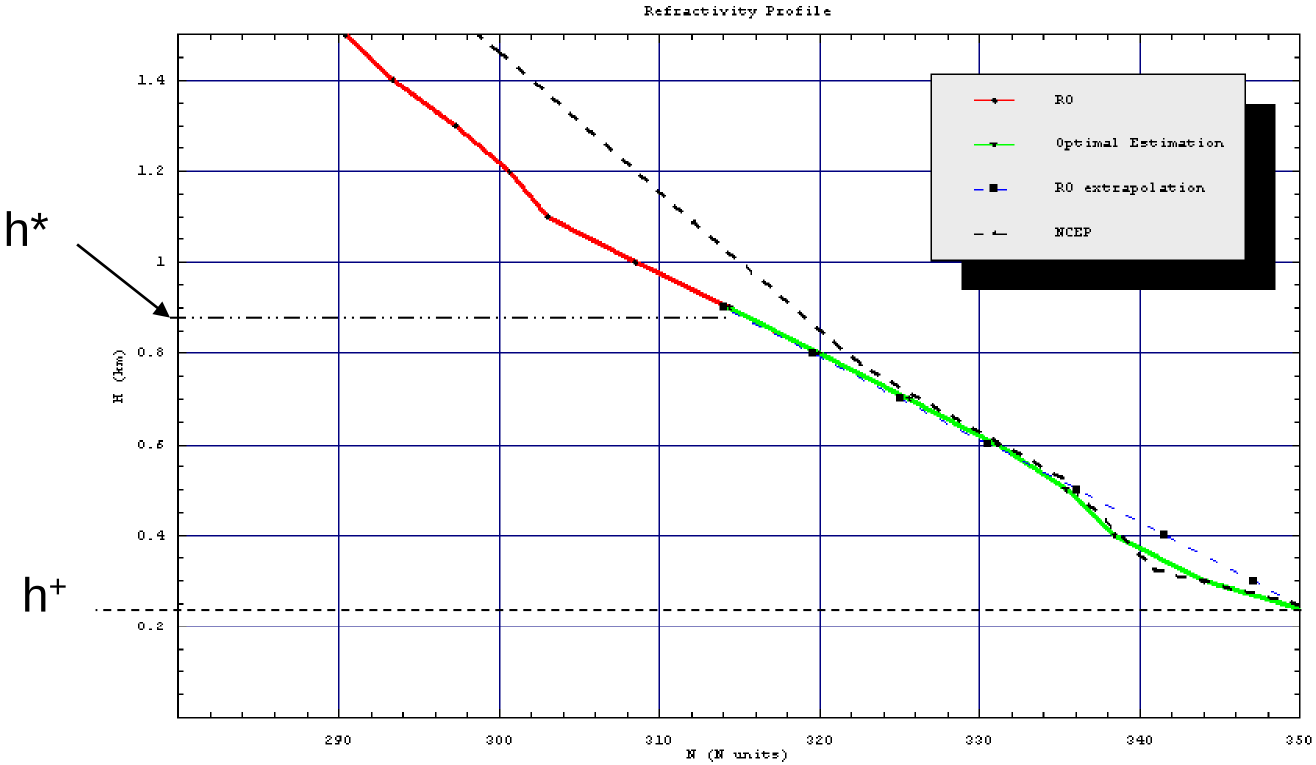
## 150.000 RO events selected !!

Spreaded and organized according the day of the year acquired. The selected events must provide profiles down at an height h < 1 km over the ground at least

COSMIC
CHAMP
SAC-C
METOP
GRACE
OCEANSAT_2



# RO vs NCEP merging



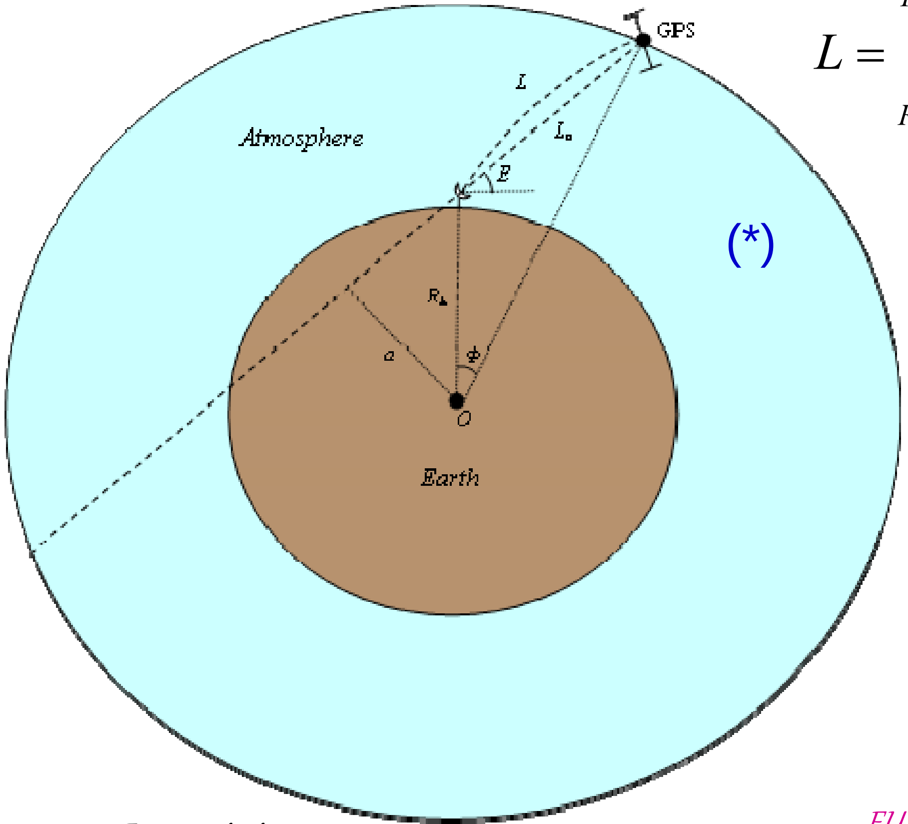

$$N(h) = k(h) \cdot N(h)_{ER} + (1 - k(h)) \cdot N(h)_{NCEP}$$

K(h*)=1 lowest layer of RO data

K(h+)=0 lowest layer of NCEP data

# Estimation of Total Delay (TD)

The tropospheric delay TD is given by the difference between the optical path of the signal (L) and the geometrical distance satellite-receiver (L0):




(*)

$$L = \int_{R_E + h_{topo}}^{R_E + h_{atm}} \frac{n^2(r) \cdot r}{\sqrt{n^2(r) \cdot r^2 - a^2}} dr \qquad a = R_E \cdot \cos(E)$$

$$L_0 = \int_{R_E + h_{topo}}^{R_E + h_{atm}} \frac{dr}{\sin(E)} = \frac{h_{atm} - h_{topo}}{\sin(E)}$$

where:

$$TD = \Delta L = L - L_0$$

- the refractive index ***n(r)*** is provided by RO data or by models (NCEP,ECMWF) ;
- the impact parameter ***a*** is related to the satellite and receiver position.

# Estimation of the MF parameters

$$m(E)=\frac{L-L_0}{ZTD}$$

$$L=\int_{R_E+h}^{R_E+h_{atm}}\frac{n^2(r)\cdot r}{\sqrt{n^2(r)\cdot r^2-a^2}}dr$$

$$L_0=\int_{R_E+h}^{R_E+h_{atm}}\frac{dr}{\sin(E)}=\frac{h_{atm}-h}{\sin(E)}$$

$$ZTD=10^{-6}\int_{R_E+h}^{R_E+h_{atm}}N(r)dr$$

$$Min\left\|M(E,\lambda_i,h_j,t_k,\mathbf{P}(\lambda_i,t_k))-\frac{L(E,\lambda_i,h_j,t_k)-L_0(E,\lambda_i,h_j,t_k)}{ZTD(E,\lambda_i,h_j,t_k)}\right\|$$

*Where*

$$\mathbf{P}(\lambda_i,t_k)=(a(\lambda_i,t_k),(b(\lambda_i,t_k),c(\lambda_i,t_k),a_h(\lambda_i,t_k),b_h(\lambda_i,t_k),c_h(\lambda_i,t_k))$$

i stands for the i$^{th}$ longitude
j “ “ “ j$^{th}$ height
k “ “ “ k$^{th}$ epoch

# *Estimation of the MF Parameters (2)*

| | |
|---|---|
| Latitude Gridding<br>Longitude Gridding | 15° (12 groups)<br>30° (12 groups) |
| Number of epochs of the year | 8 (45 days) |
| Layers selected to estimate $a_h$, $b_h$ $c_h$ | Up to 2000 meters with steps of 400 meters starting from sea level.About 150 different Elevation Angle have been selected for fitting. |

***Our parameters are not exactly the same as appear in Niell MF. So we apply the following relationship to have uniform parametetrization (important for the processing)***:

$$a_{avg}(\lambda_i) = \overline{a(\lambda_i, t)}_{Time}$$

$$a(\lambda_i, t) = a_{avg}(\lambda_i) + a_{amp}(\lambda_i)\cos\left(2\pi \frac{t - T_0}{365.25}\right)$$

$$a_h = \overline{a_h(\lambda_i, h_j, t_k)}_{\lambda, H, T}$$

$$\cdot\left(\frac{1}{\sin(\varepsilon)} - \frac{1 + \frac{a_h}{1 + \frac{b_h}{1 + c_h}}}{\sin(\varepsilon) + \frac{a_h}{\sin(\varepsilon) + \frac{b_h}{\sin(\varepsilon) + c_h}}}\right) \cdot h$$

While $a_{amp}(\lambda)$ and $T_0$ are estimated fitting the residuals: $\left[a(\lambda_i, t_k) - a_{avg}(\lambda_i)\right]$ with:

*Fit the computed coefficients in terms of an harmonic expansion truncated at 8°:*

$$a(\varphi,\lambda,t)=\sum_{n=0}^{n=8}\sum_{m=0}^{n}P_{nm}\sin(\varphi)\cdot\left(C(t)_{nm}\cos(\lambda)+S(t)_{nm}\sin(\lambda)\right)$$

$$b(\varphi,\lambda,t)=.......$$

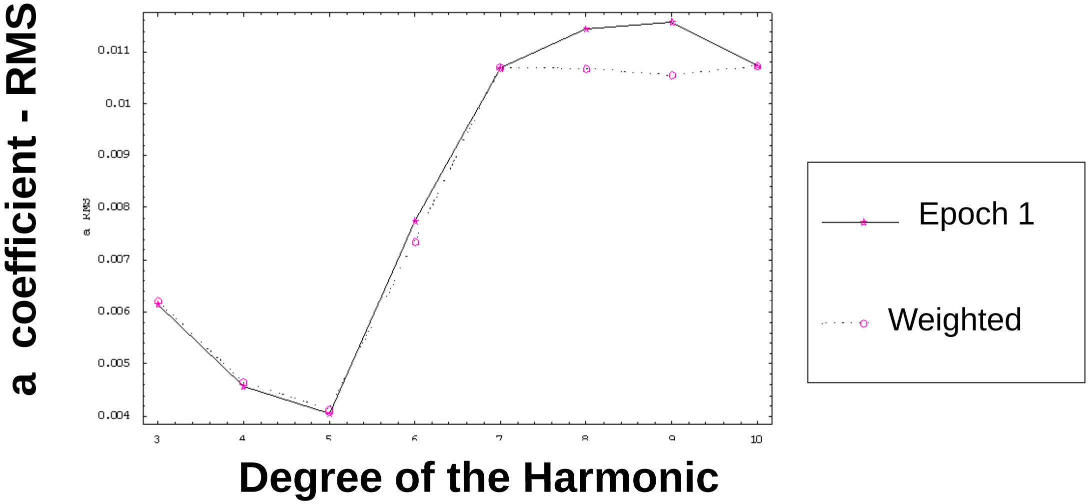
$$c(\varphi,\lambda,t)=......$$

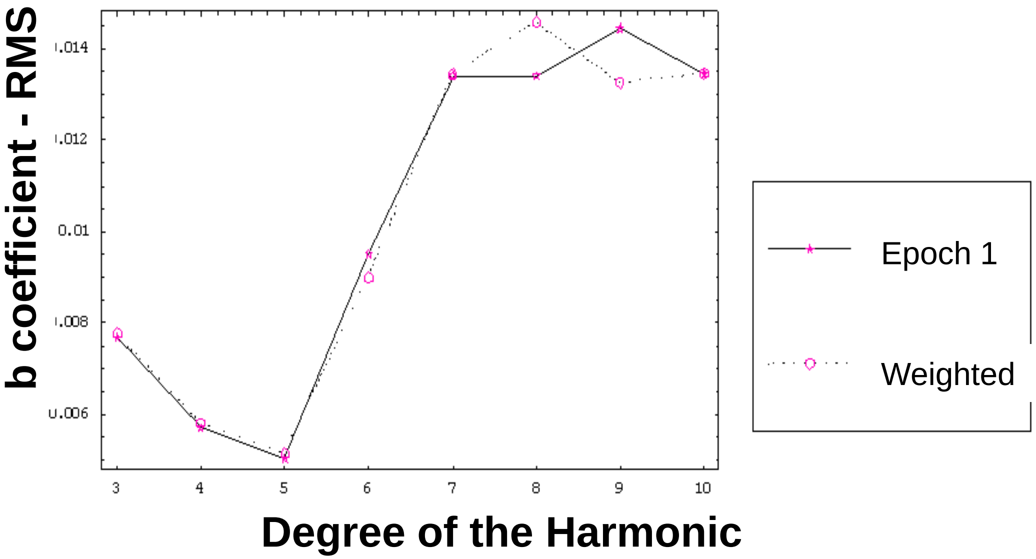
*This is the same approach applied by Boehm et al. 2006 using Numerical Weather Models for the construction of a GMF:*

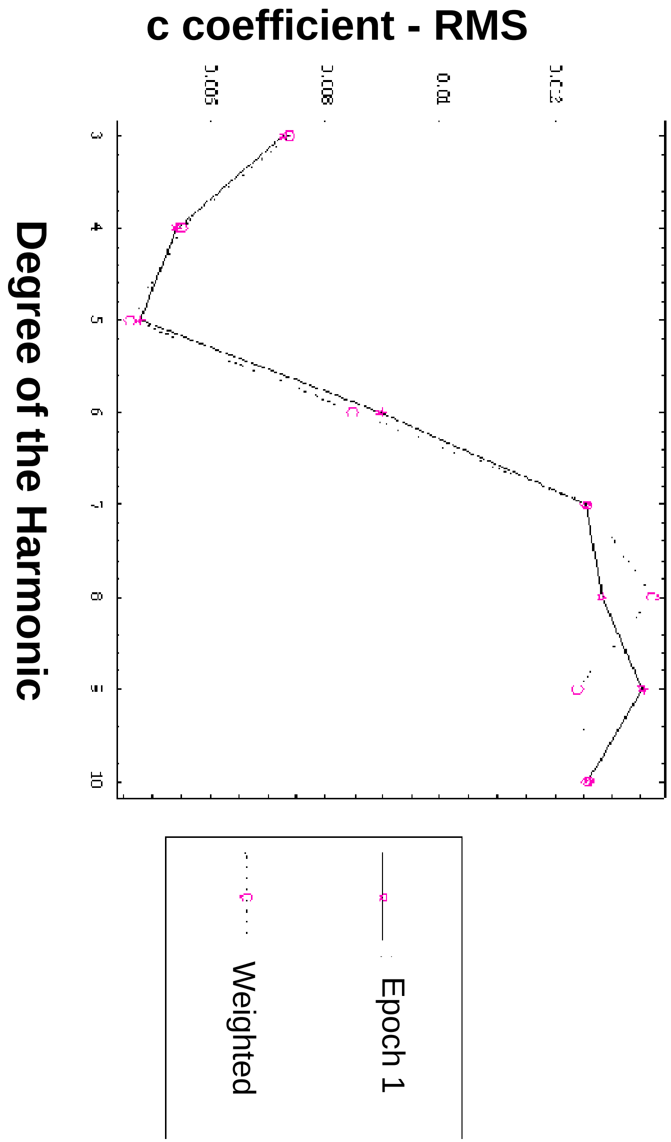
**Boehm, J.;** Niell, A.**; Tregoning, P.; Schuh, H. 2006, "Global Mapping Function (GMF): A new empirical mapping function based on numerical weather model data," Geophysical Research Letters, 33: L07304, doi:10.1029/2005GL025546.**

***Best Solution: 5th Degree***

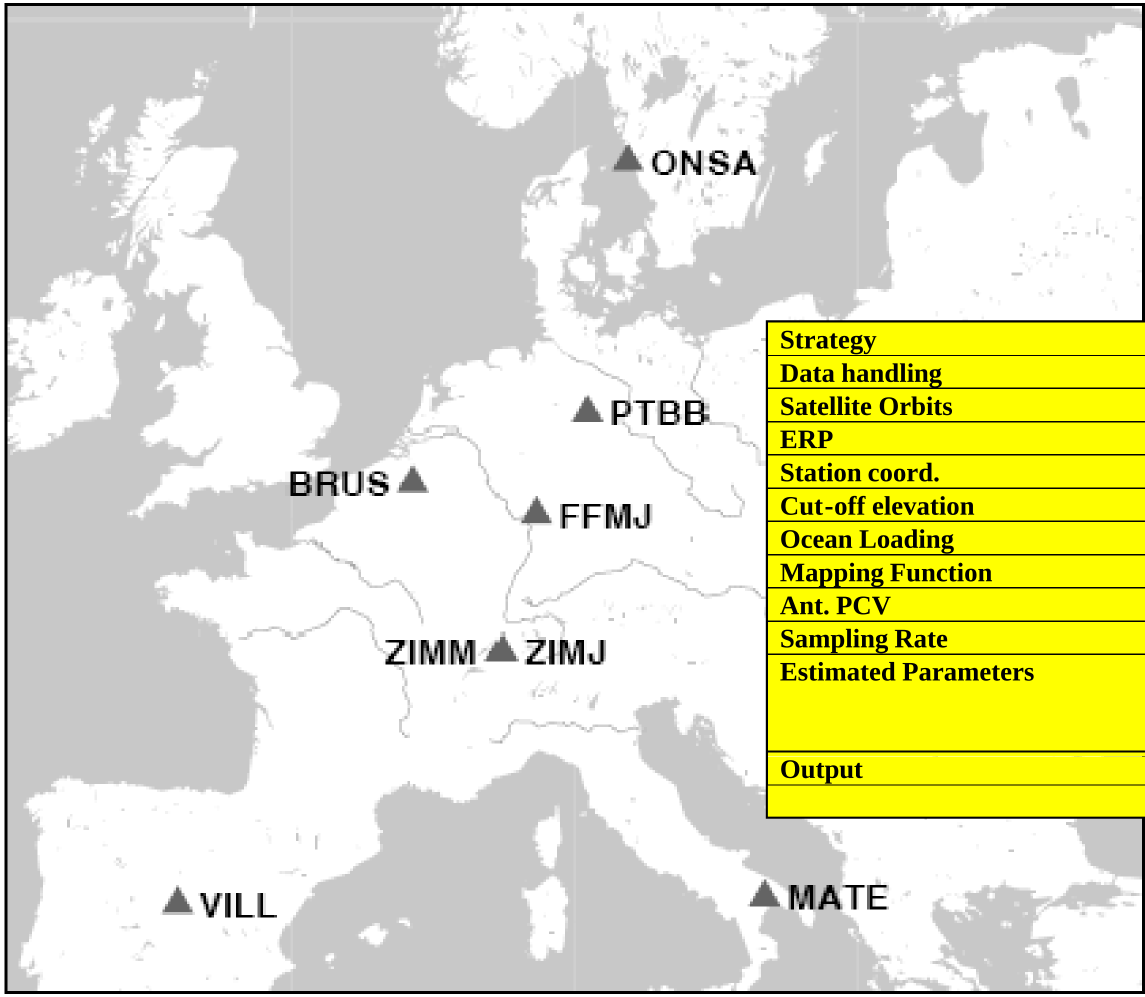
# The Optimal Degree of the HarmonicS



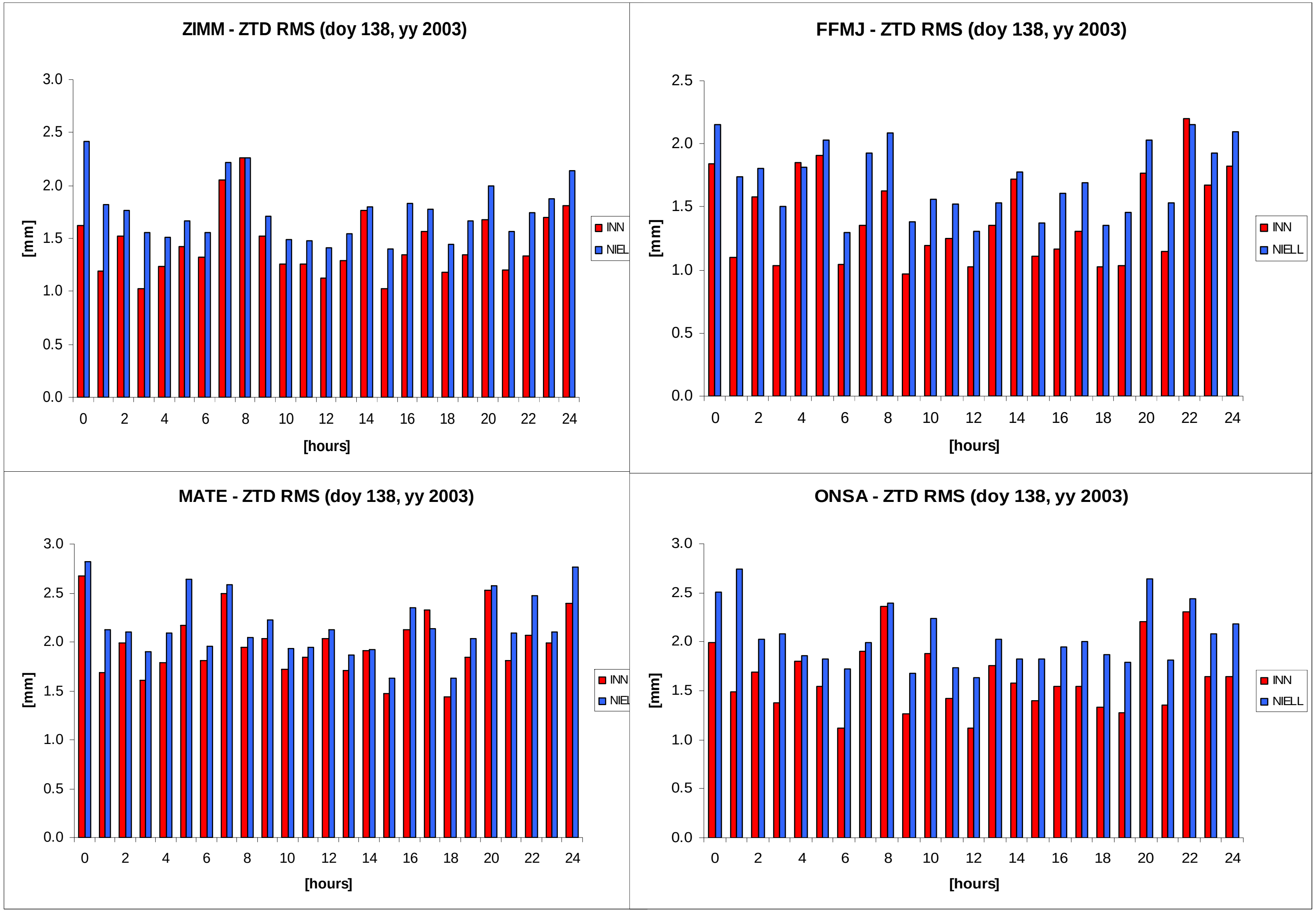



## Processing Strategy (BERNESE SW used)




| | |
|---|---|
| **Strategy** | **Network Adjustment** |
| **Data handling** | **1 week of Data of 8 Perm. GPS Stations** |
| **Satellite Orbits** | **IGS Precise** |
| **ERP** | **IERS-IGS** |
| **Station coord.** | **aligned to IGS00** |
| **Cut-off elevation** | **5°** |
| **Ocean Loading** | **Applied (H.G.Scherneck)** |
| **Mapping Function** | **Neill (1996), INN (2008)** |
| **Ant. PCV** | **Relative** |
| **Sampling Rate** | **30”** |
| **Estimated Parameters** | **Coordinates, Satel lite & station clocks w.r.t a reference one , Phase ambiguit ies (float), ZTD time res olution: every hour** |
| **Output** | **Coordinate s, ZTD** |

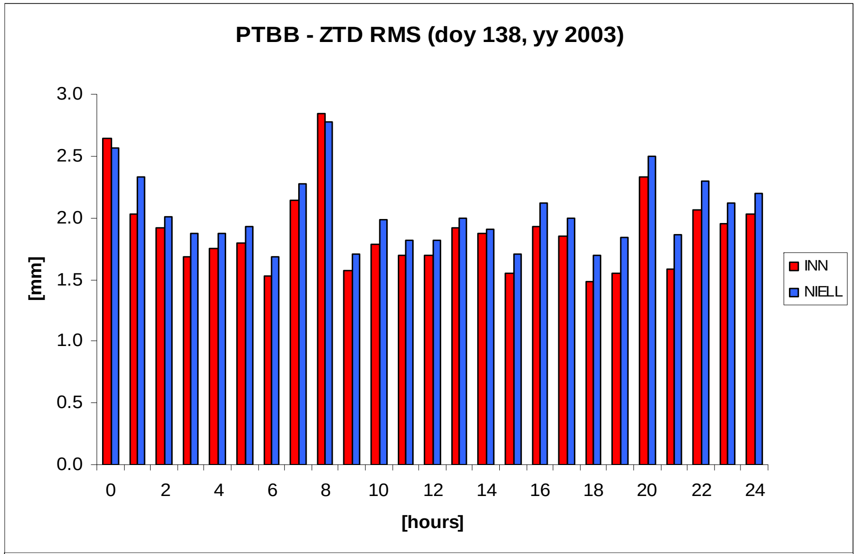
# ZTD Results

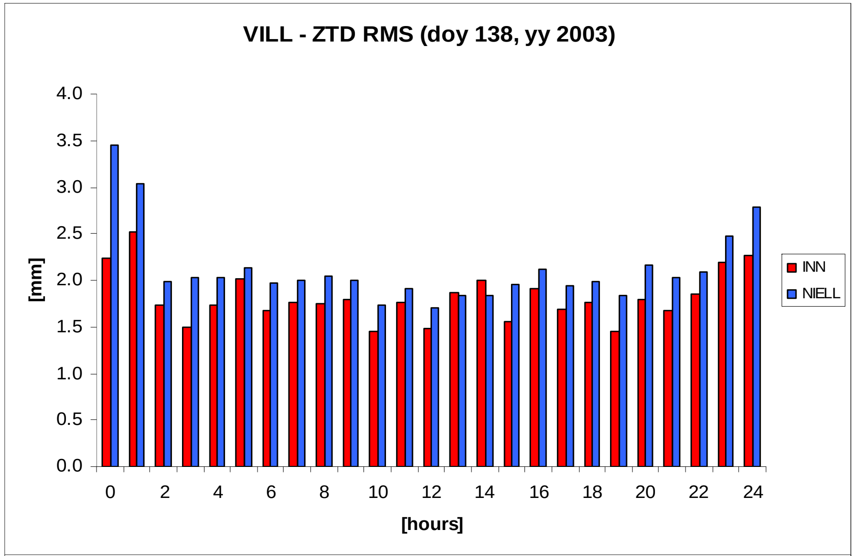


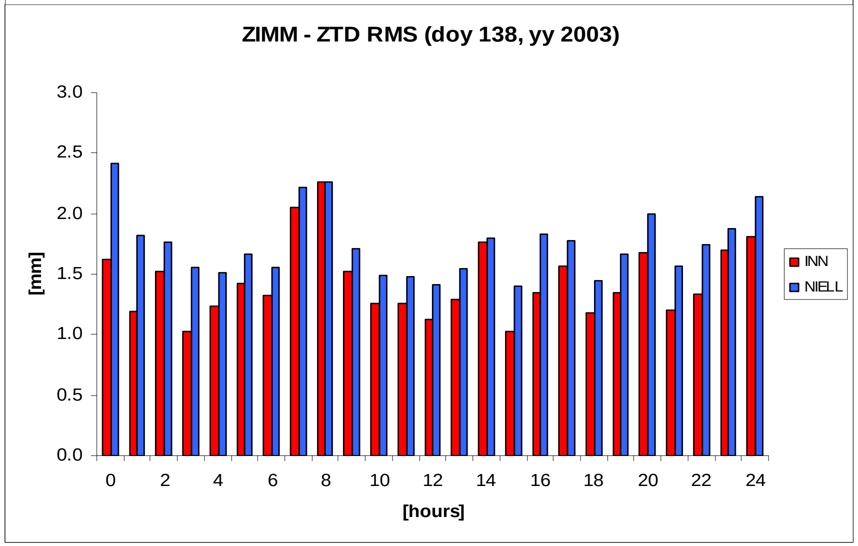
ZIMM - ZTD RMS (doy 138, yy 2003)
[mm]
[hours]
INN
NIEL
FFMJ - ZTD RMS (doy 138, yy 2003)
[mm]
[hours]
INN
NIELL
MATE - ZTD RMS (doy 138, yy 2003)
[mm]
[hours]
INN
NIE
ONSA - ZTD RMS (doy 138, yy 2003)
[mm]
[hours]
INN
NIELL

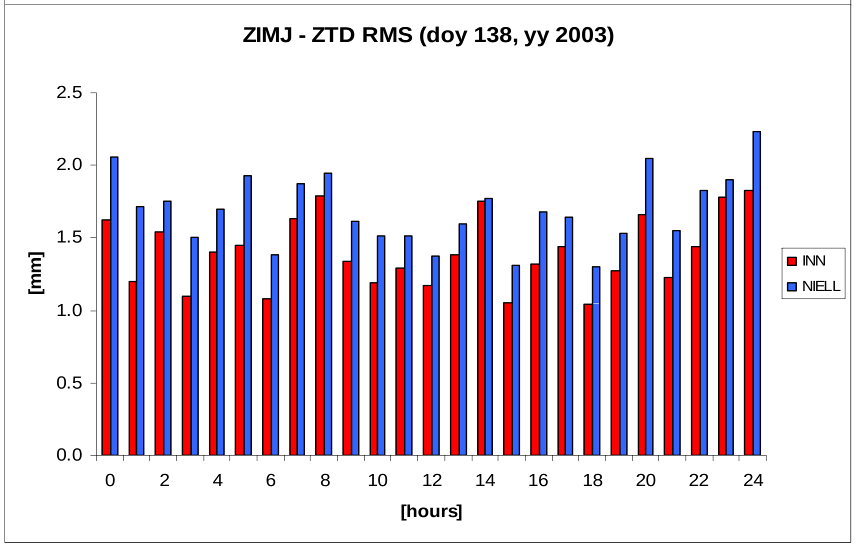
# Results_2

## Improvements up to ~20%



# Conclusions

- ***A huge amount of data are coming from RO Observations: (COSMIC, SAC-C, CHAMP, METOP, GRACE and Next OCEANSAT_2 mission)***
- ***We have replaced RAOB with RO data to construct the Niell MF***
- ***The new parameters organized in Niell MF fashion have been applied to a small network achieving encouraging improvements on ZTD estimation but not in the coordinates (?)***
- ***A better choose of the network to test the new MF (?)***
- ***A wide and extensive validation activity must be performed comparing our MF not only with NMF but also with GMF Vienna Mapping Function achieved by using Numerical Weather Models as well as against other kind of MF***