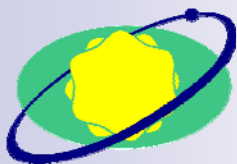


Determination of Physical Heights Using Earth Geopotential Model, GPS and Gravity Data

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Motivation

- Earth Geopotential Models, EGM96, 0.3 m
- GPS measurements, 4-7 mm/5 days,
2-3 cm/EUPOS
- Gravity measurements, 1-2 μGal /absolute,
5-10 μGal /relative

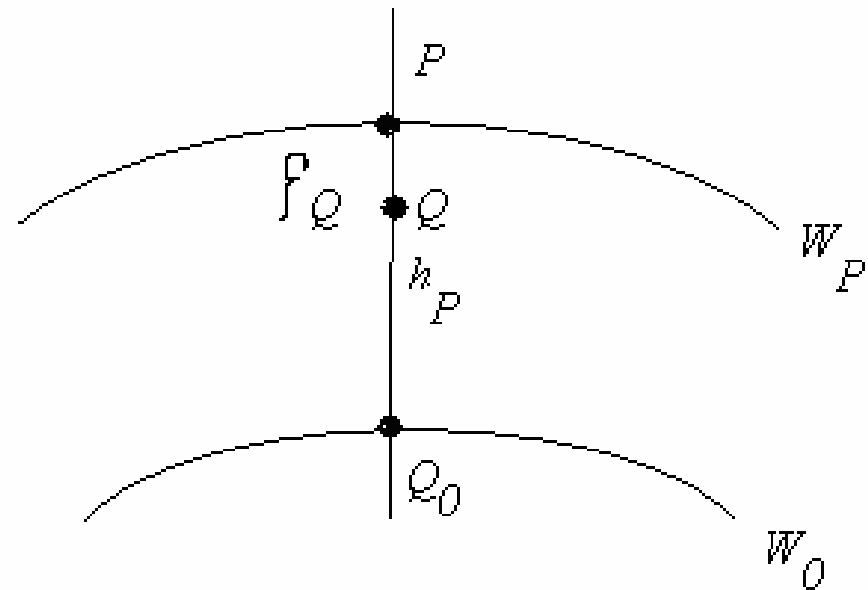
Theory of Normal Heights

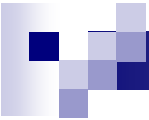
$$H_P^n = \frac{W_0 - W_P}{\bar{\gamma}}$$

W_0 — gravity potential on the geoid

W_P — gravity potential on the point P

$\bar{\gamma}$ — mean normal gravity between ellipsoid and P point





Gravity Potential

$$W_P = W_{EGM,P} + \delta W_{RES,P}$$

Earth Gravity Model

$$W_P = \frac{GM}{r_P} \left[1 + \sum_{n=1}^{n_{\max}} \left(\frac{a}{r_P} \right)^n \sum_{k=0}^n \left(\bar{C}_{n,k} \cos(k\lambda_P) + \bar{S}_{n,k} \sin(k\lambda_P) \right) \bar{P}_{n,k}(\sin \bar{\varphi}_P) \right] +$$
$$+ \frac{1}{2} \omega^2 (X_P^2 + Y_P^2)$$

$P \equiv (r_P, \bar{\varphi}_P, \lambda_P)$ - spherical coordinates of point P

Residual Gravity Potential

$$\delta W_{RES,P} = \delta W_{RES,P}^0 + \delta W_{RES,P}^1$$

$$\delta W_{RES,P}^0 = \frac{R}{4\pi} \iint_{\sigma} \delta g_{RES} H(\psi) d\sigma$$

$$\delta g_{RES} = \delta g_m - \delta g_{EGM}$$

$$\delta g_m = g_P - \gamma_P$$

$$\gamma_P = \gamma_{Q_0} + \frac{1}{h_P} \int_{Q_0}^P \gamma dh$$

$$H(\psi) = \frac{1}{\sin(\psi/2)} - \ln \left(1 + \frac{1}{\sin(\psi/2)} \right)$$

$$\delta W_{RES,P}^1 = \frac{R}{4\pi} \iint_{\sigma} \left(-\frac{\partial \delta g}{\partial h} (h - h_P) \right) H(\psi) d\sigma$$



Mean Normal Gravity

$$\bar{\gamma} = \gamma_{Q_0} + \left(\frac{\partial \gamma}{\partial h} \right)_{Q_0} \frac{h_P - \zeta_{EGM,P}}{2} + \frac{1}{2} \left(\frac{\partial^2 \gamma}{\partial h^2} \right)_{Q_0} \left(\frac{h_P - \zeta_{EGM,P}}{2} \right)^2 + \dots$$

$$\zeta_{EGM,P} = \frac{W_{EGM,P} - U_P}{\gamma_P}$$

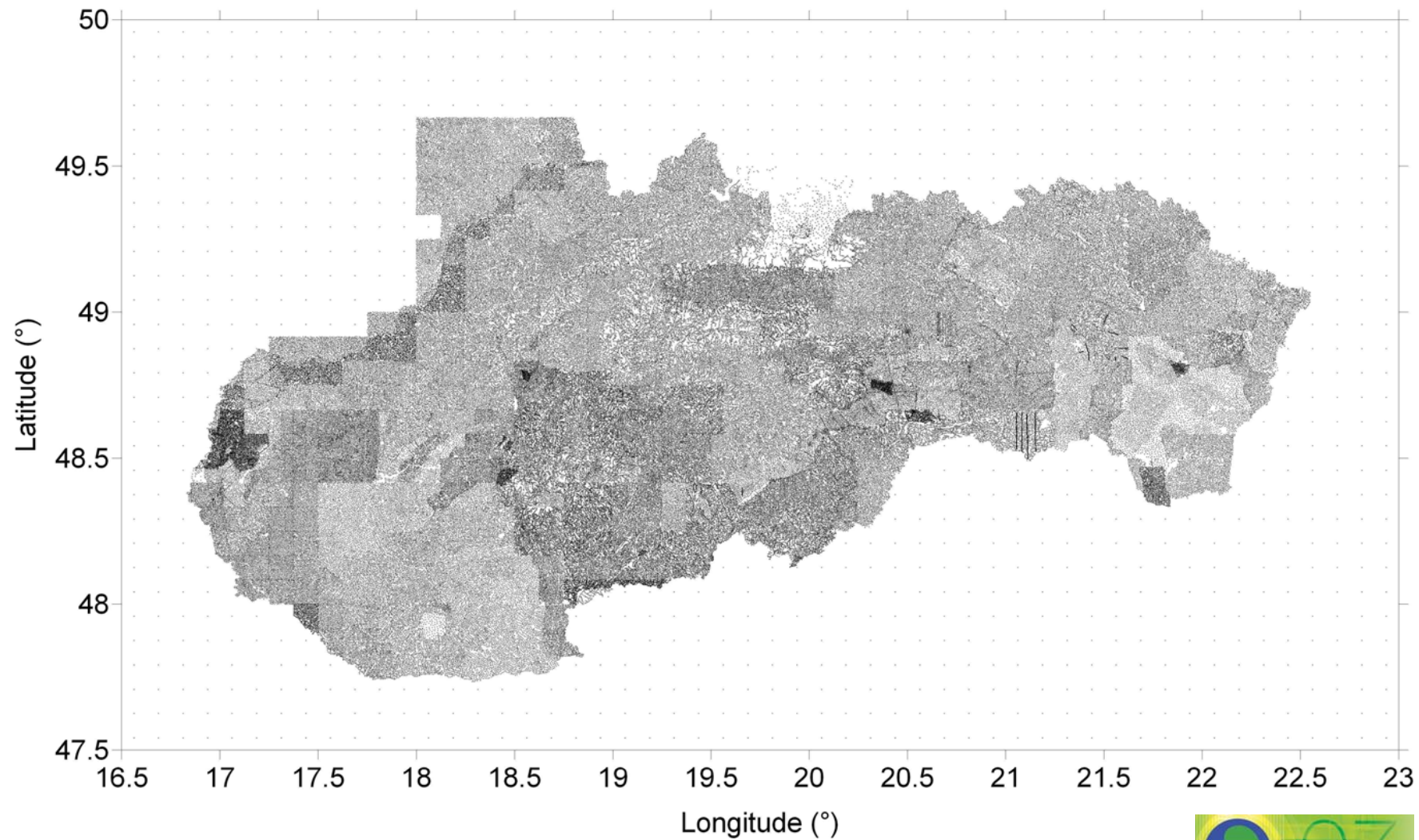


Practical Solution

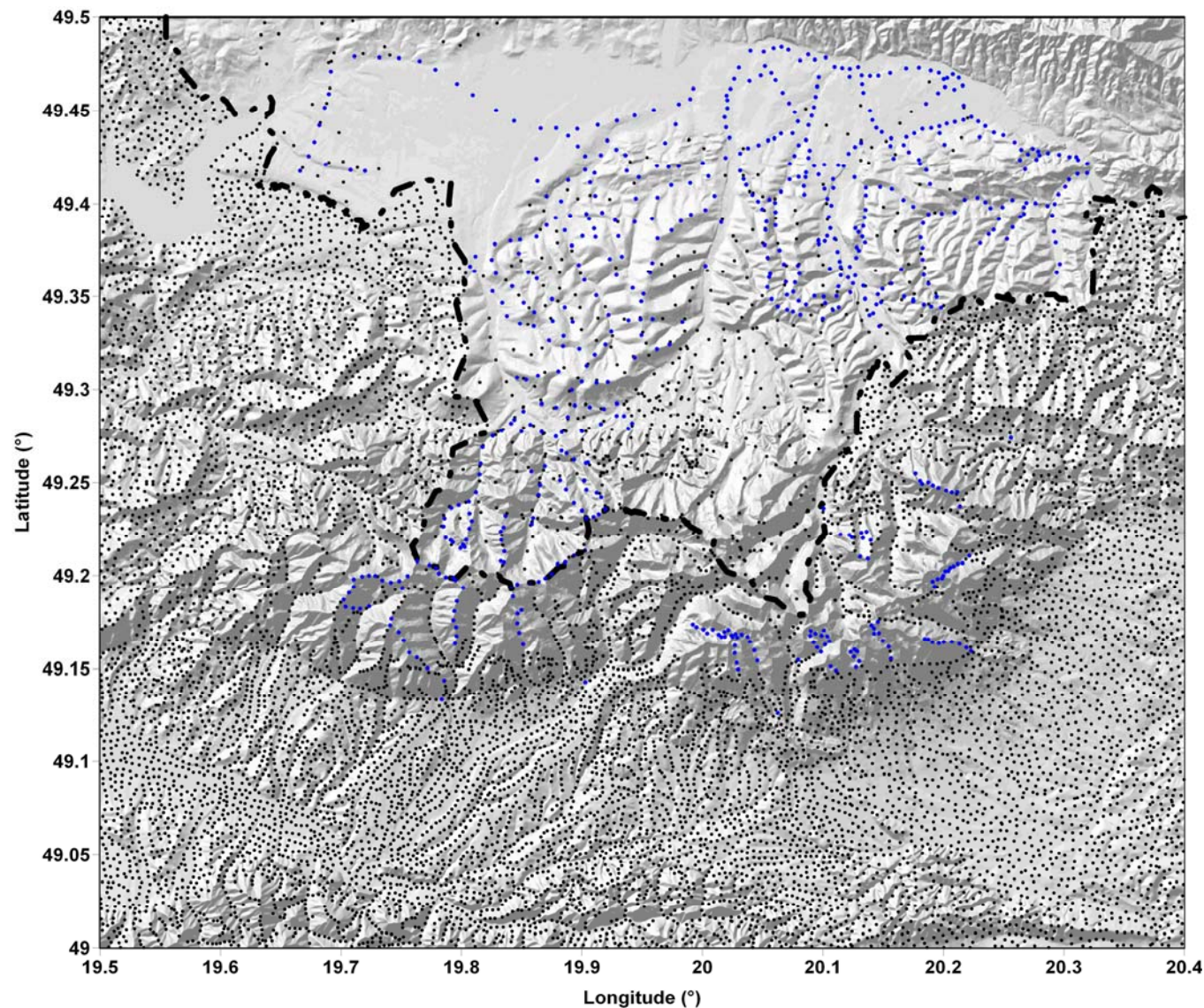
Input Models and Data

- Tide-free reference model
- EGM96, complete to $n = k = 360$
- Point gravity data, approximate 200 000 on the area of Slovakia, transformed to ETRS89 and to Absolute gravity system
- SLOVGERENET95, 41 points, ETRS89, 36 hours simultaneous observation

**Point gravity data, approximate 200 000
(4-6 points/square km)**



Detail point gravity data in the area of Tatra Mountain (black dot old data, blue dot new data)

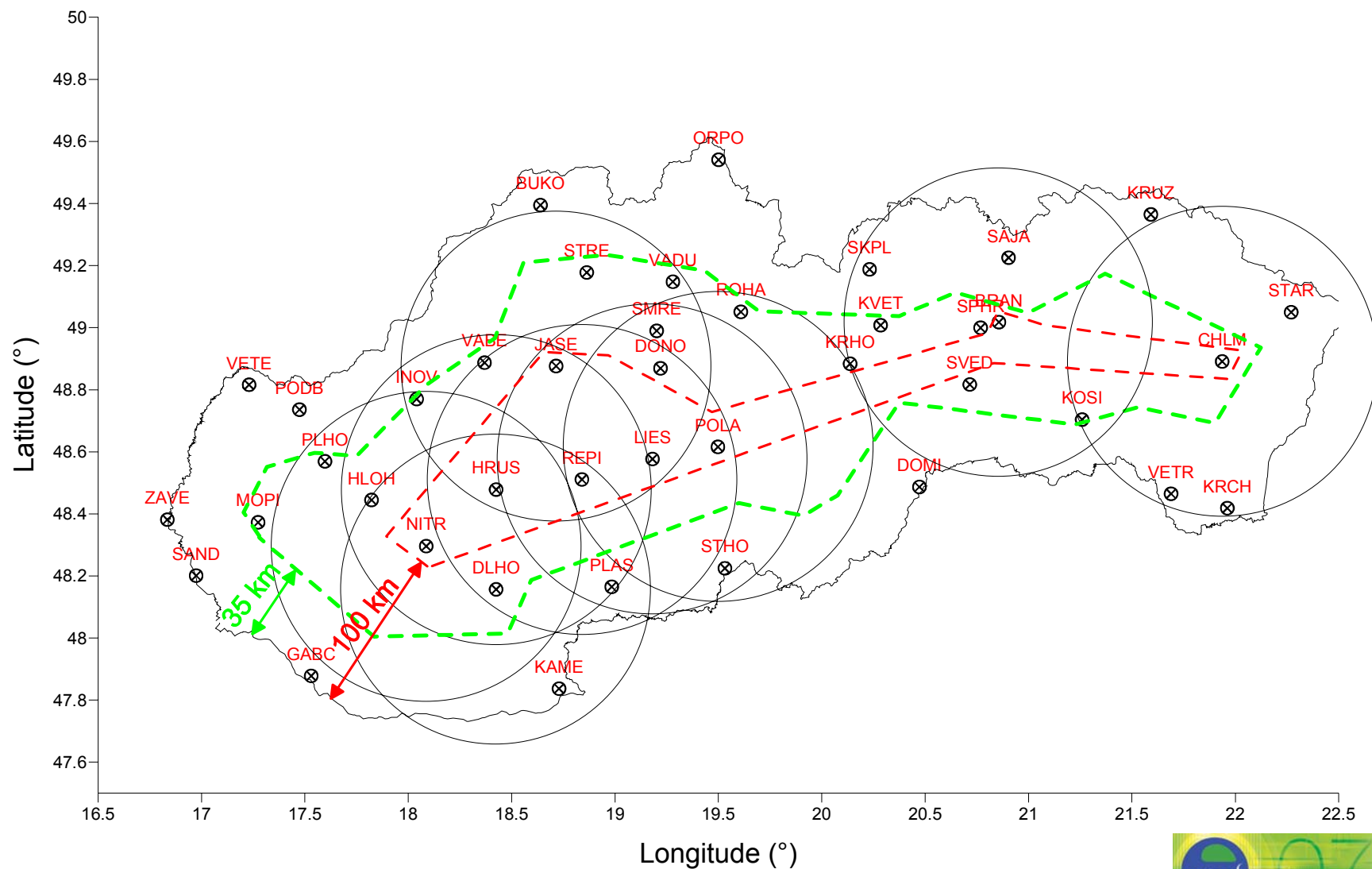




Determination of Preliminary Normal Heights

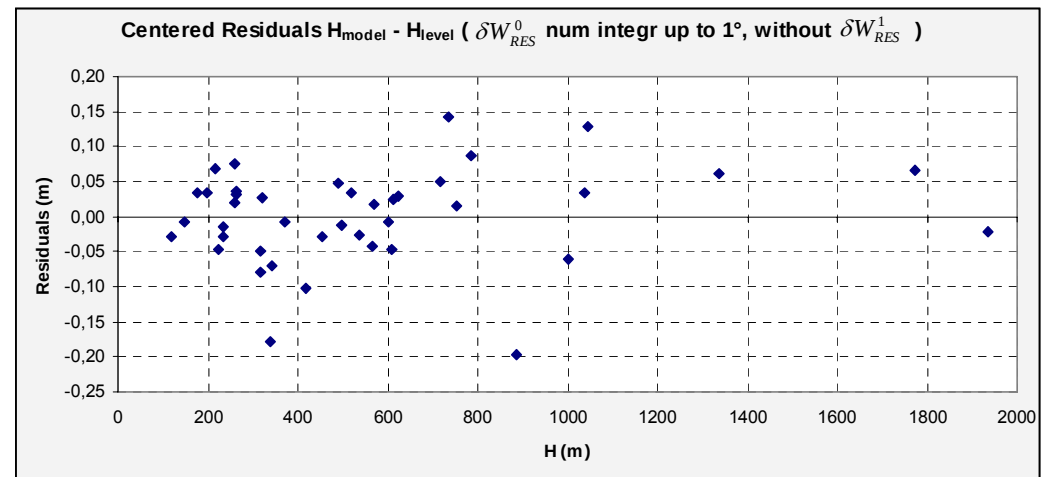
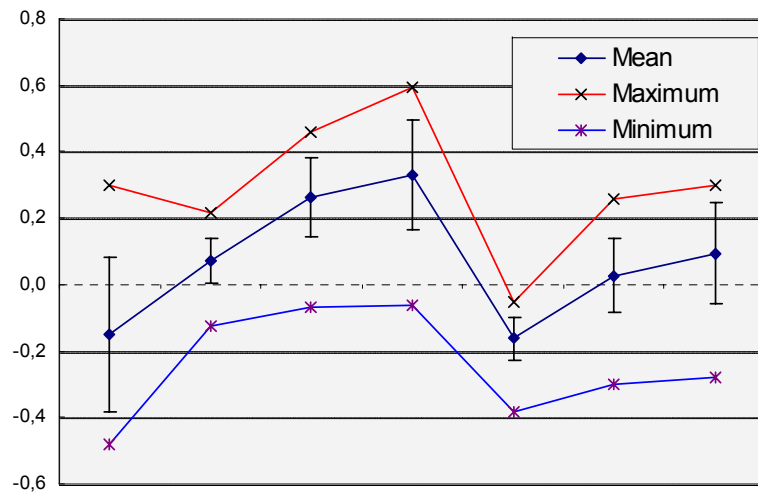
- 41 GPS points
- 24 GPS points, 35 km from abroad
- 8 GPS points, 100 km from abroad

Area of normal heights determination



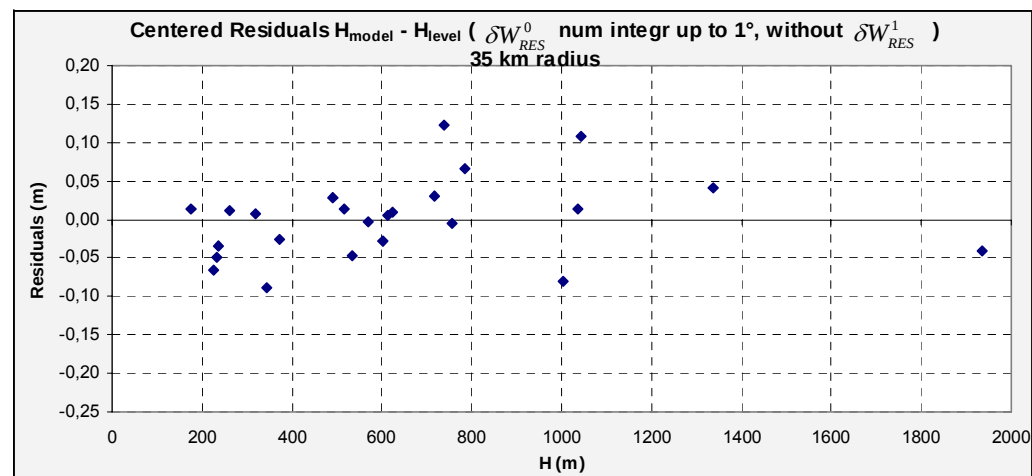
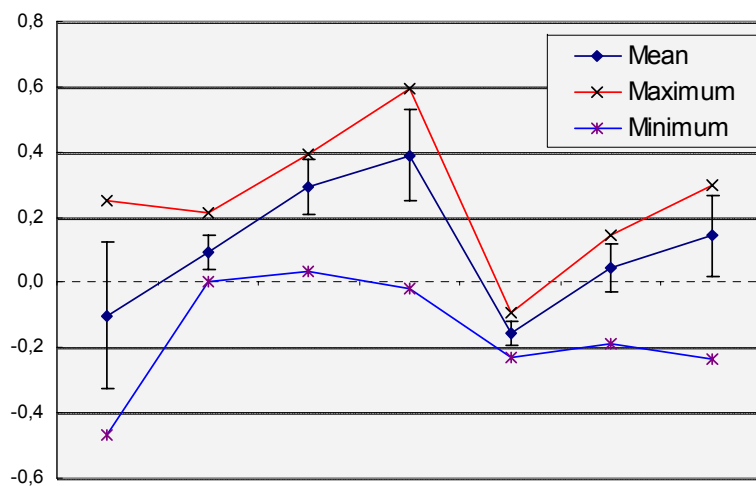
Model 41 GPS points

STAT	EGM96	only δW_{RES}^0			with δW_{RES}^1		
		1deg	2deg	3deg	1deg	2deg	3deg
MEAN	-0,151	0,072	0,262	0,329	-0,163	0,027	0,094
RANGE	0,780	0,339	0,531	0,656	0,329	0,554	0,579
ST.DEV.	0,233	0,068	0,118	0,167	0,064	0,113	0,153
MAXIMUM	0,301	0,214	0,461	0,594	-0,053	0,256	0,298
MINIMUM	-0,480	-0,125	-0,070	-0,062	-0,383	-0,298	-0,281



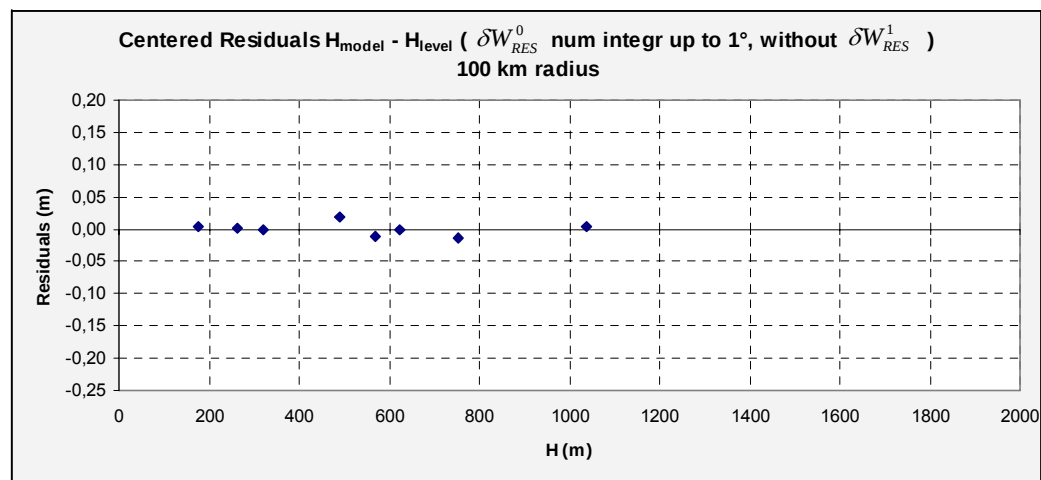
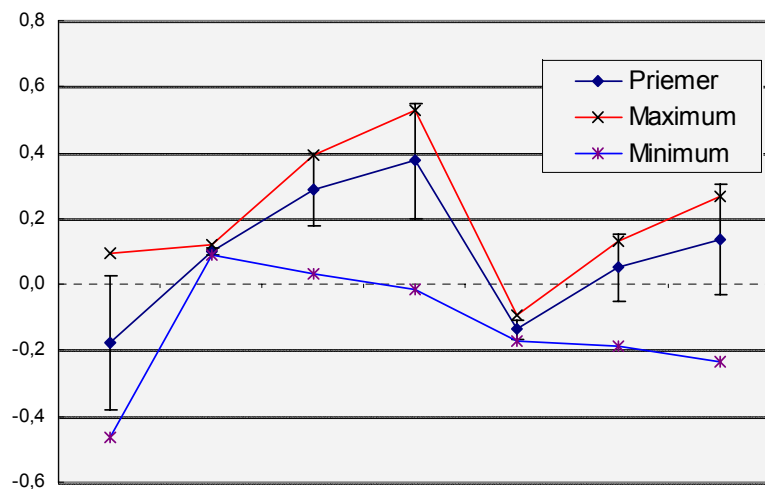
Model 24 GPS points

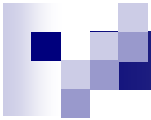
STAT	EGM96	only δW_{RES}^0			with δW_{RES}^1		
		1deg	2deg	3deg	1deg	2deg	3deg
MEAN	-0,103	0,092	0,293	0,390	-0,156	0,045	0,142
RANGE	0,717	0,211	0,362	0,611	0,140	0,331	0,534
ST.DEV.	0,225	0,052	0,087	0,138	0,037	0,073	0,123
MAXIMUM	0,251	0,214	0,394	0,594	-0,092	0,145	0,298
MINIMUM	-0,466	0,003	0,032	-0,017	-0,232	-0,187	-0,236



Model 8 GPS points

		only δW_{RES}^0			with δW_{RES}^1		
STAT	EGM96	1deg	2deg	STAT	EGM96	1deg	2deg
MEAN	-0,178	0,102	0,289	0,375	-0,136	0,051	0,137
RANGE	0,559	0,032	0,362	0,547	0,079	0,317	0,506
ST.DEV.	0,204	0,010	0,111	0,176	0,030	0,101	0,167
MAXIMUM	0,092	0,120	0,394	0,530	-0,092	0,130	0,270
MINIMUM	-0,466	0,087	0,032	-0,017	-0,171	-0,187	-0,236





Conclusion and plan for future

- Preliminary results of experiment are very optimistic
- Determined Normal Heights have a global character
- Unification of all input parameters and data are needed
- Use zero-tide reference model as geodetic standard
- Simultaneous measurement of position and gravity, perspective produce integrated instrument GNSS/Gravity
- Exchange gravity data with abroad countries, or Consortium for Gravity Data in EUREF Countries?

Thank you
for your attention

