Extension of the DFHRS approach for gravity observations and computation design for a 1cm fitted DFHRS of Europe

www.dfhbf.de

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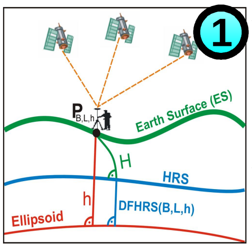
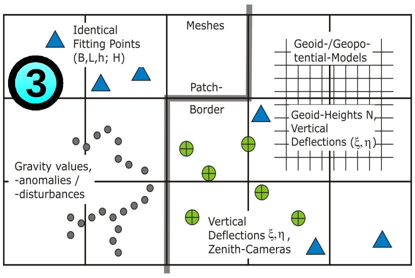
SUMMARY. The DFHRS (Digital-Finite-Element-Height-Reference-Surface) research and development project provides for GNSS positioning (GPS/GLONASS/GALILEO) a direct online conversion of ellipsoidal GNNS heights h, measured at the earth surface (ES), into standard "sea-level" heights H, referring to the height reference surface (HRS) of orthometric or normal height systems (fig. 1). The HRS is represented by a continuous polynomial surface as carrier function (fig. 4). **DFHRS_DB** are computed according to the adjustment approach fig. 4, which enables all relevant physical and geometrical observation types (fig. 3), while fitting simultaneously the HRS to the identical fitting points (B,L,h; H). DFHRS_DB provide the essential HRS parameters (p and Δm), and are offical database (DB) products in different countries and states (fig. 2, fig.9). The DFHRS_DB access and the **DFHRS**-correction **DFHRS**(p,**Δm**| B,L,h) (fig. 7) is provided by all GNSS/GPS-receiver manufacturers world-wide. So DFHRS_DB are a proved standard for precise GNSS heighting, e.g. for the ***SAPOS***® and **ascos**® GNSS services in Germany (fig. 2). Fig. 2, blue shows the German states with 1_cm DFHRS_DB, and blue&yellow togehter mean the continuous (1-3)_cm DFHRS_DB of Germany. Other (1-3)_cm DFHRS_DB were computed for sevaral states in Europe (fig. 8, fig. 9), Africa and USA. In 2004 the DFHRS-concept was applied for the evaluation of a "1_dm DFHRS for Europe (fig. 9).The extension of the DFHRS concept and software to physical observations (gravity, GPM, etc.) is based on Spherical Cap Harmonic Analysis (SCHA). Using SCHA the HRS is represented by (10) and set equal to the polynomial based DFHRS in (4), last formula.



DFHRS Computation: The DFHRS is modelled as a continuous HRS in arbitrary large areas by bivariate polynomials over an irregular grid of finite element meshes (fig. 3, 4). The accuracy of the DFHRS is controlled by the mesh size (fig. 3, 4, 8 (middle)). A size of about 5 km and a polynomial degree of 3 provide a HRS approximation quality of better than 3 mm. Geoid model heights N, deflections of the vertical, gravity anomalies Δg and identical points (B,L,h, H) are used as observations (fig. 3) in a least squares DFHRS data base (DB) computation with the observation equations fig. 4. In case of an adequate stochastical model the use of of gravity-based geoid-/geopotential model information is equivalent to that oforiginal gravity-anomalies or -disturbances Δg. Gravity observations Δg and deflections of the vertical (e.g. from zenith cameras) may be introduced however directly as original observations (fig. 4). Besides the polynomial parameters p a scale factor Δm (if significant) is set up in the DFHRS-correction DFHRS(p,Δm|B,L,h) and DFHRS_DB respectively (fig. 7) .



$$N(\overline{C}_{n(k),m}, \overline{S}_{n(k),m}) = \frac{1}{\gamma_Q}\left(\sum_{k=0}^{\infty}\sum_{m=0}^{k}\left(\frac{a}{r}\right)^{n(k)+1}(\overline{C}'_{n(k),m}\cdot\cos m\lambda' + \overline{S}'_{n(k),m}\cdot\sin m\lambda')\cdot P_{n(k),m}(\cos\theta') - \dot{V}_{ref,GRS80}\right) \quad (10)$$

$$h + v = \hat{H} + \mathbf{f}(B,L)^T\cdot\hat{\mathbf{p}} + h\cdot\Delta\hat{m}$$

$$0 + v = C(\hat{\mathbf{p}})$$

$$H + v = \hat{H}$$

$$N + v = \mathbf{f}(B,L)^T\cdot\hat{\mathbf{p}} + \partial N(\mathbf{d}^j_N)$$

$$\Leftrightarrow \quad N = \frac{a}{4\pi\gamma}\iint_\sigma \Delta g\cdot S(\psi)\cdot\psi\cdot d\psi$$

$$\xi + v = -\frac{\mathbf{f}_B(B,L)^T}{M(B)}\cdot\hat{\mathbf{p}} + \partial\xi(\mathbf{d}^j_{\xi,\eta})$$

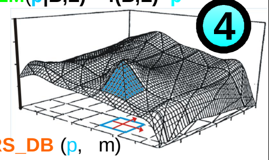
$$\eta + v = -\frac{\mathbf{f}_B(B,L)^T}{N(B)\cdot\cos(B)}\cdot\hat{\mathbf{p}} + \partial\eta(\mathbf{d}^j_{\xi,\eta})$$

$$N^j_{GPM} + v = N(\overline{C}_{n(k),m}, \overline{S}_{n(k),m}) + \partial N(\mathbf{d}^j)$$

$$g^{LGV}_{grav\ r} + v = \sum_{k=0}^{\infty}\sum_{m=0}^{k}\left(\frac{a}{r}\right)^{n(k)+1}\frac{(n(k)+1)}{r}\cdot(\overline{C}'_{n(k)),m}\cdot\cos m\lambda' + \overline{S}'_{n(k),m}\cdot\sin m\lambda')\cdot P_{n(k),m}(\cos\theta') + \partial g(\mathbf{d})$$

$$0 + v_{\Delta N} = N(\overline{C}'_{n(k),m}, \overline{S}'_{n(k),m}, V_{GRS80}) - (\mathbf{f}(x,y)^T\cdot\mathbf{p} + \Delta m\cdot h)$$

NFEM(p|B,L) = f(B,L)^T p

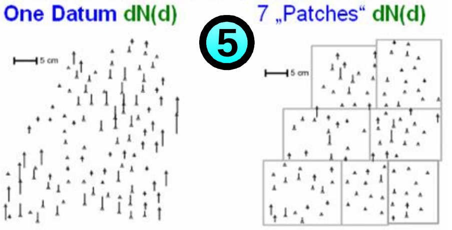


DFHRS_DB (p, m)

Example Baden-Württemberg (250 km x 300 km)

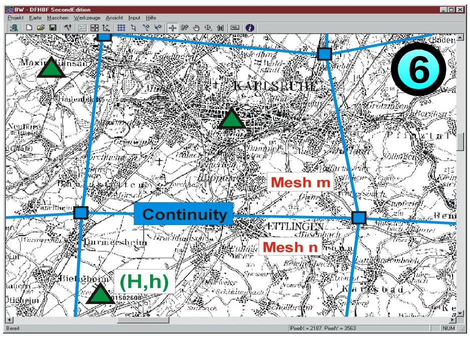
Residuals of Adjustment with EGG97 Geoid

One Datum dN(d) — **7 „Patches" dN(d)**



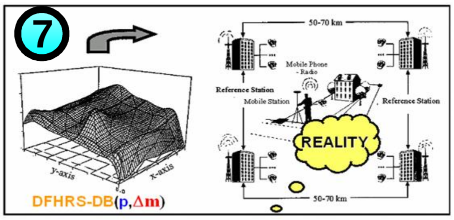
Patching Technique. Geoid-/Gepotential models may be parted into different "Geoid-Patches" with individual datum-parameters d and a respective datum parametrization part dN(d) for the geoid-model heights, as well as for vertical deflections observations (fig. 4). So the effect of long-waved systematic errors (fig. 5, left) of non-fitted geoid-/gepotential models is reduced (fig. 5, right). "Patches" see also thick blue lines in fig. 8 middle (screenshot, DFHRS-computation for Latvia).

DFHRS Production Software. The DFHRS Production Software (screenshots fig. 4, fig.8 (middle)) developed in C++ comprises several functions of visualisation and utilities for automatic and manual meshing. It provides the powerful least-squares adjustment according to the observation equations fig.4, including gross error detection and VC estimation for observation groups. In the final computation step the DFHRS_DB (p,Δm) (fig. 4, fig. 7) is set up.



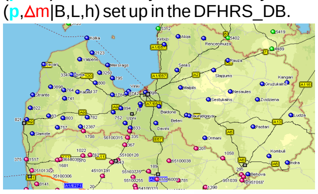
Quality Measures for the DFHRS_DB

One way to check the external accuracy of the DFHRS data base parameters (p,Δm) is to compute successively the "DFHRS-height" H(B,L,h,DFHRS) by using an individual data base DFHRS$_i$, where H$_i$ was excluded from the DB production. The so-called reproduction quality ∇H$_i$ is then simply given by the value of the difference ∇H$_i$ = H$_i$ - H$_i$ (DFHRS (p,Δm|B,L,h)), which is computed by DFHRS Production Software in the step of the DFHRS production step, where all points H$_i$ are used. Based on the adjustement concept (fig. 4) the so-called accuracy surface can be computed using the covariance-matrix of the DFHRS_DB parameters (p,Δm). In the way the accuracy surface shows the country-wide accuracy the DFHRS-correctionDFHRS (p,Δm|B,L,h) set up in the DFHRS_DB.

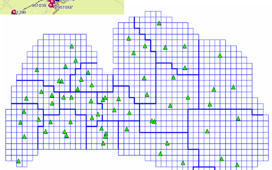


H = h - DHRS(p,Δm|B,L,h)
H = h - (NFEM(p|B,L) + Δm·h)

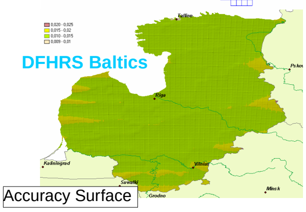
DFHRS-APPLICATION. While e.g. classical geoid models suffer from long-waved systematic errors (fig. 5, left), the parameters (p,Δm) of the DFHRS_DB are fitted to the identical points (B,L,h;H). So the so-called **DFHRS correction** reduces by **H = h - DFHRS(p,Δm|B,L,h)** the ellipsoidal GNSS height h directly into standard height H (fig. 1, 7). The correction DFHRS (p,Δm|B,L,h) consists of the FEM surface (fig. 4) of the HRS ("geoid-part") **NFEM**(p|B,L) and (if significant) a "scale part" Δm* h . DFHRS_DB access is provided by nearly all GNSS hard- and software manufacturers (see www.dfhbf.de and www.ib-seiler.de). DFHRS_DB can also be set up as RTCM_3.0 message.



Master-Thesis Lauma Lace: Computation of a Precise Height Reference Surface for Latvia and the Baltic States using the DFHRS concept and software.

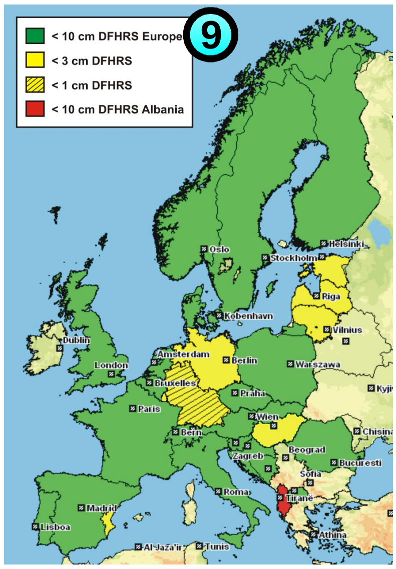


DFHRS Latvia



DFHRS Baltics

Accuracy Surface



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