

# Height transmission from mainland to the island Rab in the Republic of Croatia

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## Abstract

Leveling already employ geodesy for some centuries. In this time a constant advancement of the instruments and measuring procedures took place. Transmission of heights over waters is a special and relatively fastidious task. The conditions for error influences, like the refraction, are particularly incalculable over open water surfaces. A generally used method of height transmission by means of a millimetre-accuracy geometrical levelling, can be excluded here as suitable procedure. For this reason some decades ago a procedure was developed, particularly for those elevation transmission over larger waters and/or valleys one conceived. According this procedure in Republic of Croatia in the summer 1961 the elevation transmission over the channel, that separates the island Rab from the mainland, was made. Apart from sea-transition levelling there are further procedures, with whose assistance transfer of heights could be done. These procedures are, due to their smaller accuracy, behind this special levelling. In the meantime, because of technique development particularly regarding economy, the procedures are in a big advantage. From this reason is to be examined, to what extent this special task with appropriate accuracy by other measuring procedures could be mastered. As possible alternative to sea-transition levelling we used two procedures for transmission of heights. Firstly we used GPS measurements, which find various application possibilities into ever more fields of geodesy. With this procedure also transmissions of heights can be realized with relatively small undertaking. Secondly we used the trigonometric levelling, which is suitable in principle likewise for an elevation transmission over waters. The comparative investigation of the procedures of trigonometric and GPS levelling measurements, which we employed, will show whether we are able to find an up-to-date alternative for the verified, but complex, sea-transition levelling. The measurement locally was accomplished in November 2007 with two LEICA TC 2003 total station and four TRIMBLE R7 receiver. As a control of the measurements will be used the required difference in height between both bank sides, which was determined in the year 1961.

## Local network, time, place and meteorological conditions

On each bank side before beginning of measurement two stations were set and determined. Between these markings the height difference was determined later by a high precision levelling. The stations of sea-transition levelling were marked out during the surveying works in the year 1961. Marking of the stations served the purpose to make measurements from the different epoch comparable one to another.



Figure 1: Network for height transmission between mainland and island Rab. It was necessary to attach the heights of the network stations at the bank side to the official levelling network. The additional application of a high precision levelling between network points at the same bank side in the combination with trigonometric levelling makes possible determination of the heights with higher accuracy and serves a reliable transmission of the heights over large water surfaces.

The choice of a suitable location was made based on different factors. On the one hand the individual procedures placed their conditions against the stations environment. The horizon over the stations should be free from obstacles for the GPS measurements. At the same time however visual contact between both banks must exist for the trigonometric levelling. The restriction in the choice of stations for sea-transition levelling lies in the maximum station separation. Thus the height difference between both bank sides may be only very small. If the distance is too long, the range of the ATR could be exceeded. These restrictions made the search more difficult for a suitable location. The criterion of the visibility was fulfilled with priority, since there are no compromises here. As the suitable site the same location was selected as in the year 1961 when sea-transition levelling was made. The stations for trigonometric levelling were set in the proximity and determined with fine levelling in the reference to the stations of sea-transition levelling.

Average distance between bank sides is 2150 m.

## Devices for trigonometric and GPS levelling

The transmission of the heights from the mainland to the island Rab was made with the help of two total stations from the same manufacturer, however of different type, i.e. LEICA TC 2003 and TCA2003. Also transmission of the heights was made with the help of four GPS receiver TRIMBLE R7. The equipment TCA2003 has the automatic target recognition, which is well-known under the term ATR. Thus, the prism is to be recognized and measured automatically. The accuracy of the ATR according to manufacturer data on 1,000 m on a round prism with middle atmospheric conditions is better than 2mm. The possibility of the automatic recognition of target offers many advantages for the measurement. Fine targeting and focusing by the observer is avoided that reduces time of an individual measurement. So the productivity and effectiveness of this procedure without large expenditure can be increased. A further advantage is that the prism is targeted always with same accuracy. The measurements are made independently from the observer and the lighting conditions.



Figure 2: Devices for trigonometric and GPS levelling

## Practical measurements

The measurements are carried out with the three procedures: trigonometric levelling, GPS and sea-transition levelling. Since measurements haven't been carried out at the same time, one can not later presuppose uniform conditions in the definition of heights.

### Sea-transition levelling, summer 1961

Some decades ago a procedure of sea-transition levelling was developed by the company Zeiss/Oberkochen, particularly for height transmission over larger waters and/or valleys one conceived. According this procedure in Republic of Croatia in the summer 1961 the height transmission over the channel, that separates the island Rab from mainland, was made.

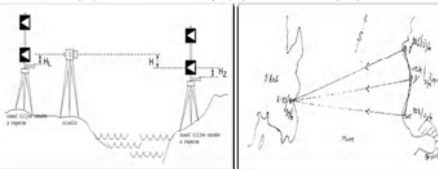


Figure 3: Sea-transition levelling mainland-island Rab, summer 1961

The measurements which were carried out were controlled by closure of formed triangles. Following results were accomplished:

$\Delta a, b, 33$	$dh = -11.3 \text{ mm}$
$\Delta b, c, 33$	$dh = -7.2 \text{ mm}$
$\Delta a, c, 33$	$dh = -18.5 \text{ mm}$

It is presupposed, due to the accomplished investigations that the height of the basic bench mark 33/576 on the island Rab was probably determined with accuracy better than 1 cm. Of it outgoing, it stands firmly that different geodetic tasks successfully can be solved.

## Conclusion

The trigonometric height transmission could be automated due to the enhanced possibilities of the instruments. The speed of the data gathering is considerable. The instruments in one hour could measure more than 40 sequences. Even with good will such quantities of observations with this quality cannot be produced manually. Since one sees already locally, to which extent the automatic target recognition fails, the sentence measurement in the desired extent can be repeated, in order to have for the evaluation sufficient data available. Observation time of the sea-transition is very long, as compared to trigonometric height transmission experience. One receives a fraction of the data in relation to the trigonometric height transmission during the same period only. In addition one needs at least two times so much personnel for this achievement. The effectiveness of sea-transition levelling is therefore by far smaller than those the trigonometric levelling. The analysis time of the trigonometric height transmission and sea-transition levelling is essentially shorter than during the GPS, since it can be accomplished by means of simple table processing programs automatically. On the other hand, in the case when the suitable software is available, analysis time and the complexity of the data processing is irrelevant. To all three procedures the statement applies that the evaluation becomes uncomplicated, if the data are high-quality. The procedures of the trigonometric levelling and sea-transition levelling, although theoretically suitable for such a task, are very dependent on meteorological conditions. With the help of GPS measurement the consistent results are obtained. It is shown that the GPS levelling with the use of local geoid model can be exact and reliable, better than it is generally acknowledged.

## Result of GPS adjustment

After observation files were imported, data from field sheets (antenna height, point number) were inserted. Precise orbits were downloaded from Internet and imported in Trimble Total Control (TTC), same as ionosphere model files. During data processing default parameters were used for static measurements of short line occupation method of GPS observations. All possible baseline combinations were calculated. Measurements passed all statistical tests so additional vector optimization was not performed for purpose of calculating.

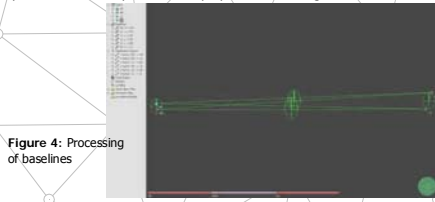


Figure 4: Processing of baselines

For GPS measurement adjustment, coordinates of one reference stations were fixed. Tau test had not detected "outliers", therefore all measurements were taken in adjustment. Measurements were adjusted using TTC software and its Network Adjustment module.

After adjustment accuracy of the final result was given as standard deviations and error ellipses for 95% probability. From the figures 4 and 5, it is possible to notice that network points coordinates were calculated with high precision, according to measurement preconditions. The reached accuracy correspond to the optimal values. Regarding the quality of the input data they can be regarded as well acceptable.

Point	$\sigma_{\text{lat}}$ [mm]	$\sigma_{\text{lon}}$ [mm]	$\sigma_{\text{h}}$ [mm]
JAB1	0.2	0.1	0.2
JAB2	0.2	0.1	0.2
RAB1	0.0	0.0	0.0
RAB2	0.2	0.1	0.2

Figure 5: Network adjustment

## GPS levelling, Geoid

From GPS observations one receives coordinates in the WGS 84-System. This is a geocentric three-dimensional Cartesian coordinate system rotating with the earth. For a practical purpose these three-dimensional-geocentric coordinates are however suitable. One uses instead a geocentrically stored earth ellipsoid as coordinate reference area, the GRS 80. Cartesian coordinates X, Y and Z are transferred into geographical coordinates width B, length L and ellipsoid height h. The ellipsoid heights h, which are independent of the gravity field, must be transferred in customs heights, in this case normal orthometric heights in Republic of Croatia. The connection between ellipsoid heights and standard heights is described by the well adapted local geoid model that must be available. In republic of Croatia for purpose of this investigation we have used our HRG2000 local geoid model.

Relative accuracy of local geoid depends on the respective survey area and amounts of 1 to 2 cm. In order to examine, whether a difference in the geoid model of 14 cm on a distance of only 2.2 km is possible, a raster was put over the area and the undulation of geoid was determined. The undulation at one point can be obtained by means of HRG2000 program. The value of undulation takes in this area in the direction south-east continuously too.

### Undulation of HRG2000 geoid

Station	N [m]
J1	43.74
J2	43.74
R1	43.60
R2	43.60

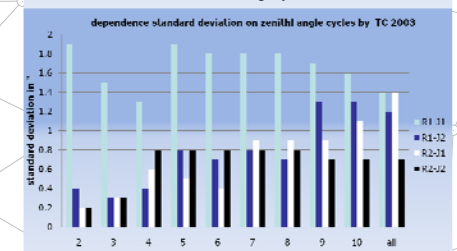
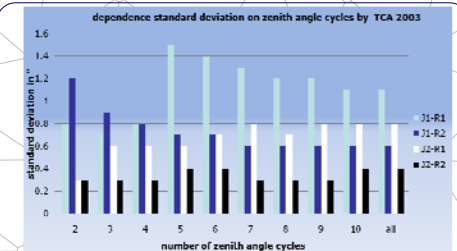


Figure 6: Undulation of HRG2000

## Trigonometric levelling

The procedure of the trigonometric height transmission is very much older than special sea-transition levelling. Execution is beyond that also less complex and the principle more understandable. During the trigonometric height transmission from angle and distance measurements height differences are computed. Zenith angles z must be introduced here with clearly higher accuracy into the computation than the inclined distance. The larger the distance is, propagation of the measured zenith angle error is also larger. Thus if the height difference with very high accuracy is to be determined, the angle measuring accuracy of the instrument is a crucial factor. In the meantime, significant development of the instruments is achieved. Hence, accuracy of angle measurement up to 0.5" is guaranteed. With a carefully accomplished measurement and an appropriate distance, one can reach theoretically the millimetre accuracy. A producer-determined accuracy of the total station is the index deviation. On the basis a measurement in two telescope faces one can determine the index deviation. By attaching the simple index deviation to the measured zenith angle, this instrumental error is eliminated. In addition, in view of the large distance measured, the problem of Earth's curvature must be considered. This problem could be eliminated by using of the bilateral trigonometric levelling. Also the refraction influence can be neglected, if the measurements take place at the same time. With this procedure on each bank a total station as well as a reflector are needed. The stations, between which also some kilometres of distance may lie, are freely selectable. Only condition is the direct visual contact.

Increasing number of zenith angle cycles from 1 to 10 does not change significantly the standard deviation by TCA2003 and TC2003. Average standard deviation of zenith angle from all measured zenith angle cycles is 0.7" measured by TCA2003 and 1.1" by TC2003.



## Result

The results of the trigonometric levelling are satisfying. Mutual deviation of the results and deviation from the previously determined value are acceptable. The computed standard deviation from constrained adjustment of 3,0 cm (table 1), and 2,7 cm from constrained adjustment with condition of unknowns (table 2) does not show this problem in full measure. During constrained adjustment with condition of unknowns the heights of two points from mainland bank sides and high difference between two points on the island Rab which were measured by geometric levelling were fixed. It was assumed that the height differences measured by geometric levelling were without errors. If one does not have comparative measurements or a knowledge of the height difference of stations, then the standard deviation pretends here accuracies, which are missing.

Table 1. Constrained network adjustment

zenith cycles	J1-Fixed [m]	J2-Fixed [m]	R1 [m]	R2 [m]	σ [m]
1	38.1164	47.7001	0.0212	21.5851	0.0303
2	38.1164	47.6984	0.0211	21.5812	0.0301
3	38.1164	47.6987	0.0214	21.5804	0.0306
4	38.1164	47.6995	0.0214	21.5808	0.0307
5	38.1164	47.7001	0.0212	21.5832	0.0304
6	38.1164	47.7003	0.0212	21.5835	0.0303
7	38.1164	47.7003	0.0213	21.5835	0.0305
8	38.1164	47.7006	0.0213	21.5846	0.0304
9	38.1164	47.7004	0.0212	21.5853	0.0303
10	38.1164	47.7006	0.0210	21.5852	0.0301
all	38.1164	47.7003	0.0211	21.5850	0.0301
RANGE		0.0022	0.0049	0.0040	

Table 2. Constrained network adjustment with condition of unknowns

zenith cycles	J1-Fixed [m]	J2-Fixed [m]	R1 [m]	R2 [m]	σ [m]
1	38.1164	47.7032	21.5867	0.0270	0.0270
2	38.1164	47.7032	21.5835	0.0269	0.0269
3	38.1164	47.7032	21.5826	0.0273	0.0273
4	38.1164	47.7032	21.5826	0.0274	0.0274
5	38.1164	47.7032	21.5846	0.0271	0.0271
6	38.1164	47.7032	21.5849	0.0270	0.0270
7	38.1164	47.7032	21.5849	0.0272	0.0272
8	38.1164	47.7032	21.5858	0.0271	0.0271
9	38.1164	47.7032	21.5866	0.0270	0.0270
10	38.1164	47.7032	21.5864	0.0269	0.0269
all	38.1164	47.7032	21.5864	0.0269	0.0269
RANGE		0.0041	0.0033		

conditions an accuracy of some millimetres. Nevertheless, one should choose equipment, with angle measurement accuracy better than 0.5". With an appropriate number of measurements, the necessary accuracy can be achieved, table 3.

Table 3. Comparison of the heights

Point	H Sea-Transition [m]	H_GPS [m]	H_Trigon. [m]	H_ST-GPS [m]	H_ST-T [m]	H_GPS-T [m]
J1	38.1164	38.1164	38.1164	0	0	0
J2	47.7032	47.7063	47.7032	-0.0031	0	0.0031
R1	21.5807	21.5684	21.5864	0.0123	-0.0057	-0.0180
R2	19.5077	19.4940	19.5144	0.0137	-0.0067	-0.0204

measurement. There are always asymmetrical remainder portions, which affect the height difference in unknown quantities way. They cannot be eliminated by formation of middle, since they are not completely detectable. They result here from unbalances of the bank sides and the special atmospheric conditions over the water surface. Also different angles of incidence of the sun on the individual bank affect themselves. Additional problem is that different distance of the instrument to the bank has an influence on the process of the ray of light. However, above all the meteorological conditions over the water surface have a very large portion of the process of the ray of light. Here the conditions of the temperature stratifications turned around. Above the ground the radiant heat proceeds from the soil and decreases with the height. Over the water the lower layer is colder than the layers lying over it. Only in larger height again original conditions are reached.

