# Terrestrial reference systems from theory to implementation

- Basic Theory of RS & RF
- Principles for TRF Implementation
- Application to ITRF/ETRF
- Illustrating Examples

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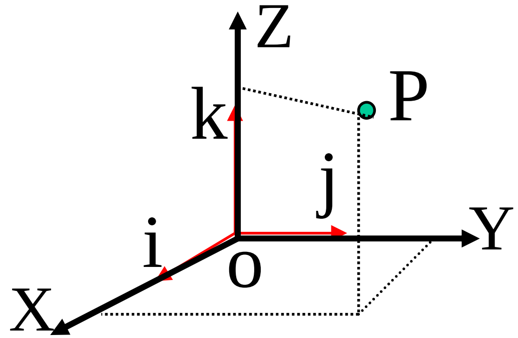
# Defining a Reference System & Frames:

- **Three conceptual levels [*Kovalevsky et al*., 1989]:**
  - **Ideal Terrestrial Reference System (TRS) is a mathematical, theoretical system**
  - **The Conventional TRS is the sum of all conventions (models, constants,...) that are necessary to realize the TRS**
  - **A Conventional TRF, which uses above to realize the TRS.**
- **In effect: Use only TRS and TRF terms**
  - **The TRS is an ideal, conventional model**
  - **The TRF is a materialization of the TRS inheriting the mathematical properties of the TRS**
  - **As the TRS, the TRF has an origin, scale & orientation**
  - **TRF is constructed using space geodesy observations**

# Ideal Terrestrial Reference System

**A tridimensional reference frame (mathematical sense)**
**Defined in an Euclidian affine space of dimension 3:**




**Affine Frame (O,E) where:**

**O: point in space (Origin)**
**E: vector base: orthogonal with the same length:**

- **unit vectors co-linear to the base (Orientation)**
- **unit of length (Scale)**

$$\lambda = \left\| \vec{E}_i \right\|_{i=1,2,3}$$

$$\vec{E}_i . \vec{E}_j = \lambda^2 \delta_{ij}$$

$$(\delta_{ij} = 1, \quad i = j)$$

# "Motions" of the deformable Earth

- **Nearly linear motion:**
  - Tectonic motion: horizontal
  - Post-Galcial Rebound: Vertical & Horizontal

- **Non-Linear motion:**
  - Seasonal: Annual, Semi & Inter-Annual caused by loading effects
  - Rupture, transient: uneven motion caused by Earthquakes, Volcano Eruptions, etc.

# Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch $t$ could be written as :

$$X(t) = X_0 + \dot{X}.(t - t_0) + \sum_i \Delta X_i(t)$$

$X_0$ : point position at a reference epoch $t_0$

$\dot{X}$ : point linear velocity

$\Delta X_i(t)$ : high frequency time variations:

- Solid Earth, Ocean & Pole tides
- Loading effects: atmosphere, ocean, hydrology, Post-glacial-Rebound
- ... Earthquakes

# TRF Representations

- "Quasi-Instanteneous" Frame: mean station positions at "short" interval:
  - One hour, 6-h, 12-h, one day, one week

  ==> Non-linear motion embedded in time series of instanteneous frames
- Long-Term Secular Frame: mean station positions at a reference epoch ($t_0$) and station velocities: $X(t) = X_0 + V^*(t - t_0)$
- Non-Linear Frame does not exist, otherwise it is an "Earth model"

# TRF Datum Definition

- A TRF should be clearly and unambiguously defined through 7 (14) parameters:
  - 3 (6) origin parameters
  - 1 (2) scale parameters
  - 3 (6) orientation parameters
- The 7 (14) parameters are relative quantities
  - e.g. if we say SLR origin is selected to define ITRF2005 origin, it means zero translations/rates btw SLR and ITRF2005

# TRF implementation

- Using Space Geodesy data of a <u>single</u> technique or <u>multiple</u> techniques
- Combination of SG Solutions (ex. ITRFs)

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==> Construct a singular Normal Equation

==> Add constraints to define the TRF in origin, scale or/and orientation

==> Produce SSC + Other parameters (e.g.EOPs)

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- By transformation formula / 7(14) parameters

==> Adjusted (by LS) or Computed parameters

==> Ex. ITRF-to-ETRF transformation

# Classes of Sets of Station Coordinates (SSC)

- "Primary" SSC: results from LS adjustment:
  - Space Geodesy solutions at the observation level
  - Combined solutions: e.g. ITRFs
- "Secondary" SSC: Expressed/Transformed in a given TRF:
  - IGS00, IGS05, EPN weekly/cumulative solutions
  - National, Regional or Local GPS network solutions
  - ETRFyy lists as results from ITRF-to-ETRF transformation

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- CTRF: Set of physical points with coordinate numerical information (values, derivatives) expressed in a selected coordinate system linked to a specific TRS

# CATREF Software

**Station Positions & Velocities**

$$\begin{cases} X_s^i = X_c^i + (t_s^i - t_0)\dot{X}_c^i \\ \quad + T_k + D_k X_c^i + R_k X_c^i \\ \quad + (t_s^i - t_k)\left[\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i\right] \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{cases}$$

**EOPs**

$$\begin{cases} x_s^p = x_c^p + R2_k \\ y_s^p = y_c^p + R1_k \\ UT_s = UT_c - \frac{1}{f} R3_k \\ \dot{x}_s^p = \dot{x}_c^p + \dot{R}2_k \\ \dot{y}_s^p = \dot{y}_c^p + \dot{R}1_k \\ LOD_s = LOD_c + \frac{\Lambda_0}{f} \dot{R}3_k \end{cases}$$

**Derived from relationship btw Celestial & Terrestrial Systems :**

$$X_{TRS} = S.N.P.X_{CRS}$$

$$LOD = \int_t^{t+\Lambda_0} dUT$$

# Time Series Stacking: Frame Definition (1/3)

- (1) Define the frame at a given epoch $t_0$

==> 7 degrees of freedom to be selected/fixed

- (2) Define a linear (secular) time evolution

==> 7 degrees of freedom to be selected/fixed

==> Assume linear motion both for stations and frame parameters:

  - Add break-wise approach for discontinuities
  - Investigate the non-linear part in the time series of the residuals

# Time Series Stacking: Frame Definition (2/3)

- **(1) Minimum Constraints Approach: Select an external frame as a "reference" ($X_R$)**

$$X_R = X_c + A\theta \xrightarrow{\theta = 0} (A^T A)^{-1} A^T (X_R - X_c) = 0$$

- **(2) Internal (Intrinsic) Constraints (See next)**

# Time Series stacking: Frame Definition (3/3) (Intrinsic Conditions: CATREF Software)

- **Consider Transfo. Param. as unknowns in Normal Eq. Sys.**
- **Estimate time series of Transfo. Param. & long-term solution**
- **Considering linear transf. parameter *P* :**

$$P(t) = P(t_0) + \dot{P}.(t - t_0) \qquad \textbf{(1)}$$

- **Eq. 1 could be solved by linear regression:**

$$\begin{pmatrix} K & \sum_{k \in K} (t_k - t_0) \\ \sum_{k \in K} (t_k - t_0) & \sum_{k \in K} (t_k - t_0)^2 \end{pmatrix} \begin{pmatrix} P_k(t_0) \\ \dot{P}_k \end{pmatrix} = \begin{pmatrix} \sum_{k \in K} P_k \\ \sum_{k \in K} (t_k - t_0) P_k \end{pmatrix}$$

**Intrinsic conditions:**

$$P(t_0) = 0 \quad \textbf{\&} \quad \dot{P} = 0 \quad \textbf{or} \quad \left\{ \begin{array}{lcl} \sum_{k \in K} P_k(t_k) & = & 0 \\ \sum_{k \in K} \frac{P_k(t_k)}{(t_k - t_0)^{-1}} & = & 0 \end{array} \right.$$

# Intrinsic Conditions

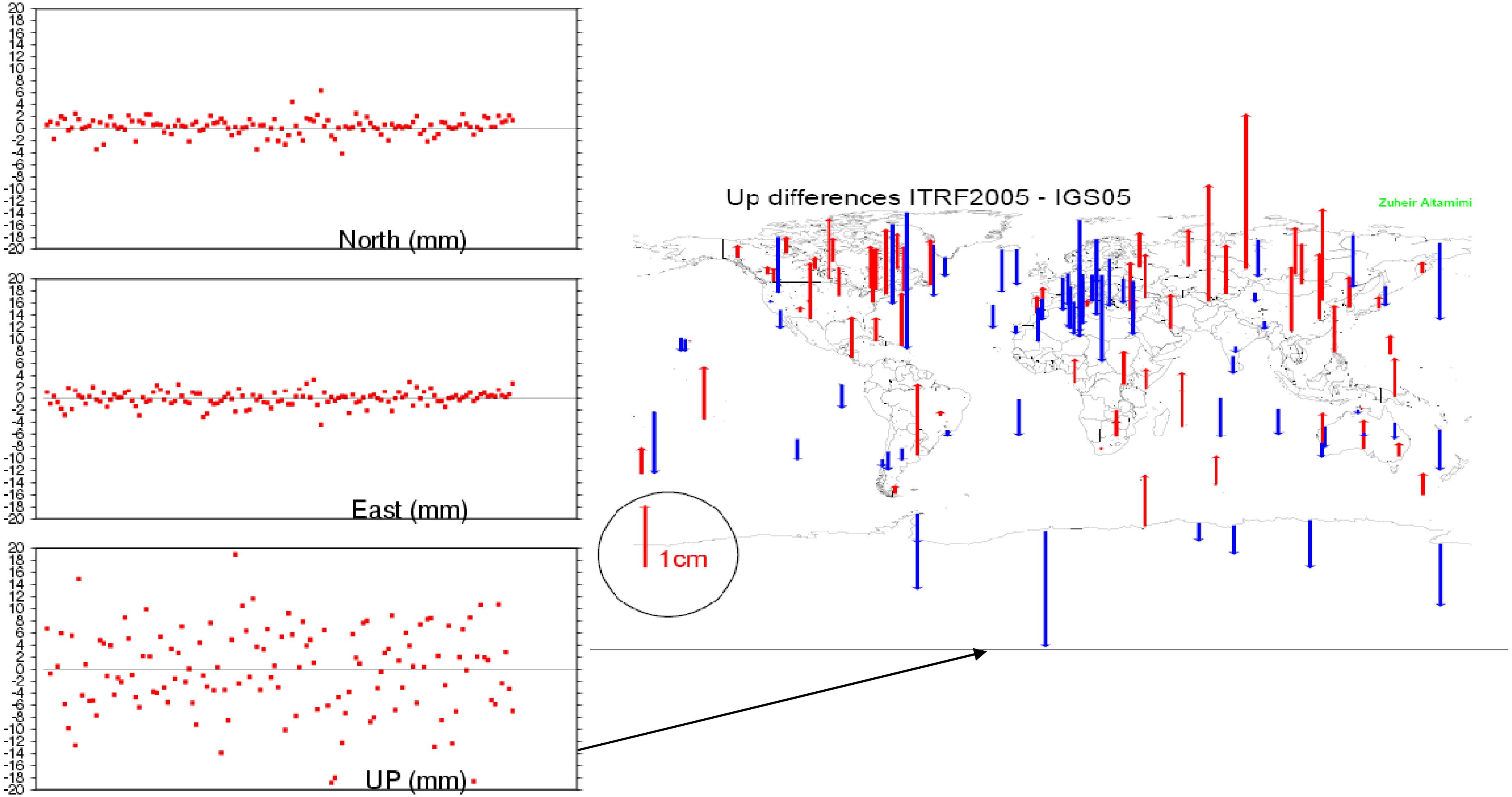
$$P(t_0) = 0 \quad \& \quad \dot{P} = 0$$

$$\begin{cases} \sum_{k \in K} P_k(t_k) & = & 0 \\ \sum_{k \in K} \frac{P_k(t_k)}{(t_k - t_0)^{-1}} & = & 0 \end{cases}$$

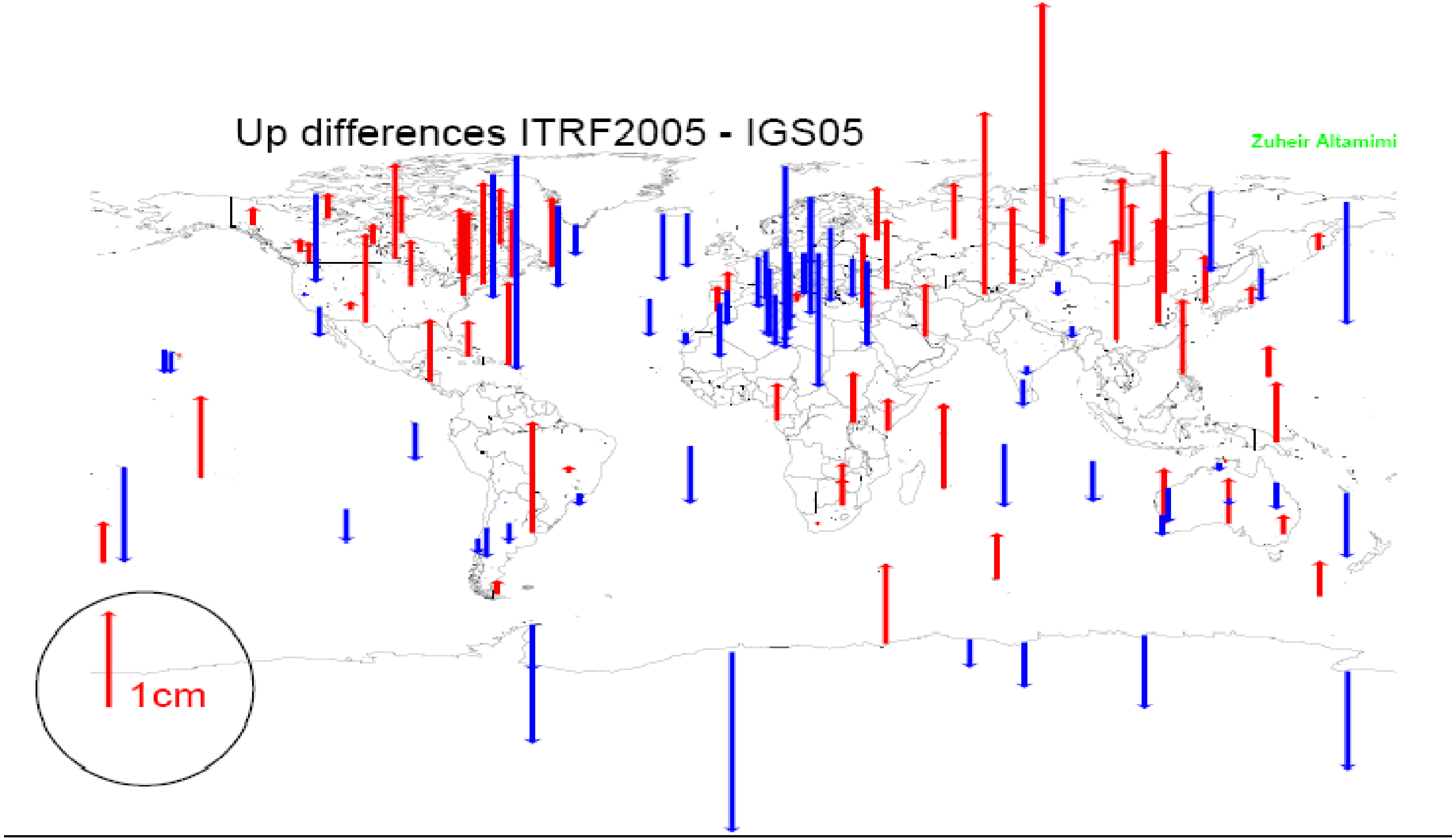
- **Preserve the intrinsic origin of SLR**
  - **Seen as No-Net-Translation condition**
  - **Preserve/Realize the long-term CoM as sensed by SLR**
- **Preserve the intrinsic scale of SLR & VLBI**

# Some illustrating examples

# ITRF2005-IGS05 differences



# ITRF2005-IGS05 differences

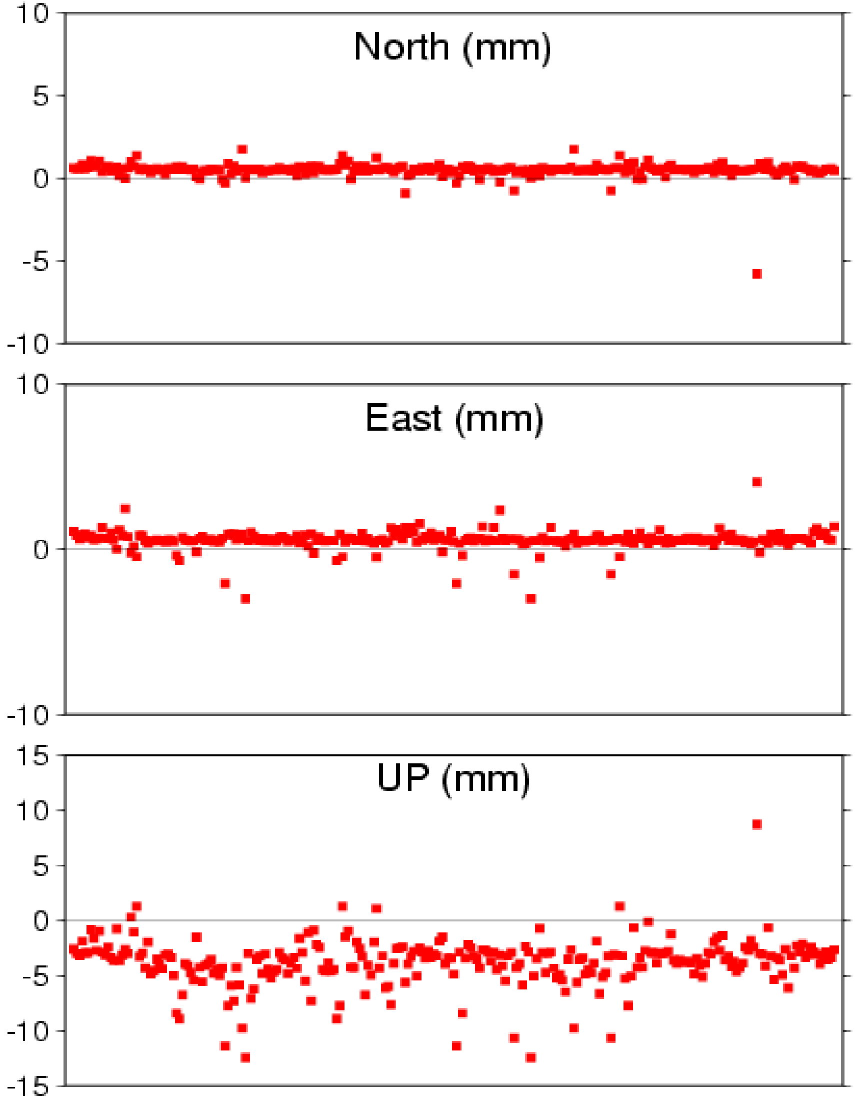


# Example of an EPN cumulative solution

**Differences when using IGS05 or ITRF2005 RF stations**

**If relative PCV:**
**Use ITRF2005**

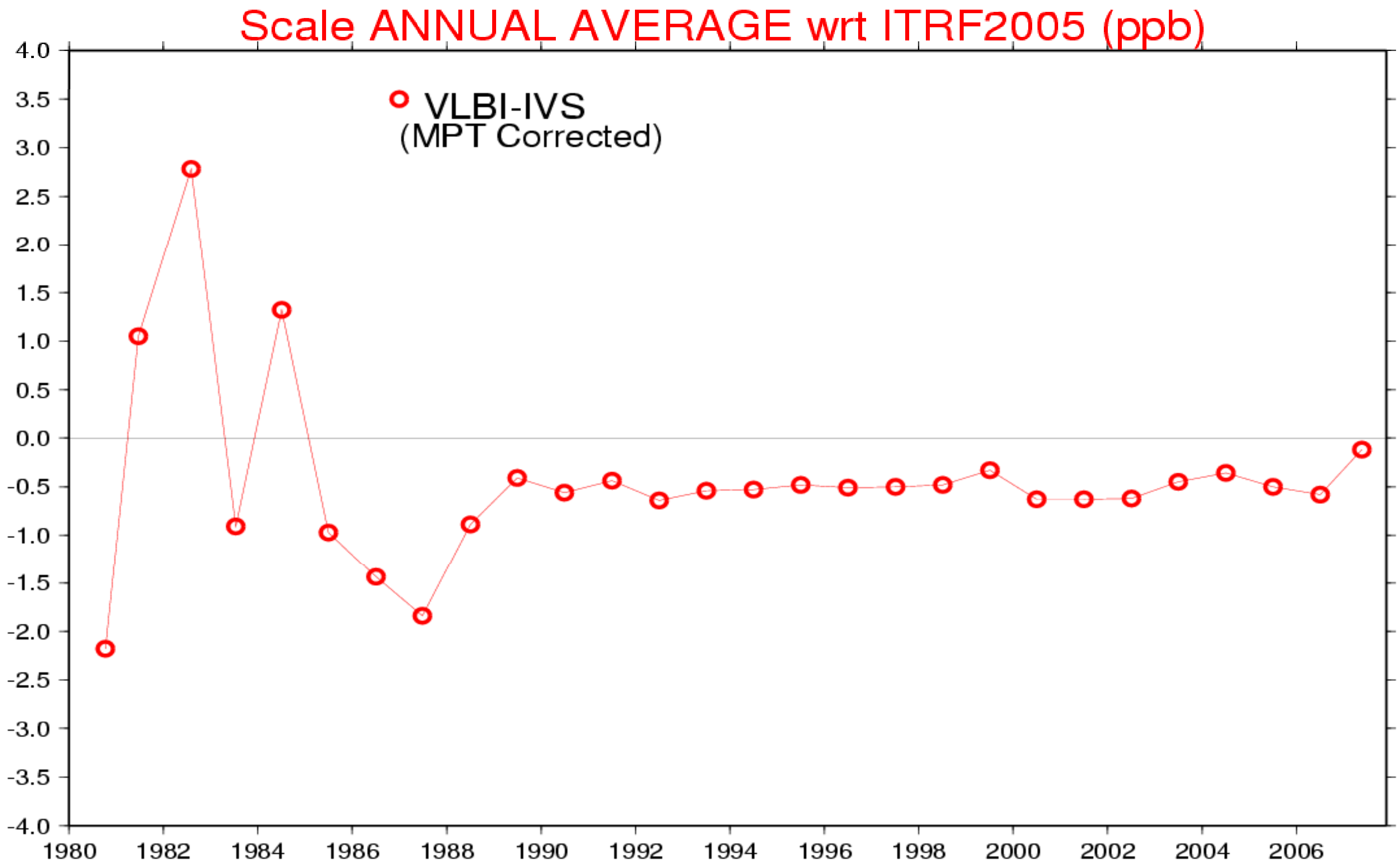
**If absolute PCV:**
**Use IGS05, but if a user wants to be in ITRF2005, what to do ?**



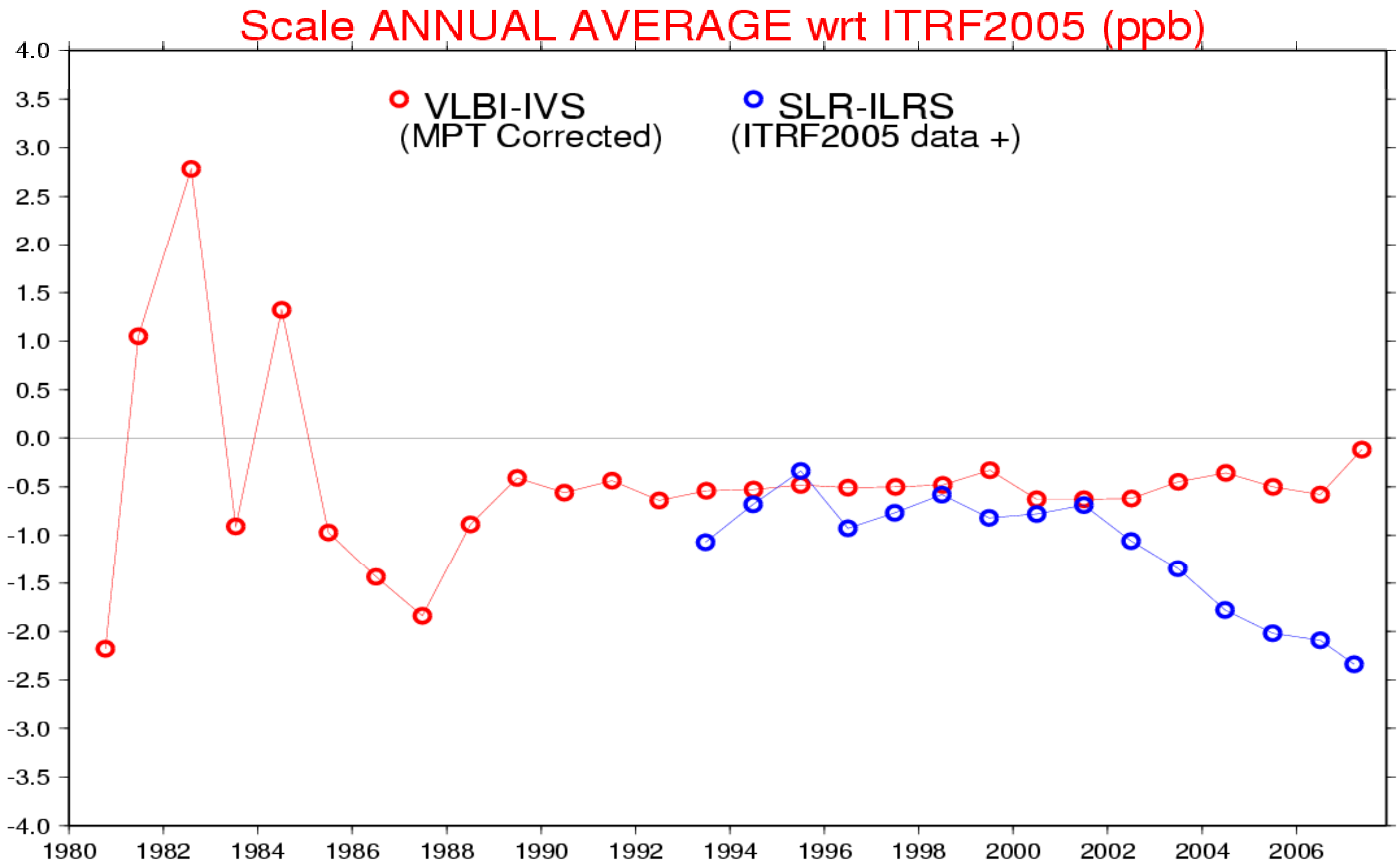
# TRF Scale

- GM adopted (or estimated) value in case of satellite techniques
- Relativistic corrections (all techniques)
- Troposphere modelling (most of techniques)
- Technique-specific effects
  - VLBI, GPS and DORIS antenna-related effects
  - SLR range and time biases

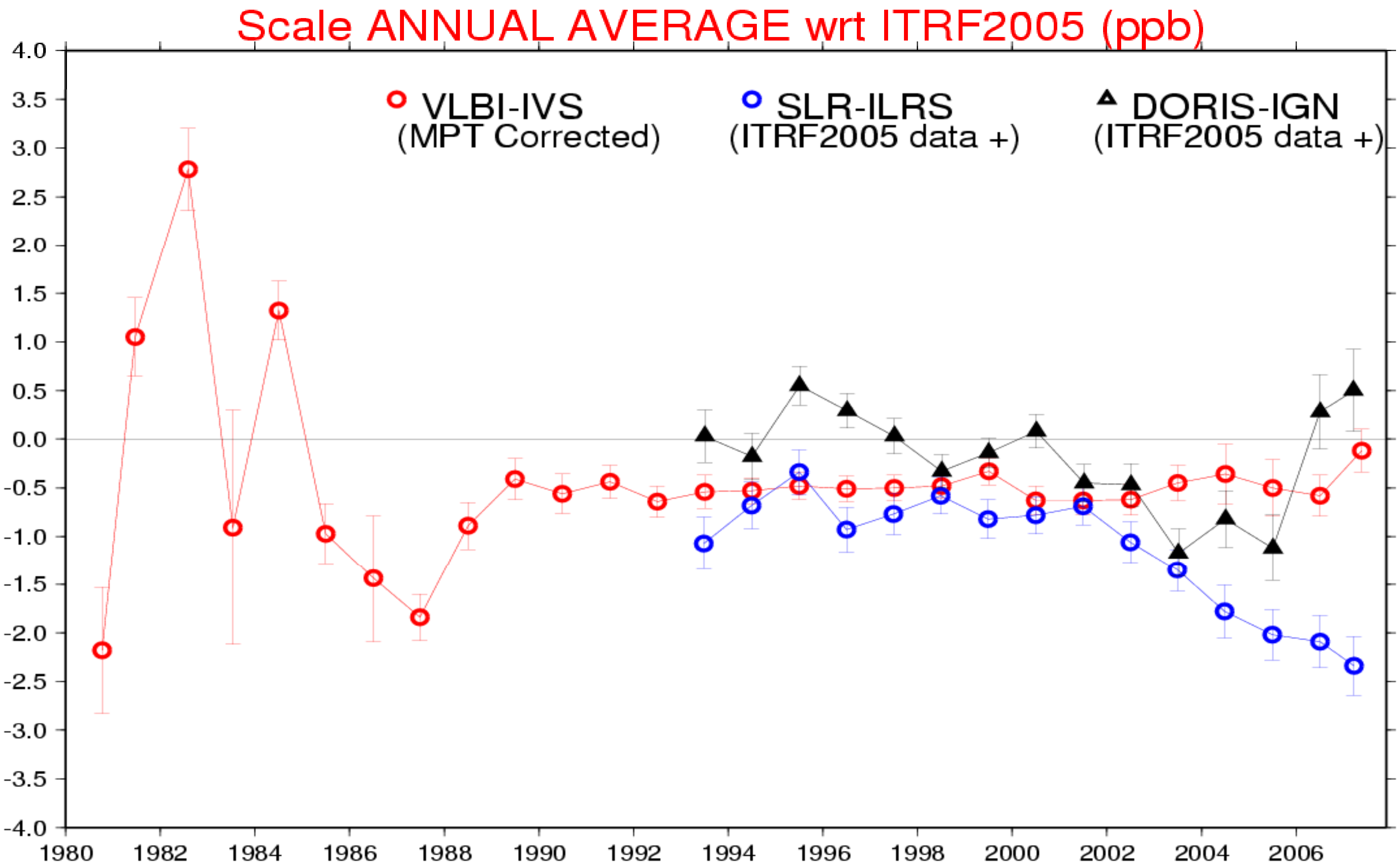
# Scale Annual Average



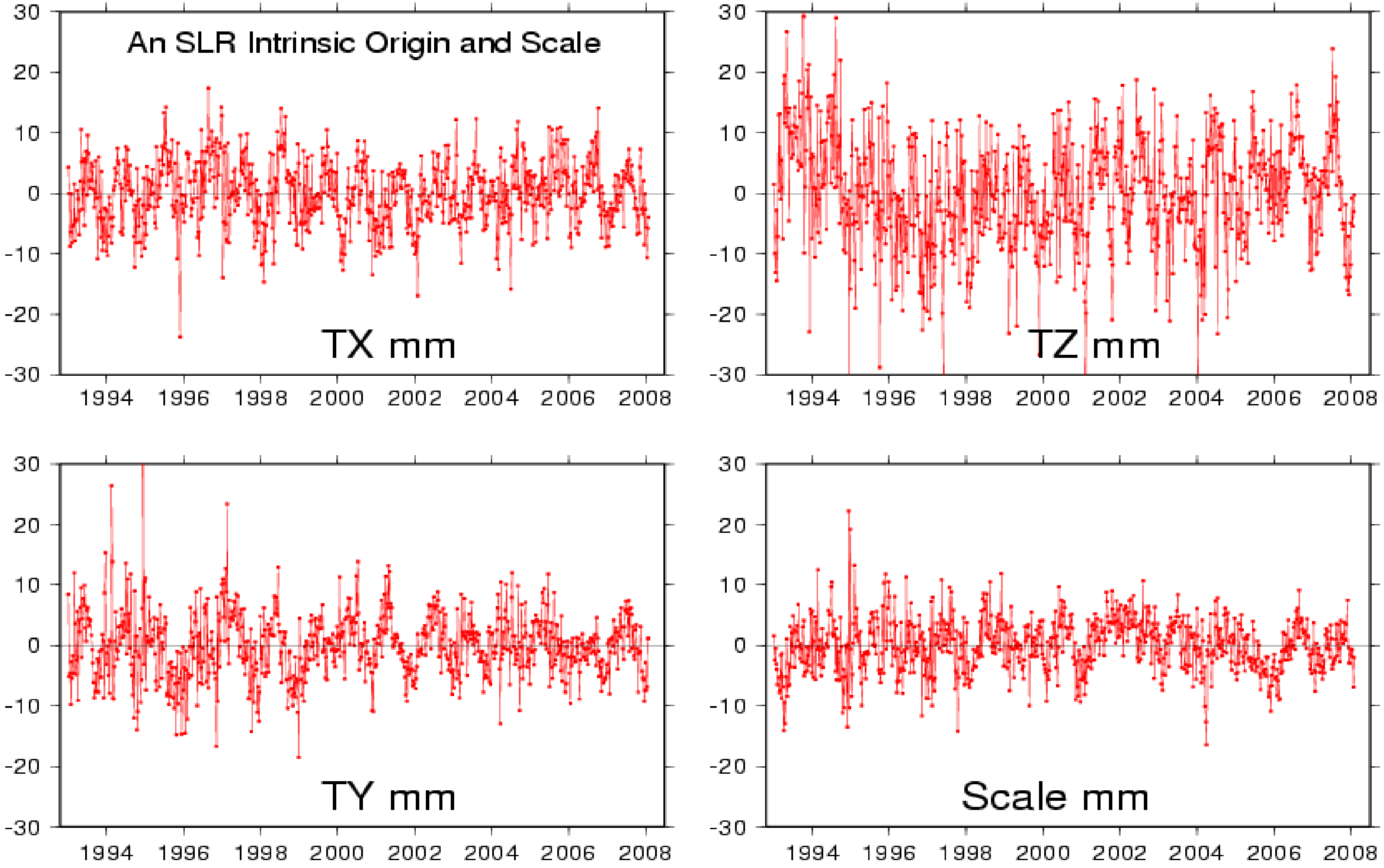
# Scale Annual Average



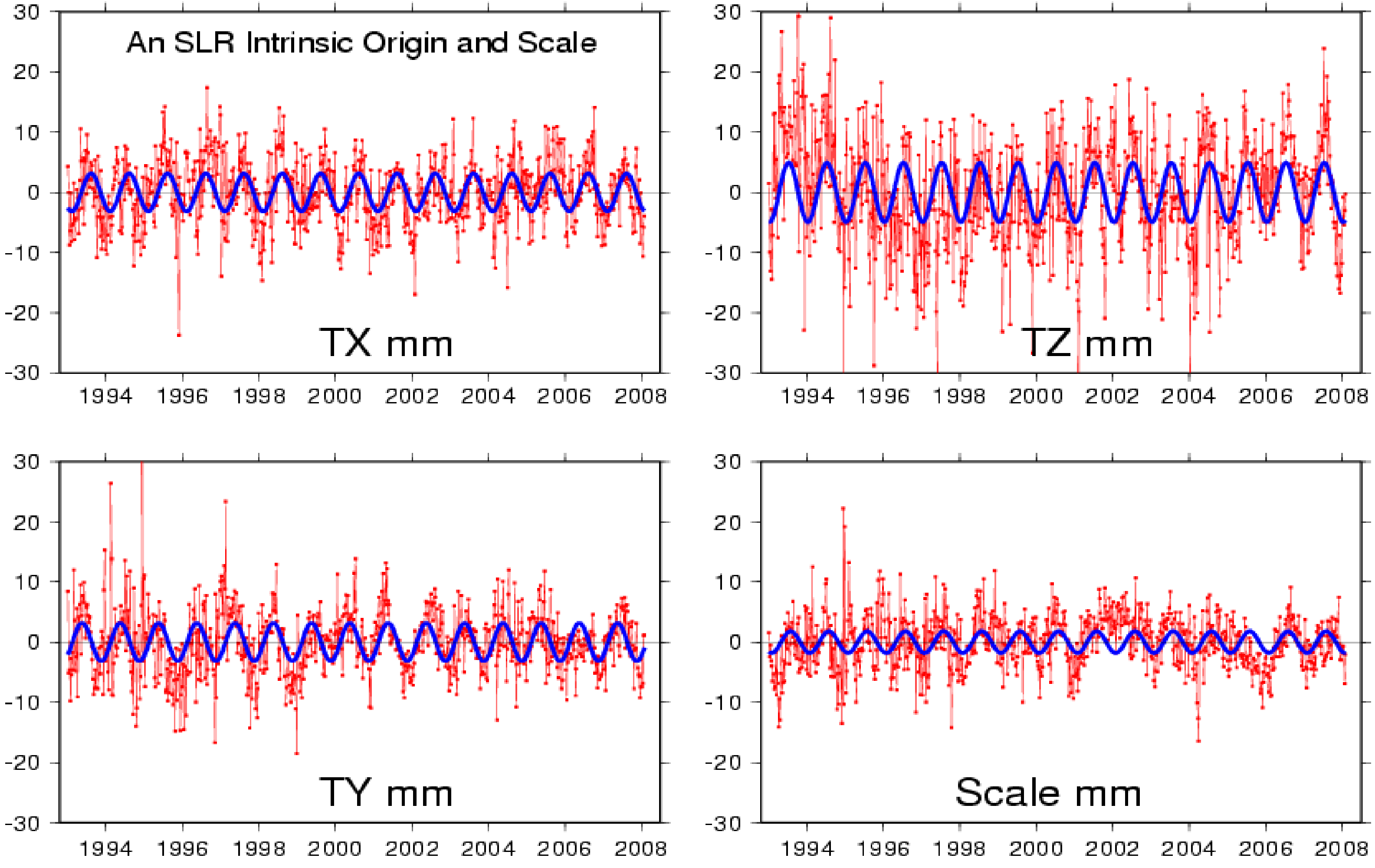
# Scale Annual Average



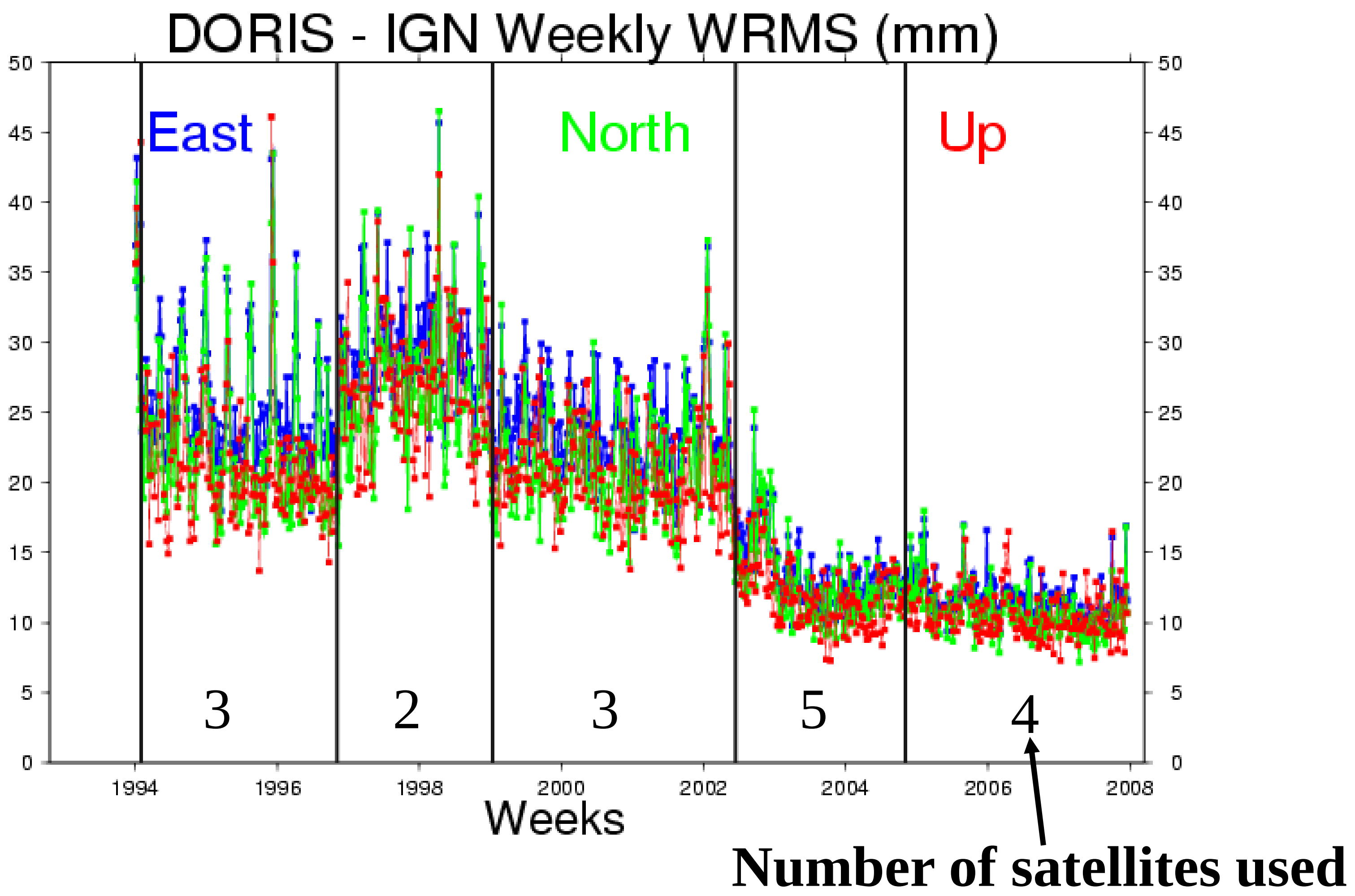
# An SLR Solution
# Intrinsic Origin & Scale



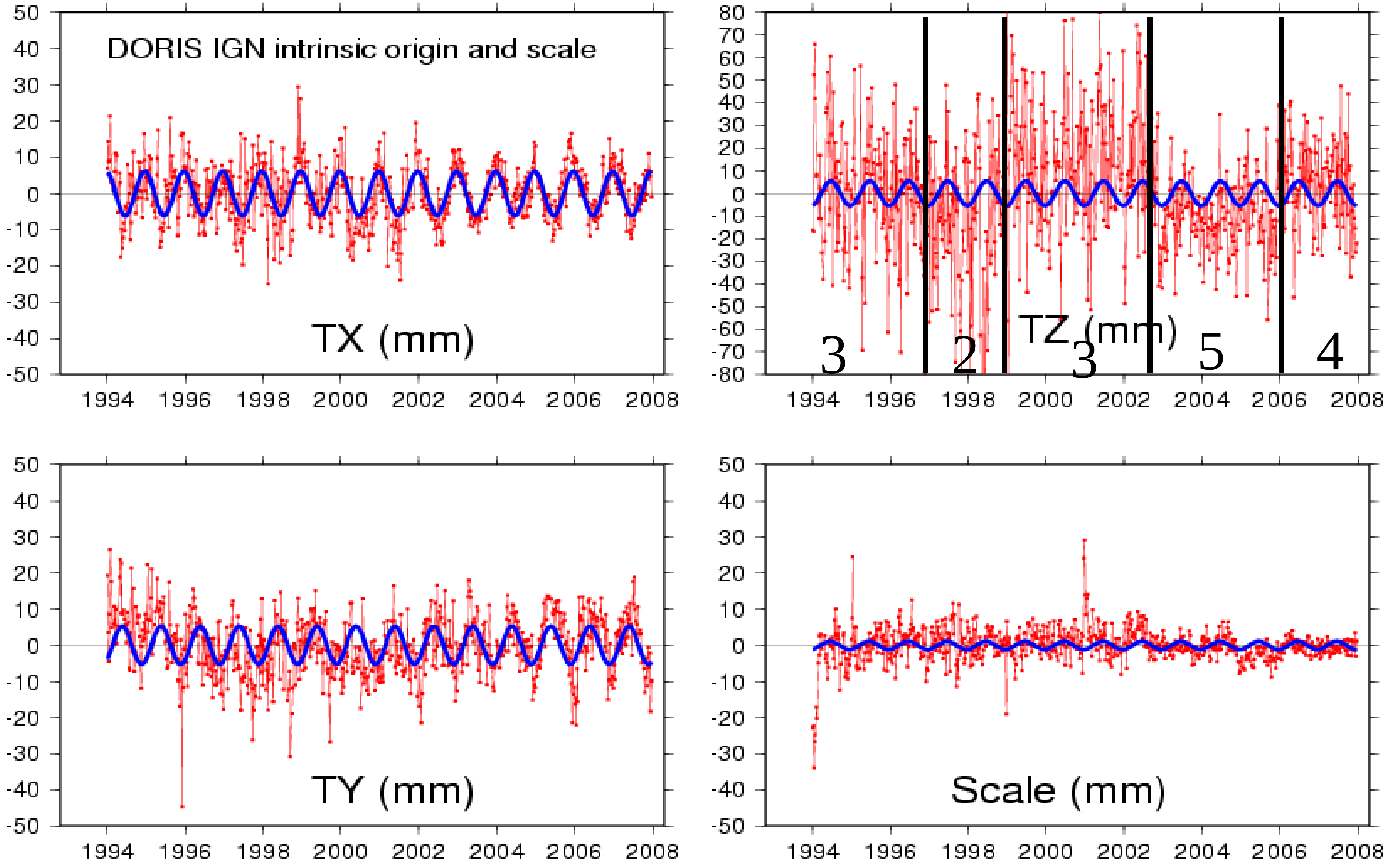
# An SLR Solution
# Intrinsic Origin & Scale



# New DORIS Solution (IGN): WRMS

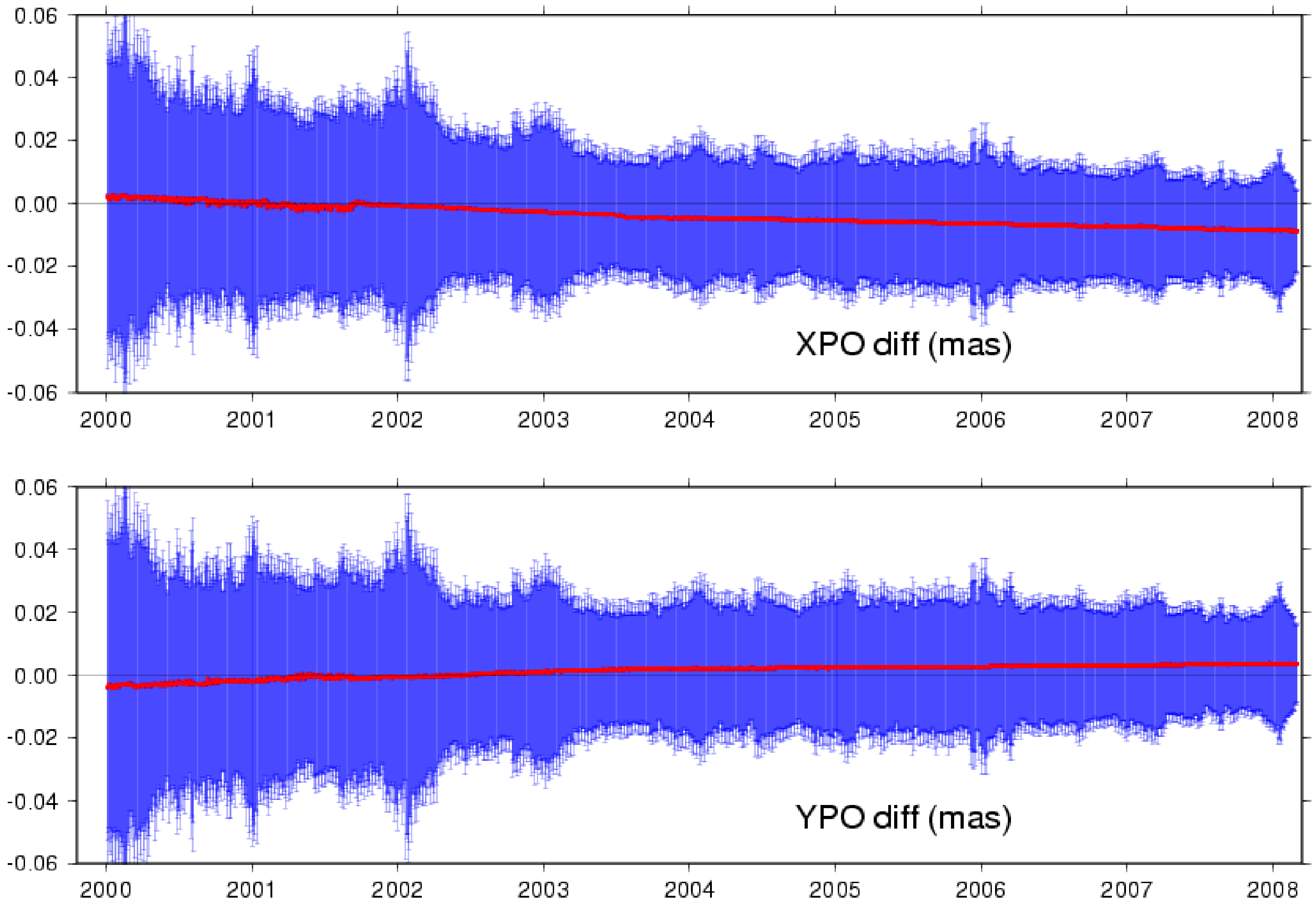


# DORIS IGN Solution
# Intrinsic Origin & Scale



# Network effect on EOPs
# Change the RF Core sites by 10 sites

**Main difference Drift:**
**Xp: 2 µas/yr**
**Yp: 1 µas/yr**



# Concluding Remarks

- Simplify the terminology: use TRS and TRF
  - Work to be done within IAG Commission 1
- Time series analysis for station positions and TRF parameters (origin & scale) are critical for TRF implementation. Two types of TRF are needed:
  - Quasi-Instantaneous TRF
  - Secular TRF
- Objective quality assessment of technique systematic errors
- Continuous observations by space techniques are fundamental
- Equally fundamental is the improvement of the geodetic infrastructure