Leveling adjustment and land uplift

Calculation of a land uplift model

- **Observations**
- **The method**
- **Results**

Riga

June 2006

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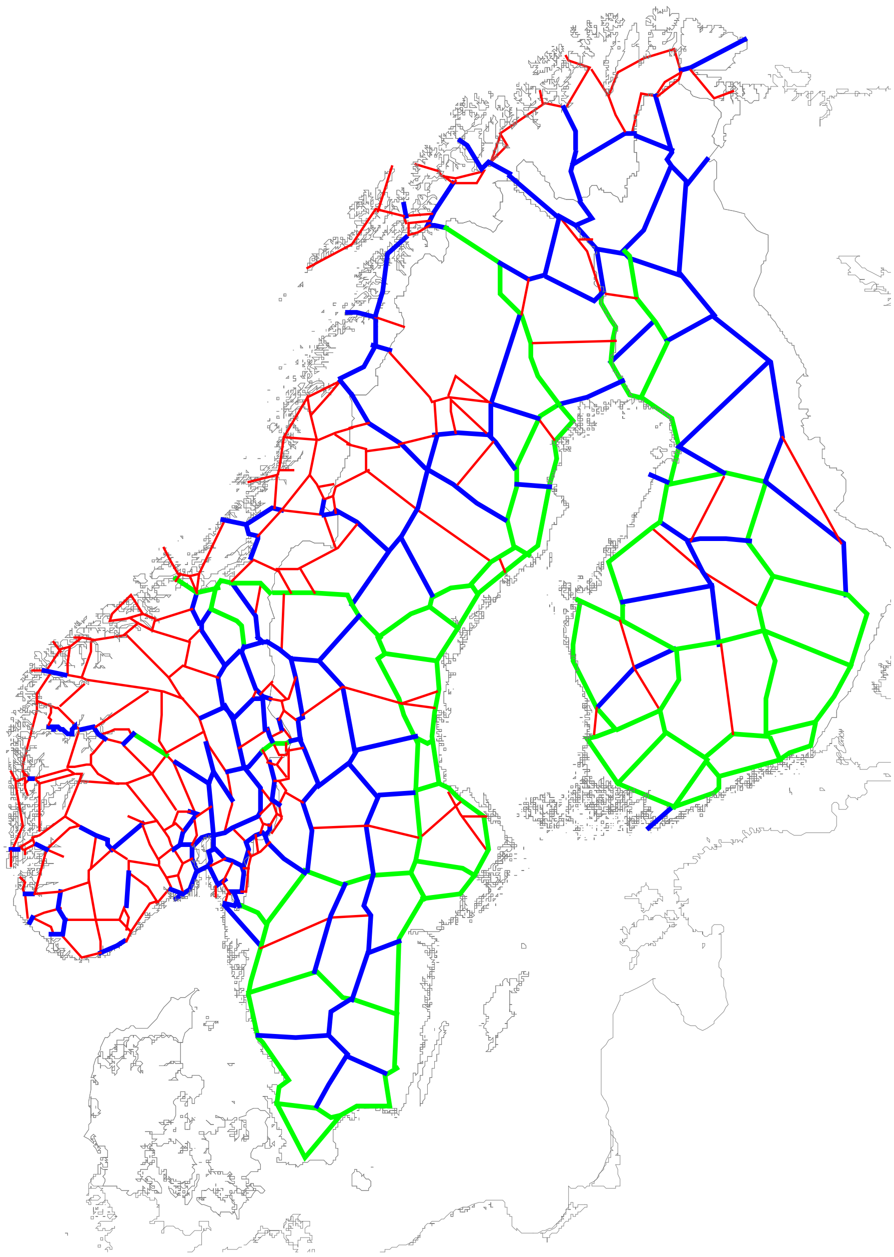
Observations

- **Leveling observations**
  - *From Norway, Sweden and Finland*
  - *Geopotential differences between nodal points*
  - *From 1890 - 2003*
- **Tide gauges**
  - *58 stations*
  - *Martin Ekman's values, published in 1996.*
- **cGPS-rates**
  - *55 stations in BIFROST*
  - *Results from Martin Lidberg's Licentiate Thesis*

Leveling network



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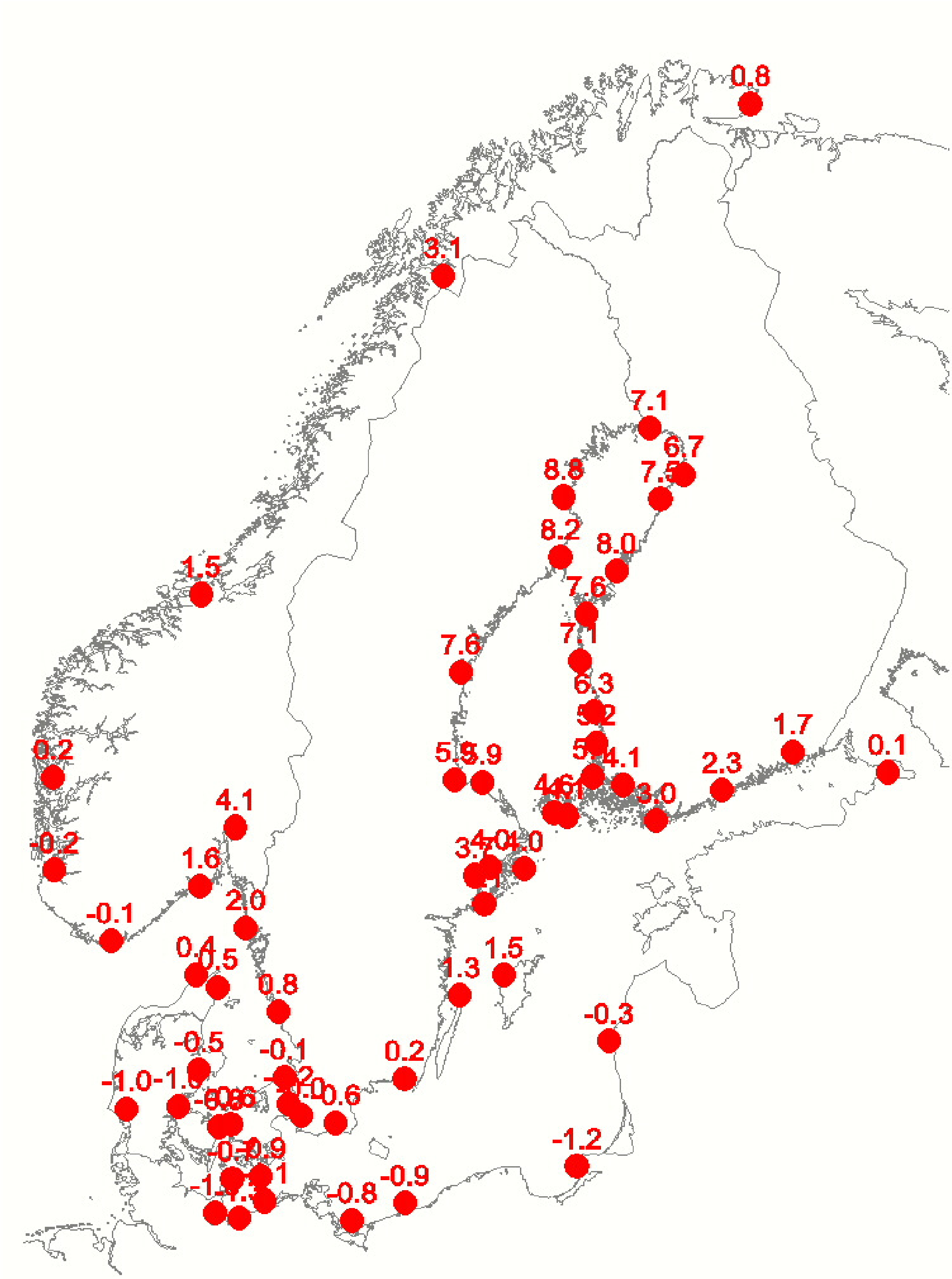
Tide gauges



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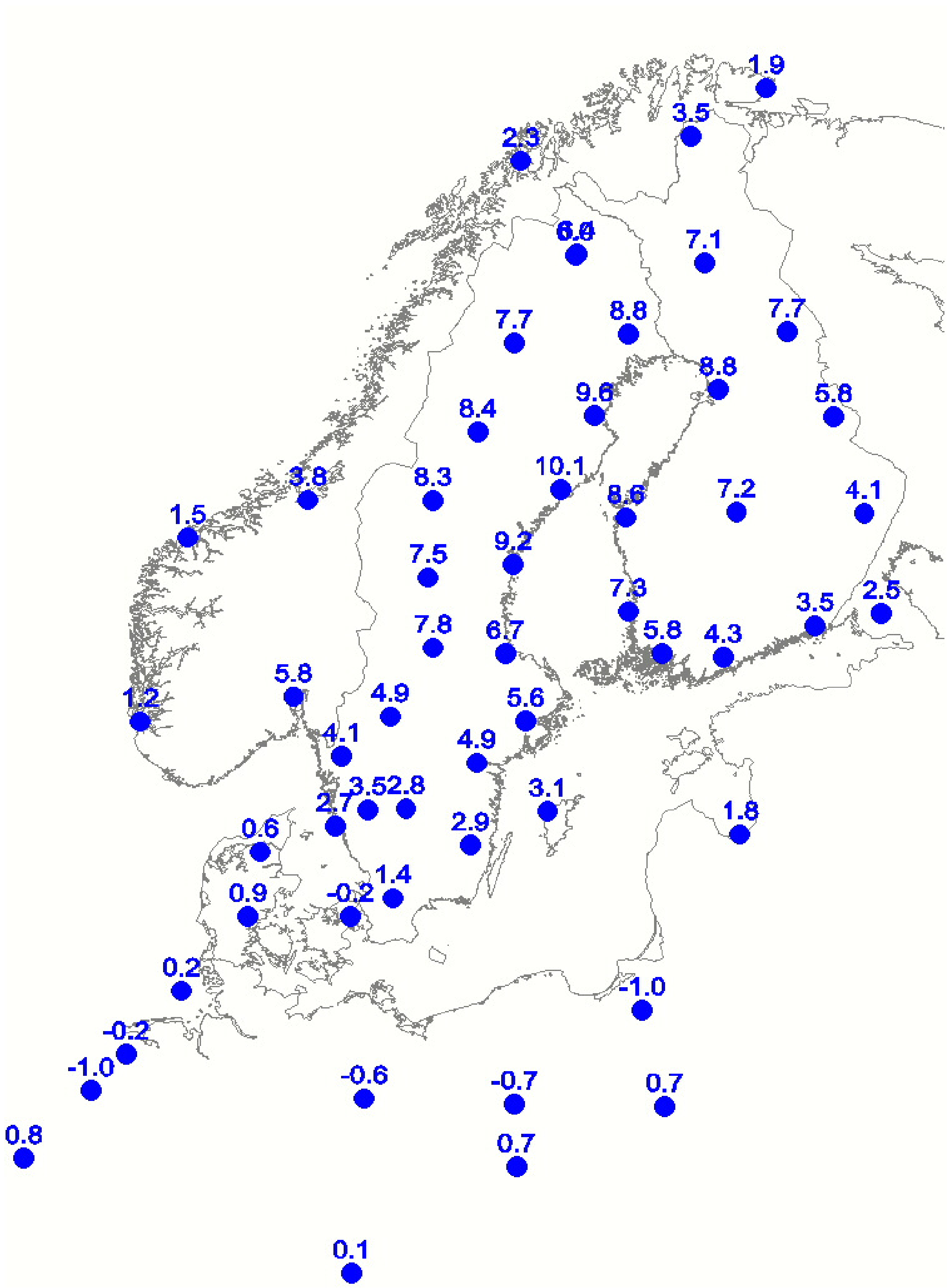
Permanent cGPS-stations



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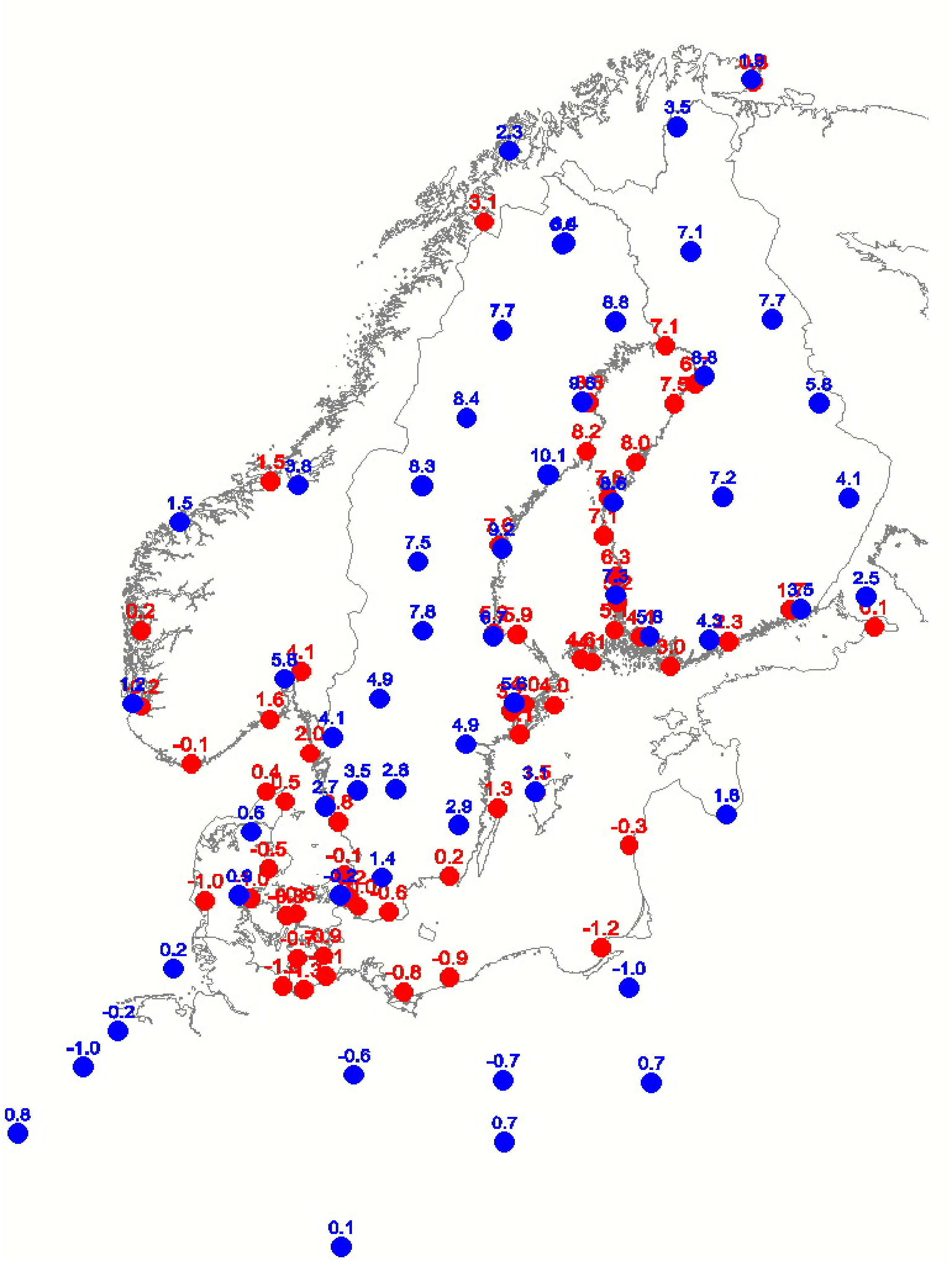
cGPS and tide gauges



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The data
A summery

- **Leveling**
  - ***Relativ land uplift values between nodal points.***
  - ***Some points are measured one time only.***
  - ***Influenced by the rise of the geoid***

- **Tide gauges**
  - ***Apparent land uplift values. Values relative to a rising mean see level.***
  - ***Influenced by the rise of the geoid***

- **cGPS-stations**
  - ***'Absolute' values observed in a geodetic frame work we assume is stable.***
  - ***Not influenced by the rise of the geoid***

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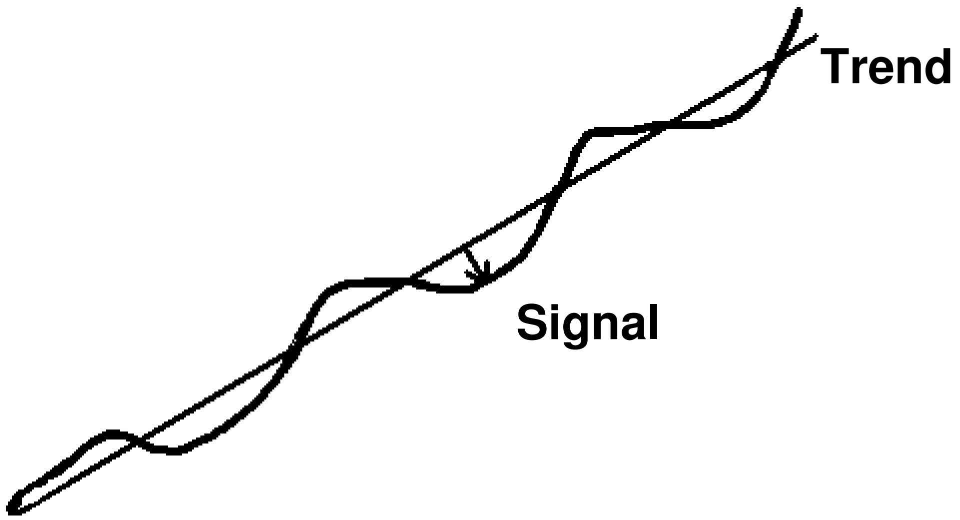
How to model the land uplift



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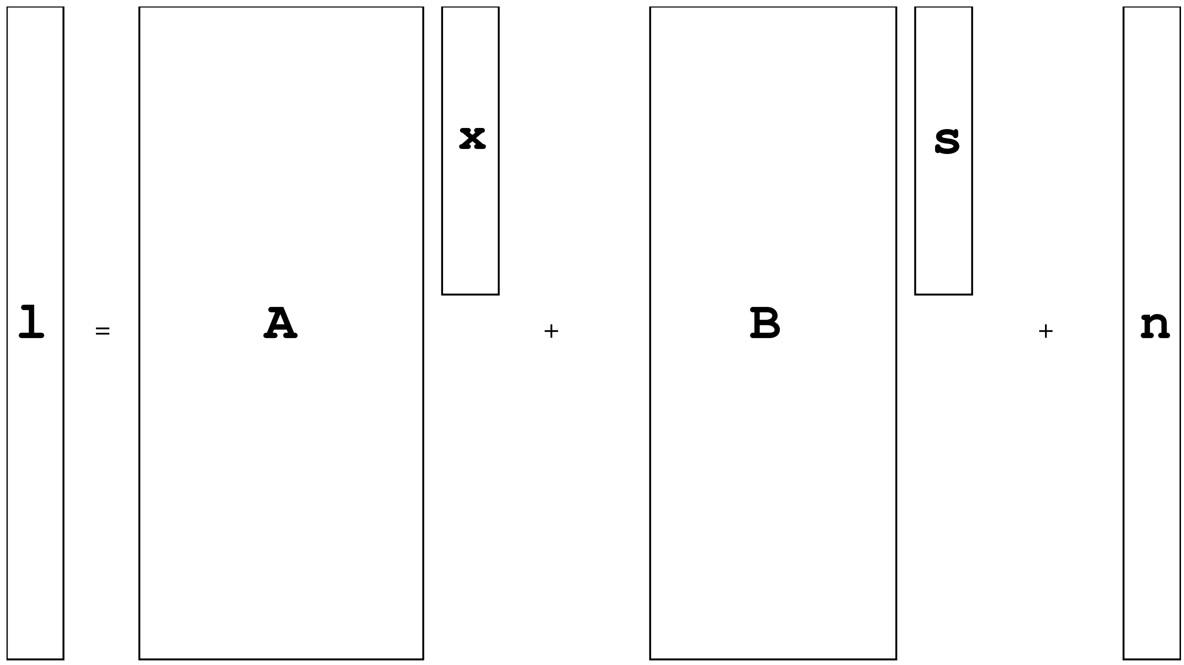
The model



Where	**l**	=	**Observations**
	A	=	**Design matrix**
	X	=	**Unknown heights and trend coefficients**
	B	=	**Design matrix for the signals**
	s	=	**Signals (unknown land uplift.)**
	n	=	**Noise**

Observation equations

For leveling

$-1\mathbf{h_a} + 1\mathbf{h_b} + t_i\{(x_b-x_a)\mathbf{b} + (y_b-y_a)\mathbf{c} + (x_by_b-x_ay_a)\mathbf{d} + (x_b^2-x_a^2)\ \mathbf{e} + \ldots\} \quad + t_i\mathbf{s_b} - t_i\mathbf{s_a} \quad + n_i$

For tide gauges

$l_i = \quad 1\mathbf{a} + x\mathbf{b} + y\mathbf{c} + xy\mathbf{d} + x^2\mathbf{e} + \ldots \quad + 1\mathbf{s_i} \quad + n_i$

For GPS-stations

$l_i = \quad 1\mathbf{a} + x\mathbf{b} + y\mathbf{c} + xy\mathbf{d} + x^2\mathbf{e} + \ldots + 1\mathbf{GPS_{const}} + u_{appr}\mathbf{GPS_{scale}} + 1\mathbf{s_i} \quad + n_i$

Where

h_a	=	the height of the start point of the leveling line
h_b	=	the height of the end point of the leveling line
a – e	=	coefficients on the trend surface. Only the five first are listed here.
GPS_{const}	=	unknown constant in the GPS-rates
$\mathbf{GPS_{scale}}$	=	unknown scale factor in the GPS-rates
$\mathbf{s_a}$	=	signal in the start point of the leveling line
$\mathbf{s_b}$	=	signal in the end point of the leveling line
$\mathbf{s_i}$	=	signal in the station point
$x,\ y$	=	coordinates of the points involved
t_i	=	reference year – observation year
u_{appr}	=	Approximated uplift (= the observed GPS-rate)
n_i	=	Noise
l_i	=	An observation

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The ordinary solution

$$\hat{\mathbf{x}} = \left(\mathbf{A}^{T}\mathbf{C}_{xx}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{T}\mathbf{C}_{xx}^{-1}\mathbf{l}$$

$$\hat{\mathbf{s}} = \mathbf{C}_{ss}\mathbf{B}^{T}\mathbf{C}_{xx}^{-1}\left(\mathbf{l} - \mathbf{A}\hat{\mathbf{x}}\right)$$

Where:

$$\mathbf{C}_{xx} = \mathbf{C}_{ll} + \mathbf{B}\mathbf{C}_{ss}\mathbf{B}^{T}$$

Schwarz method

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{12} & N_{22}+C_{ss}^{-1} \end{bmatrix} \begin{bmatrix} X \\ S \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Where

- **X** - Unknown heights and trend surface
- **S** - Unknown land uplift
- N_{11} - Normal equation matrix for X
- N_{12} - Normal equation matrix for S
- C_{ss}^{-1} - Inverse Co-variance matrix

Variance component estimation

- **Four types of input to the normal equations**
  - *Leveling*
  - *Tide gauges*
  - *GPS-rates*
  - *Inverse co-variance matrix (artificial observations)*

- **The leveling observations are separated into eight groups**
  - *New and old Norwegian*
  - *1. 2. and 3. Swedish leveling*
  - *1. 2. and 3. Finnish leveling*

- **For each group we calculate:**
  - *Sum of squares* $\mathbf{n}_i{}^T \mathbf{C}_i{}^{-1} \mathbf{n}_i$
  - *Redundancy* $\mathbf{r}_i$
  - *Variance component* $\hat{\sigma}_{0i}{}^2 = \dfrac{\mathbf{n}_i{}^T \mathbf{C}_i{}^{-1} \mathbf{n}_i}{\mathbf{r}_i}$

Results of the variance component estimation

Calculated standard deviations

- Leveling data:
  - Old Norway: 1.3 mm/km
  - New Norway: 1.1 mm/km
  - Finland 1.: 1.1 mm/km
  - Finland 2.: 0.9 mm/km
  - Finland 3.: 0.8 mm/km
  - Swedish 1.: 2.0 mm/km
  - Swedish 2.: 1.4 mm/km
  - Swedish 3.: 1.1 mm/km
- GPS-rates (average): 0.5 mm/yr
- Tide gauges (average): 0.1mm/yr

Results
of the test of outliers I

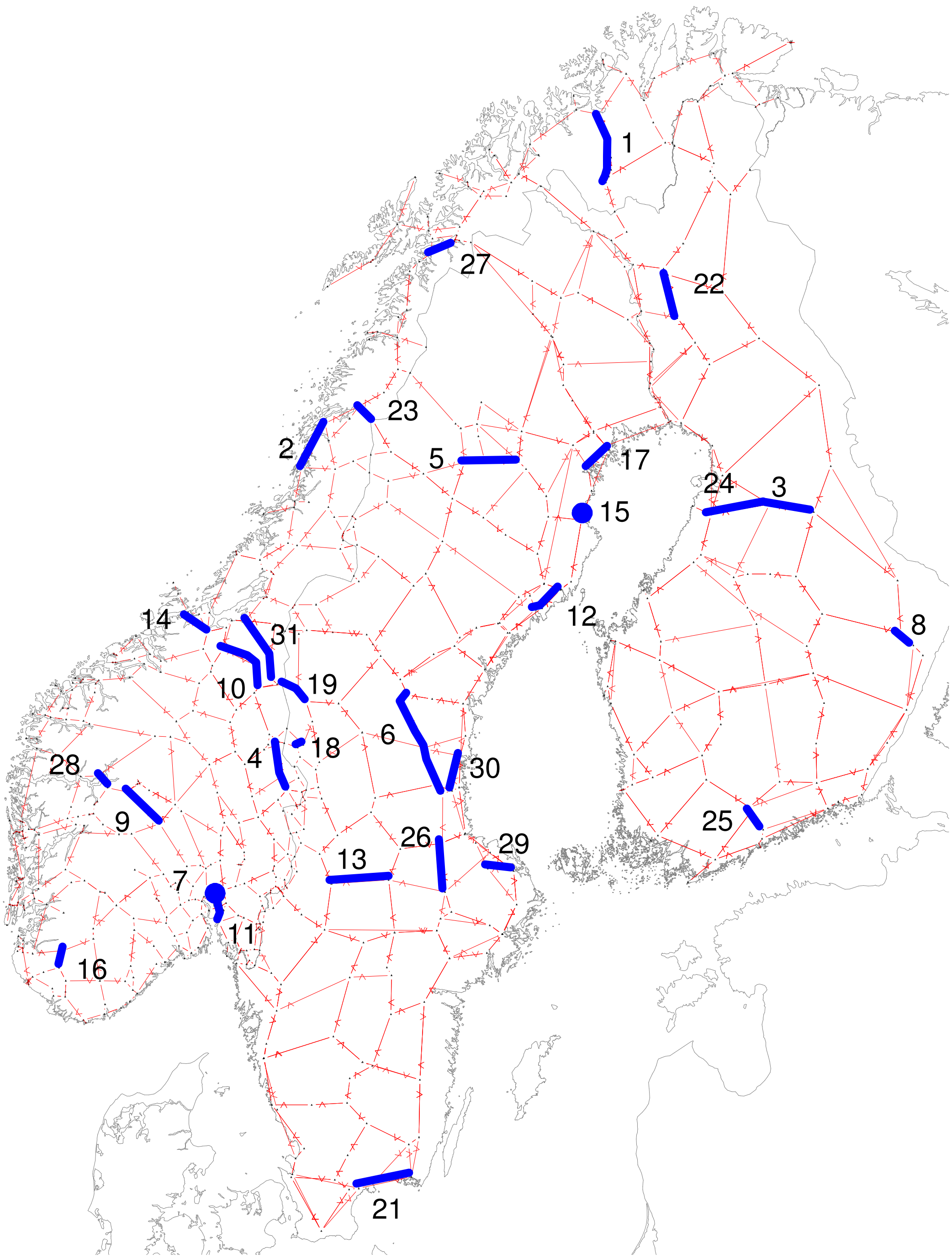
Rejected outliers

- Norway
  - Old: 7
  - New: 6
- Sweden
  - 1. Leveling: 6
  - 2. Leveling: 2
  - 3. Leveling: 3
- Finland
  - 1. Leveling: 2
  - 2. Leveling: 1
  - 3. Leveling: 2

- Tide gauges
  - Oslo 0.9 mm/yr
  - Furuøgrund 0.9 mm/yr

- No GPS-rates are rejected

Results
of the test of outliers II



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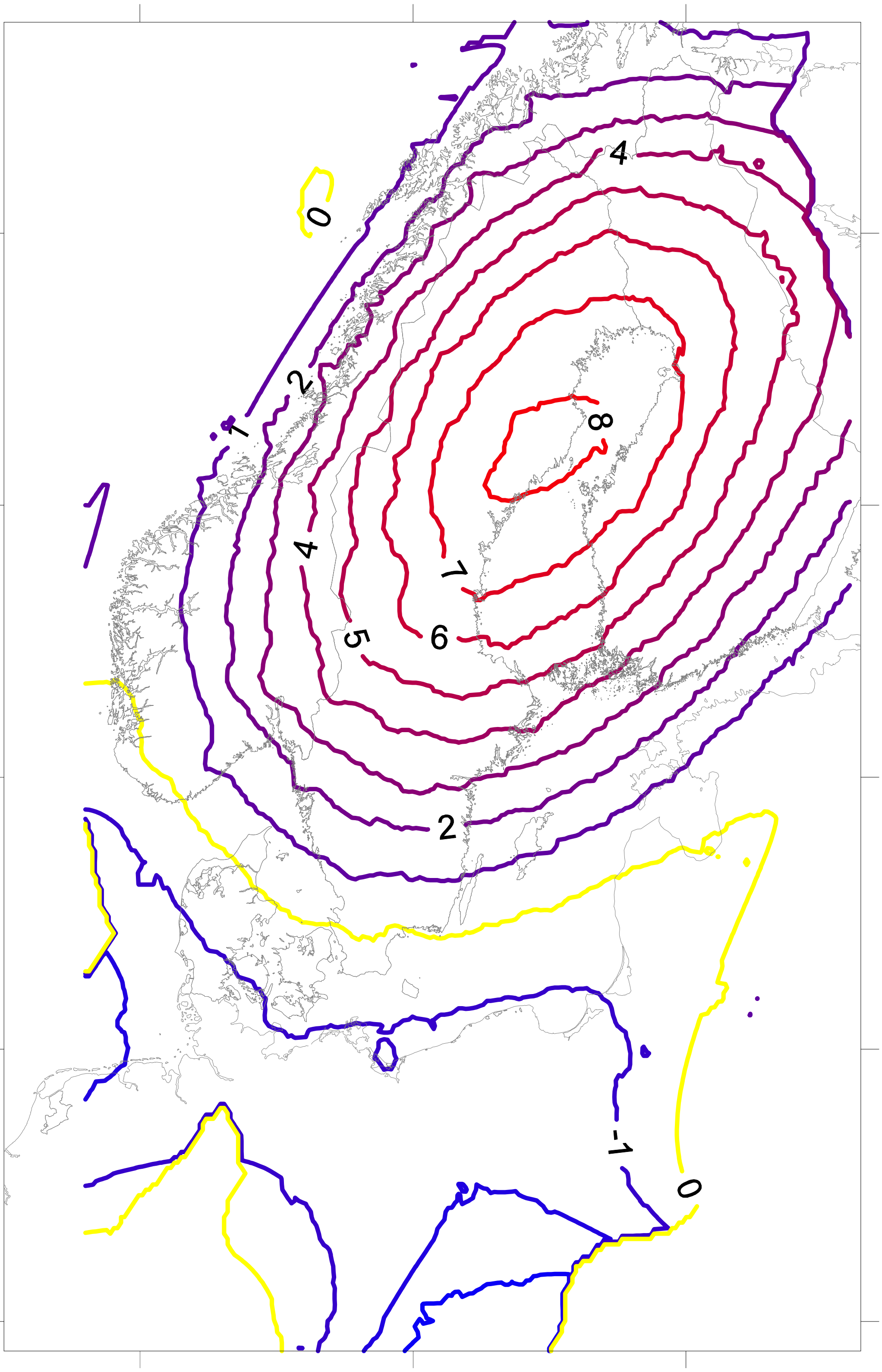
Land uplift



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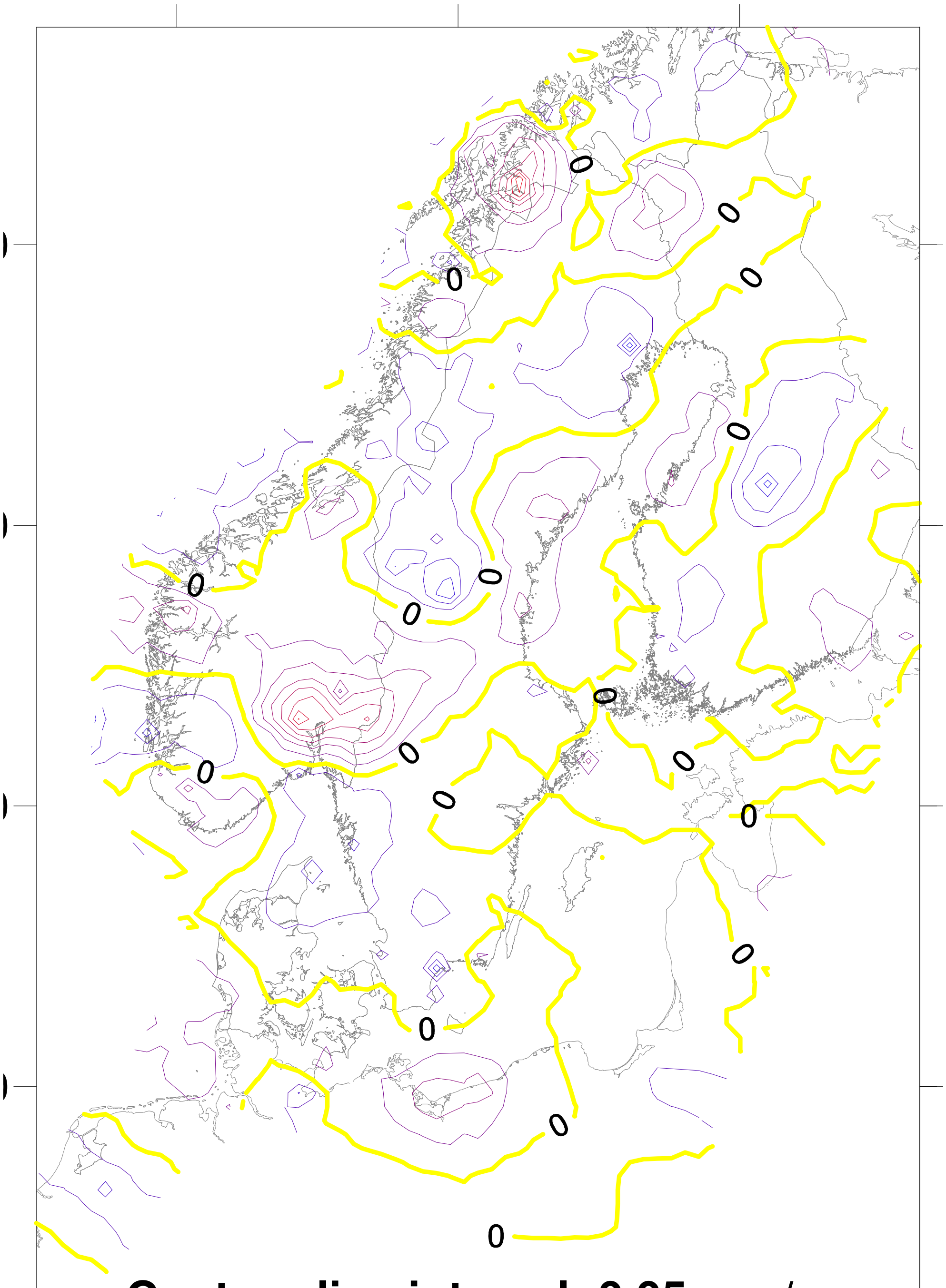
Land uplift signals



Contour line interval: 0.05 mm/yr

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The fit of the GPS-rates

- Constant and scale
  - -1,32 mm/yr +/- 0.14 mm/yr
  - 5.7% of the absolute land uplift +/- 2.3%

- Residuals (mm/yr)
  - Five larges
    - WROC (Poland) 1,54
    - BUDP (Denmark) -1,24
    - OSLS (Norway) 1,22
    - LAMA (Poland) -1.05
    - TRDS (Norway) -0.99
  - In total
    - RMS 0,44
    - Mean -0,01

- Outlier test
  - No station rejected
  - Mar6 (Sweden) suspect
    - T-value 2,7
    - Outlier value -0,78 mm/yr

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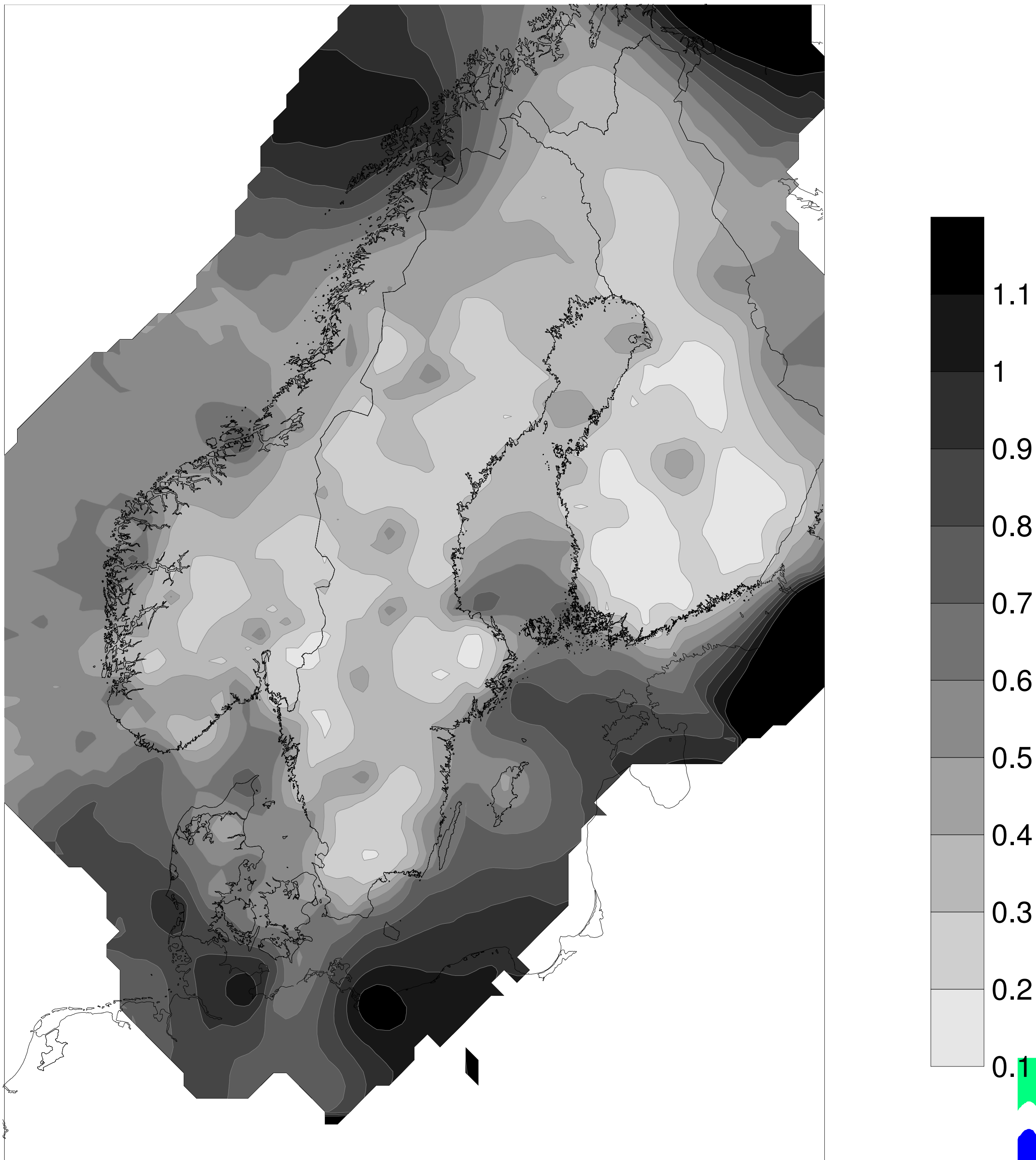
Reliability of the land uplift model



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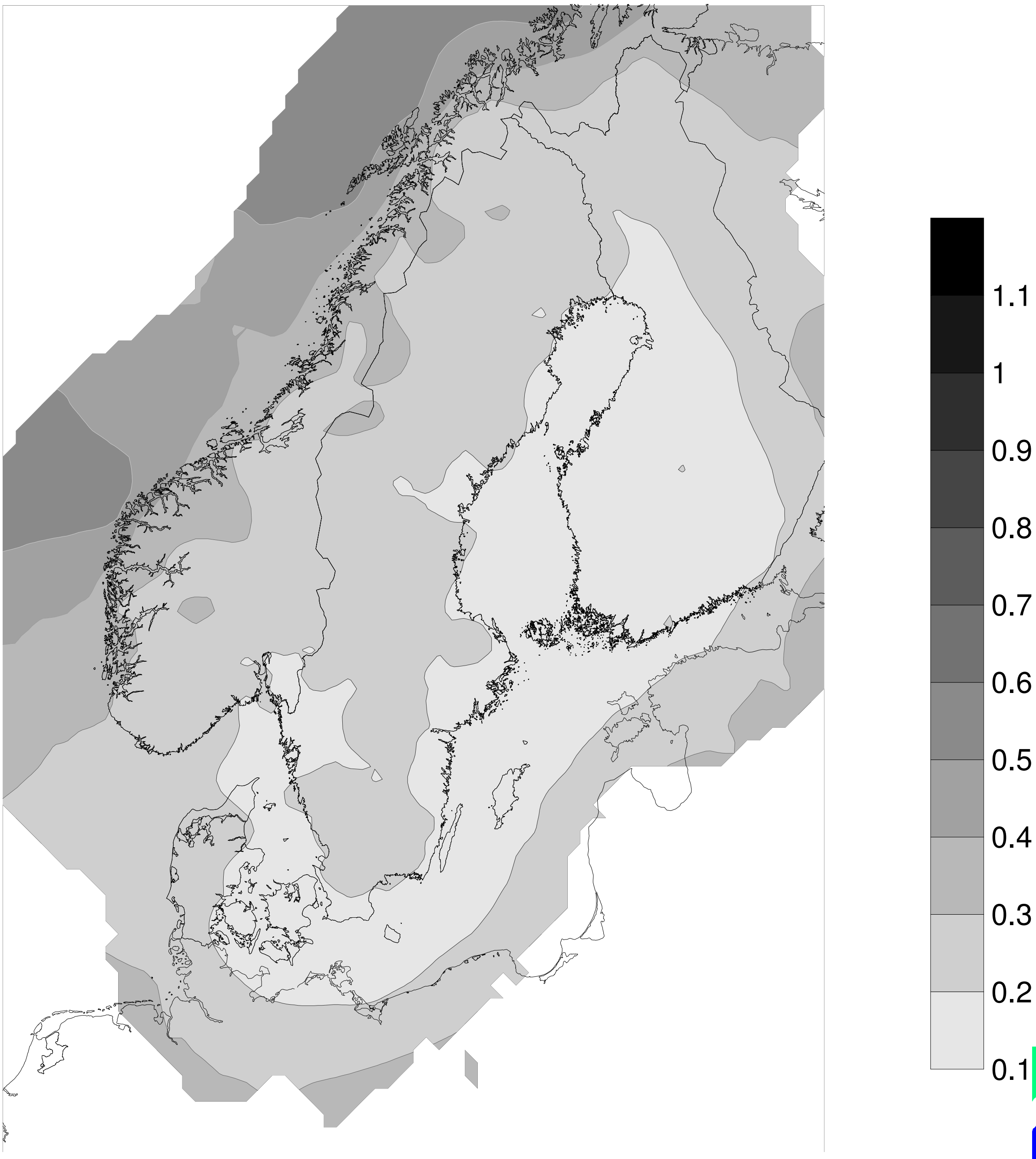
Standard deviations on the uplift values



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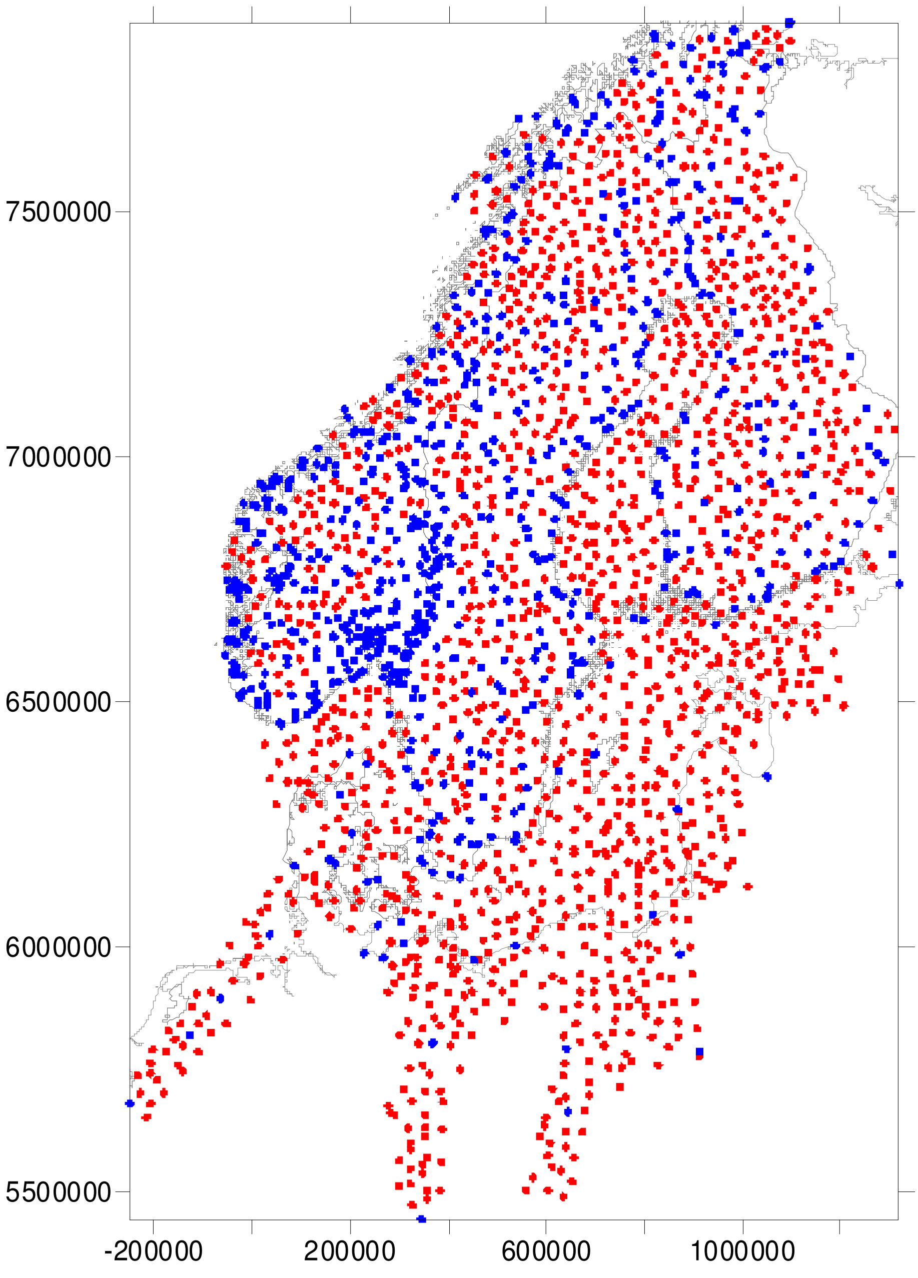
Gridding of the model



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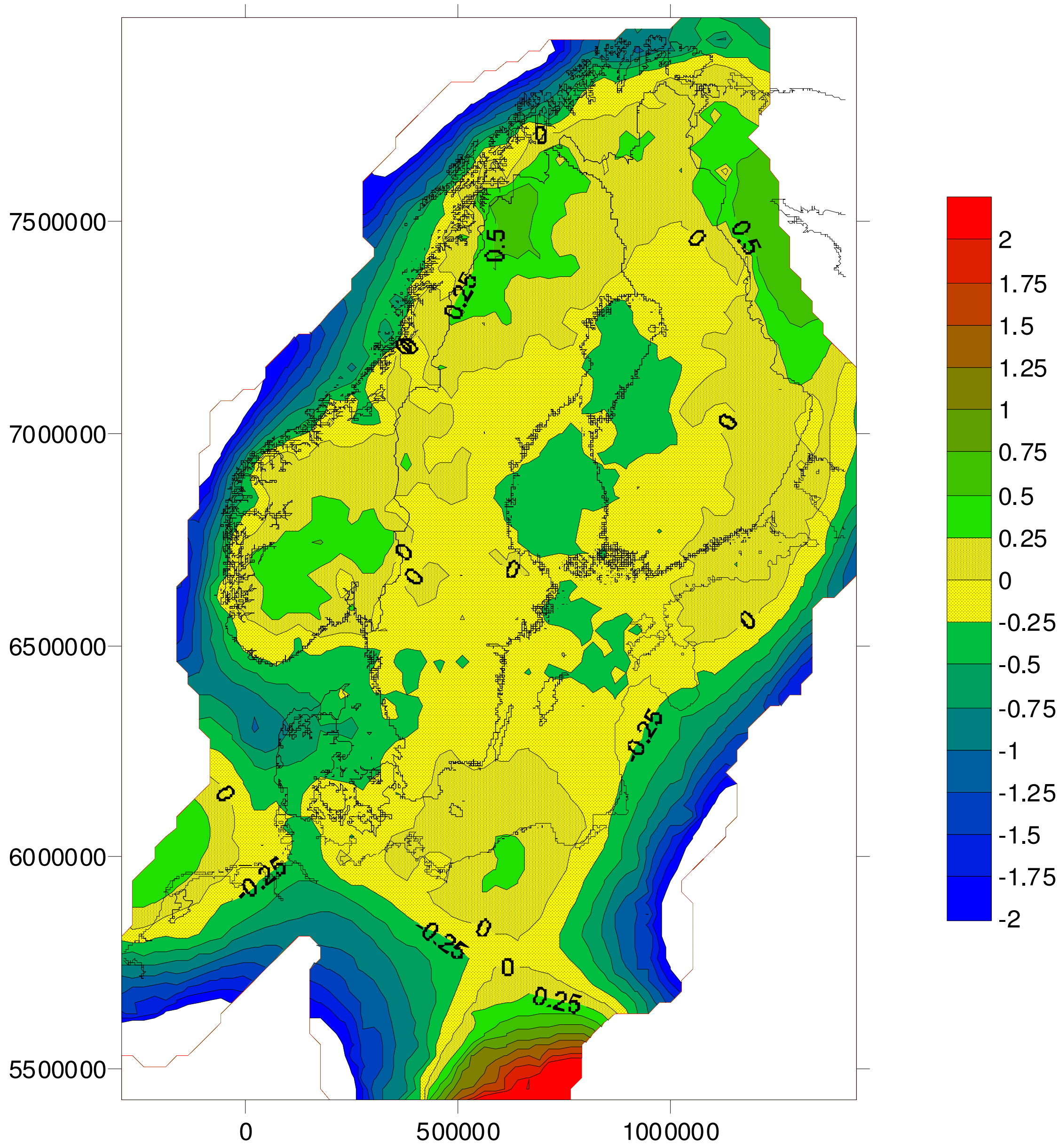
Names

- *This model is referred to as 'VESTOL-model'*
- *The modified version, used in the new Swedish height system and in the 'Baltic leveling ring' is called RH 2000 LU*
- *The 'official' name of RH 2000 LU is* **NKG2005LU**

NKG2005LU-VESTOL



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Thank you for the attention!

An article will be published in next number of Journal of Geodesy:

"DETERMINATION OF POSTGLACIAL LAND UPLIFT

IN FENNOSCANDIA FROM LEVELING, TIDE-GAUGES

AND CONTINUOUS GPS STATIONS

USING LEAST SQUARES COLLOCATION"

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