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Interpolation of the European velocity field using least squares collocation method

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Outline

- Stacking of EPN combined time series of station positions
- Euler Pole Estimation
- Interpolation: Least squares collocation method
- Conclusion

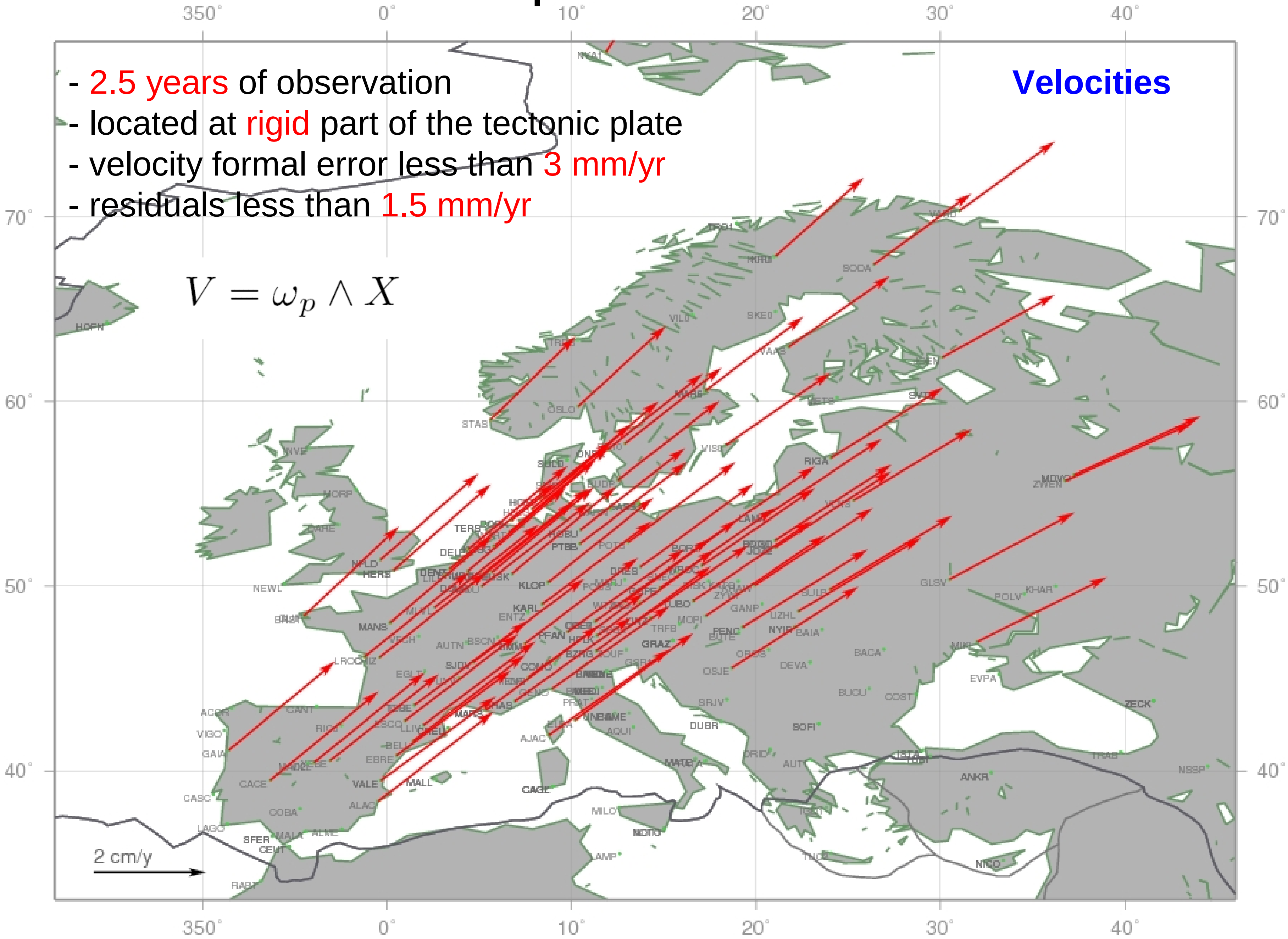
European Velocity Field Estimation

- EPN weekly solution from 1996 to 2006
- Stacked with CATREF Software [Altamimi]
  - before december 2004 : Remove original constraints and apply minimum constraints
  - after december 2004 : Use as they are minimally constrained solutions
  - Reject outliers and properly handle discontinuities
  - Combination ==> Global solution (Pos&Vel)
- Datum definition : MC over a reference set of 16 EPN stations (ITRF2005P)

Euler pole estimation

- 2.5 years of observation
- located at rigid part of the tectonic plate
- velocity formal error less than 3 mm/yr
- residuals less than 1.5 mm/yr

$$V = \omega_p \wedge X$$



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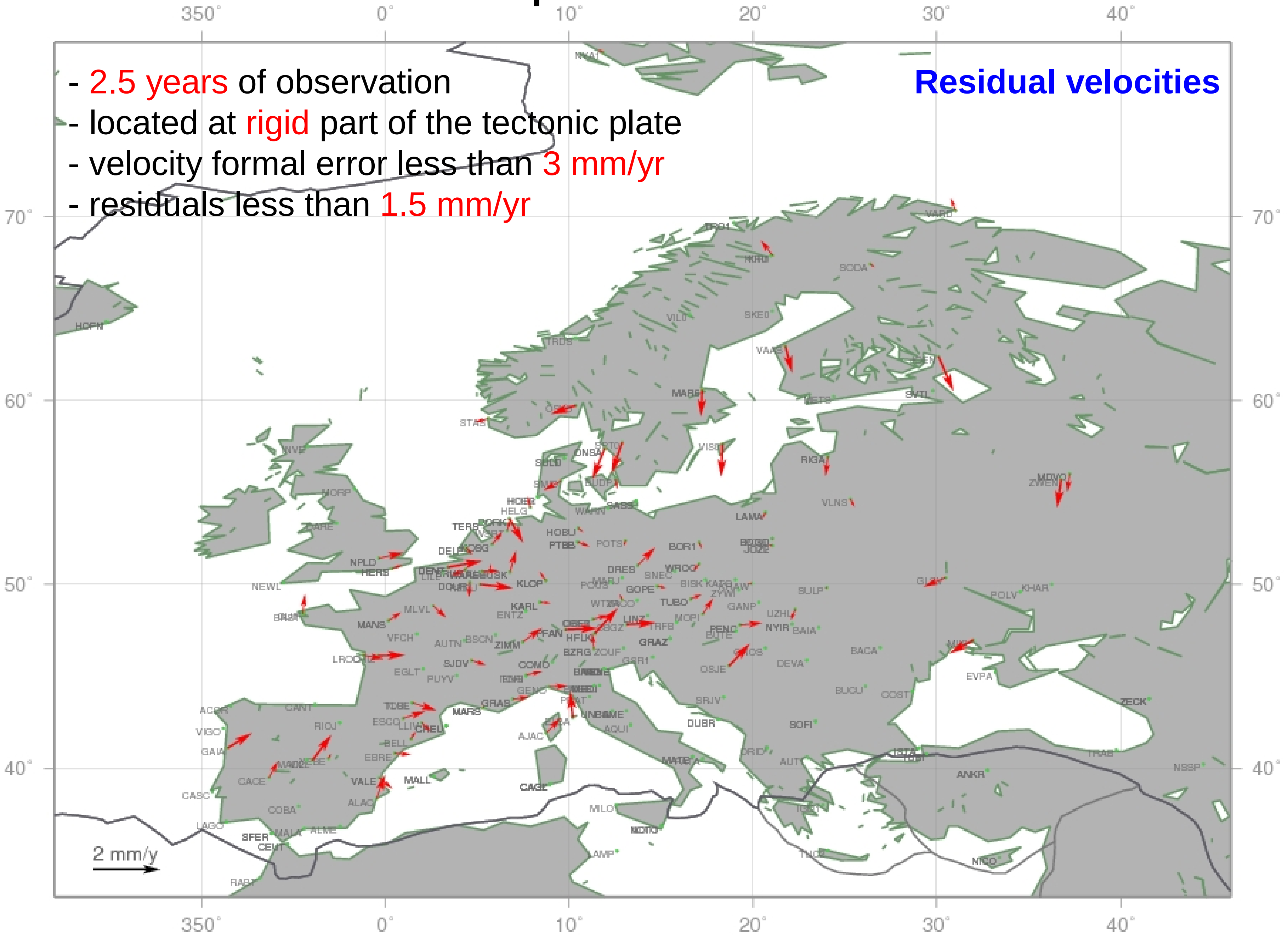
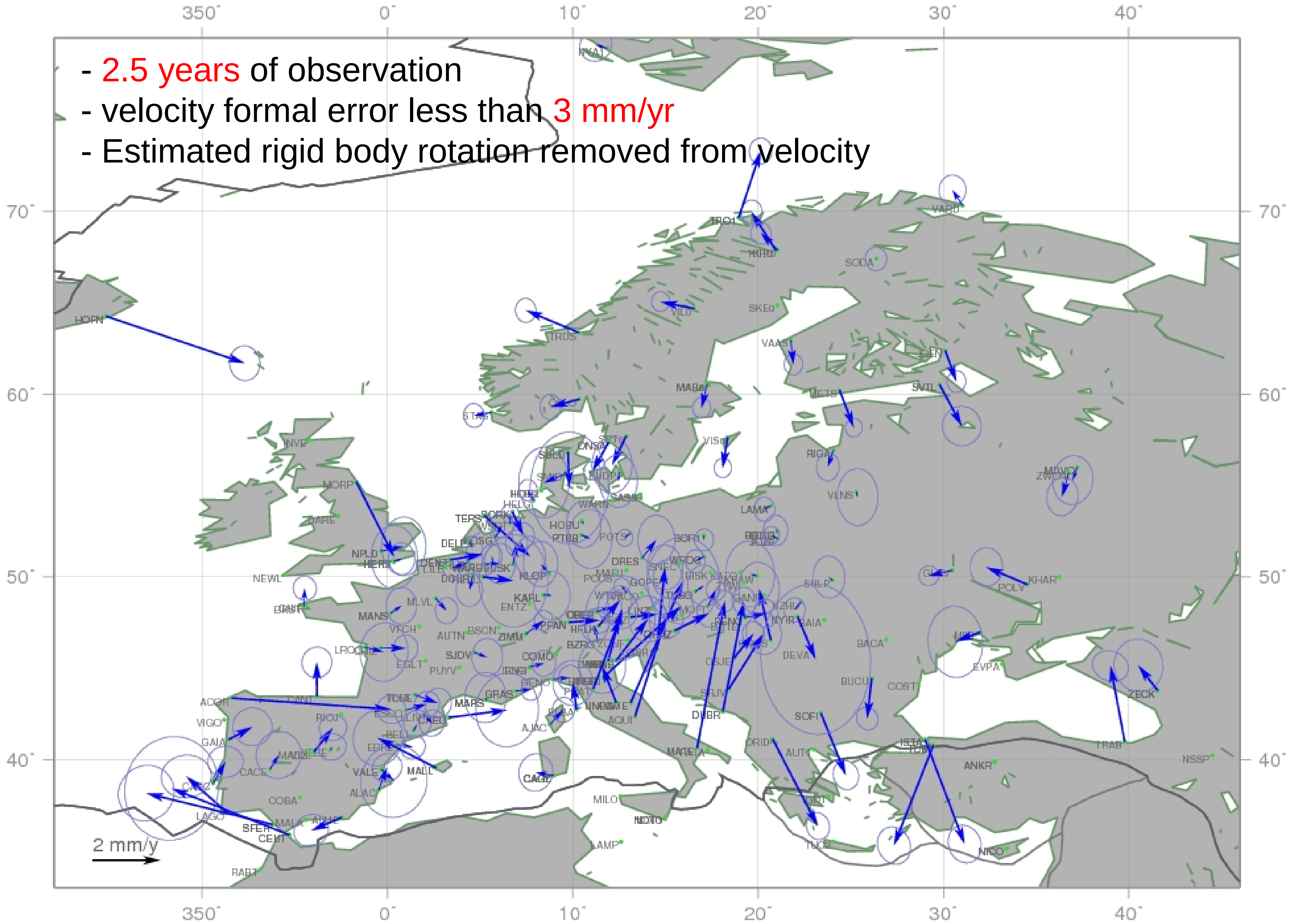


Interpolation: Residual velocity Field

- 2.5 years of observation
- velocity formal error less than 3 mm/yr
- Estimated rigid body rotation removed from velocity



Least-squares collocation method

We have the horizontal velocities of p stations P_i

$$V_{P_i} = {}^t[V^e_{P_i}, V^n_{P_i}]$$

We want to estimate the horizontal velocities at q points Q_j

$$W_{Q_j} = {}^t[W^e_{Q_j}, W^n_{Q_j}]$$

The “measurement” is decomposed into

– a signal $W_{P_i} = {}^t[W^e_{P_i}, W^n_{P_i}]$

– and a noise $\eta_{P_i} = {}^t[\eta^e_{P_i}, \eta^n_{P_i}]$

$$\left.\begin{array}{l} W_{P_i} = {}^t[W^e_{P_i}, W^n_{P_i}] \\ \eta_{P_i} = {}^t[\eta^e_{P_i}, \eta^n_{P_i}] \end{array}\right\} \quad V_{P_i} = W_{P_i} + \eta_{P_i}$$

At each point, the signal can be linearly expressed as a function of the angular velocity.

$$\begin{bmatrix} W^e_{P_i} \\ W^n_{P_i} \end{bmatrix} = A_{P_i} \begin{bmatrix} \omega^X_{P_i} \\ \omega^Y_{P_i} \\ \omega^Z_{P_i} \end{bmatrix}$$

with

$$A_{P_i} = \begin{bmatrix} -\sin(\phi_{P_i})\cos(\lambda_{P_i}) & -\sin(\phi_{P_i})\sin(\lambda_{P_i}) & \cos(\phi_{P_i}) \\ \sin(\lambda_{P_i}) & -\cos(\lambda_{P_i}) & 0 \end{bmatrix}$$

Least-squares collocation method

The best (unbiased minimum variance) linear estimate of the signal vector $W(Q)$ in terms of the "measurement" vector $V(P)$ is given by [Moritz, 1989] :

$$\boxed{\hat{W}_Q = A_Q C_{\omega_Q \omega_P}{}^t A_P [A_P C_{\omega_P \omega_P}{}^t A_P + C_{\eta_P \eta_P}]^{-1} V_P}$$

The covariance matrix of the interpolated velocity field is :

$$\boxed{C_{\epsilon\epsilon} = A_Q C_{\omega_Q \omega_Q} A_Q^T - A_Q C_{\omega_Q \omega_P} A_P^T [A_P C_{\omega_P \omega_P} A_P^T + C_{\eta_P \eta_P}]^{-1} A_P C_{\omega_P \omega_Q} A_Q^T}$$

Least-squares collocation method

The covariance matrices

$$\hat{W}_Q = A_Q C_{\omega_Q \omega_P}{}^t A_P [A_P C_{\omega_P \omega_P}{}^t A_P + C_{\eta_P \eta_P}]^{-1} V_P$$

Covariance Model

Noise covariance matrix

$$C_{\omega_Q \omega_P} = E[\omega_Q{}^t \omega_P]$$

$$C_{\omega_P \omega_P} = E[\omega_P{}^t \omega_P]$$

$$E[\omega_{P_i}^R \omega_{P_j}^S]_{(R,S=X,Y,Z)} = \begin{cases} 0 & \text{if } R \neq S \\ K(d_{P_i P_j}) & \text{if } R = S \end{cases}$$

No correlation between
2 different components

Spherical symmetry

Covariance function

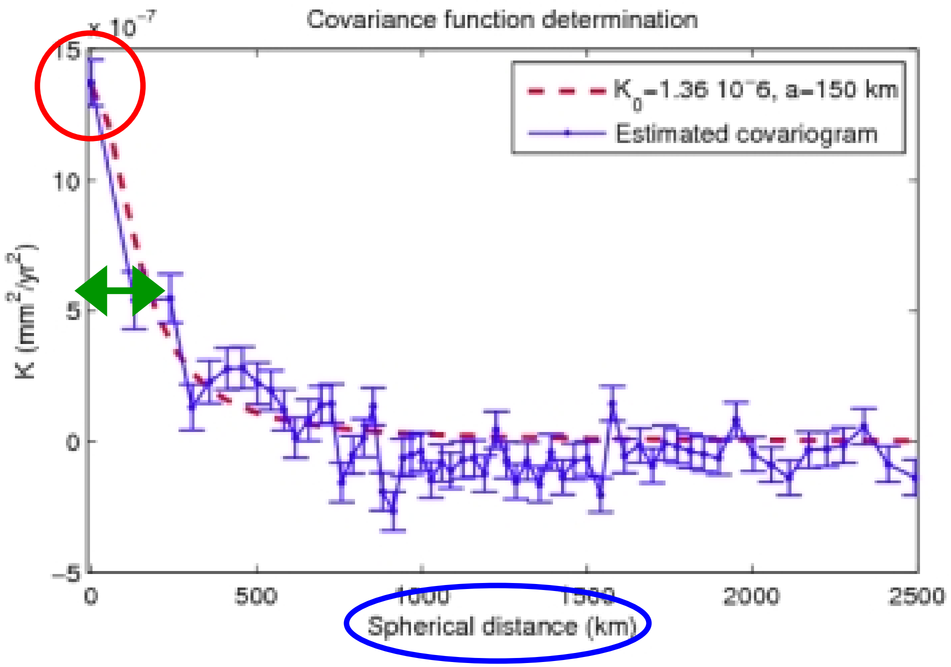
Least-squares collocation method

Covariance function determination

$$K(d_{P_iP_j}) = E\left[\frac{V^s_{P_i}V^t_{P_j} - E[\eta^s_{P_i}\eta^t_{P_j}]}{f_{st}(P_iP_j)}\right]_{(s,t=e,n)}$$

$K(d)$ is estimated as the weighted mean of the $\left[\frac{V^s_{P_i}V^t_{P_j} - E[\eta^s_{P_i}\eta^t_{P_j}]}{f_{st}(P_iP_j)}\right]_{\substack{d_{P_iP_j}\approx d \\ (s,t=e,n)}}$

Isotropic covariance function with zero derivative at the origin [**Kahle et al., 2000**]



$$K(d) = \frac{K_0}{1 + (d/a)^2}$$

Signal covariance for d =0

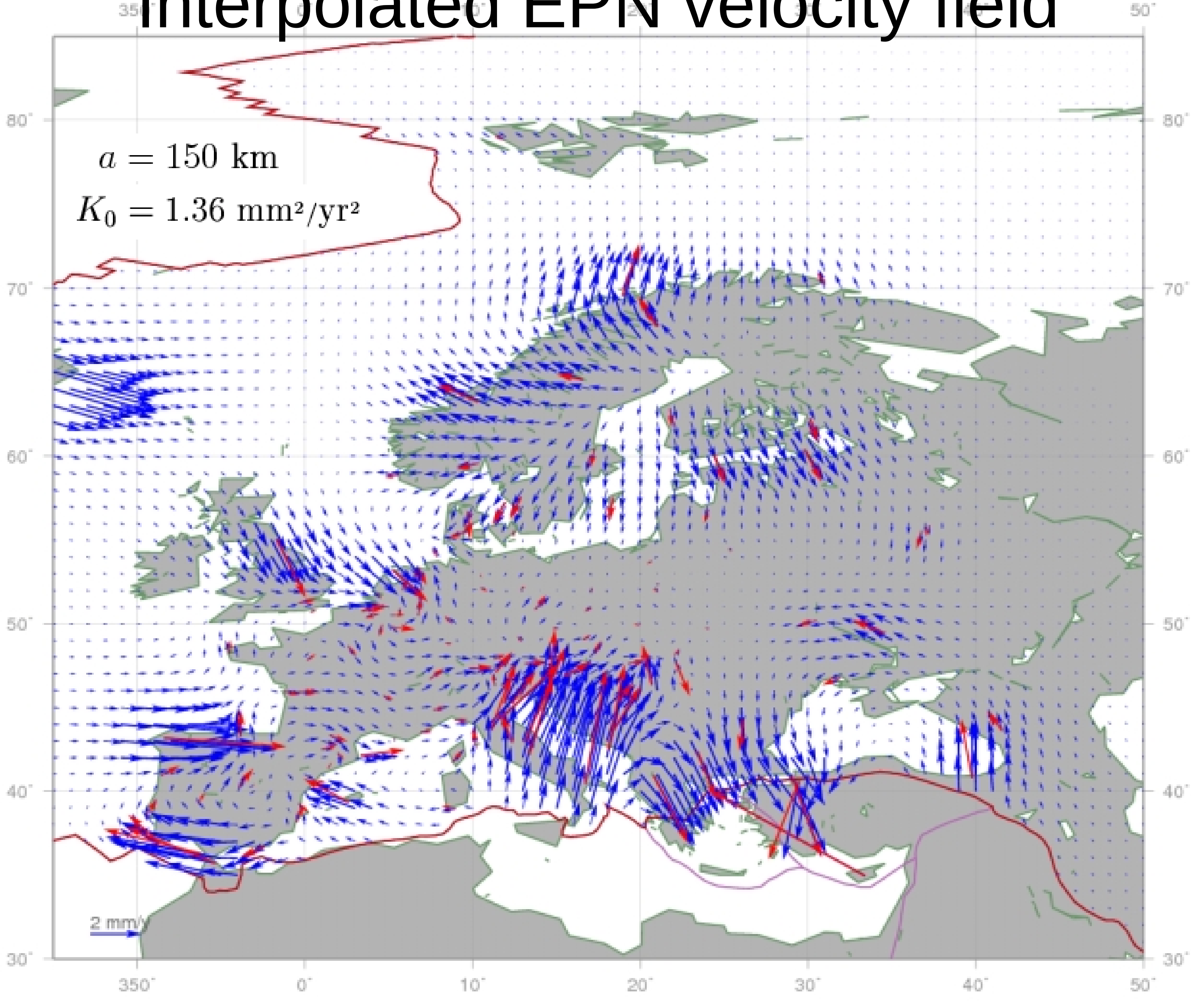
Spherical distance

Correlation length

$$a = 150 \text{ km}$$

$$K_0 = 1.36 \text{ mm}^2/\text{yr}^2$$

Interpolated EPN velocity field



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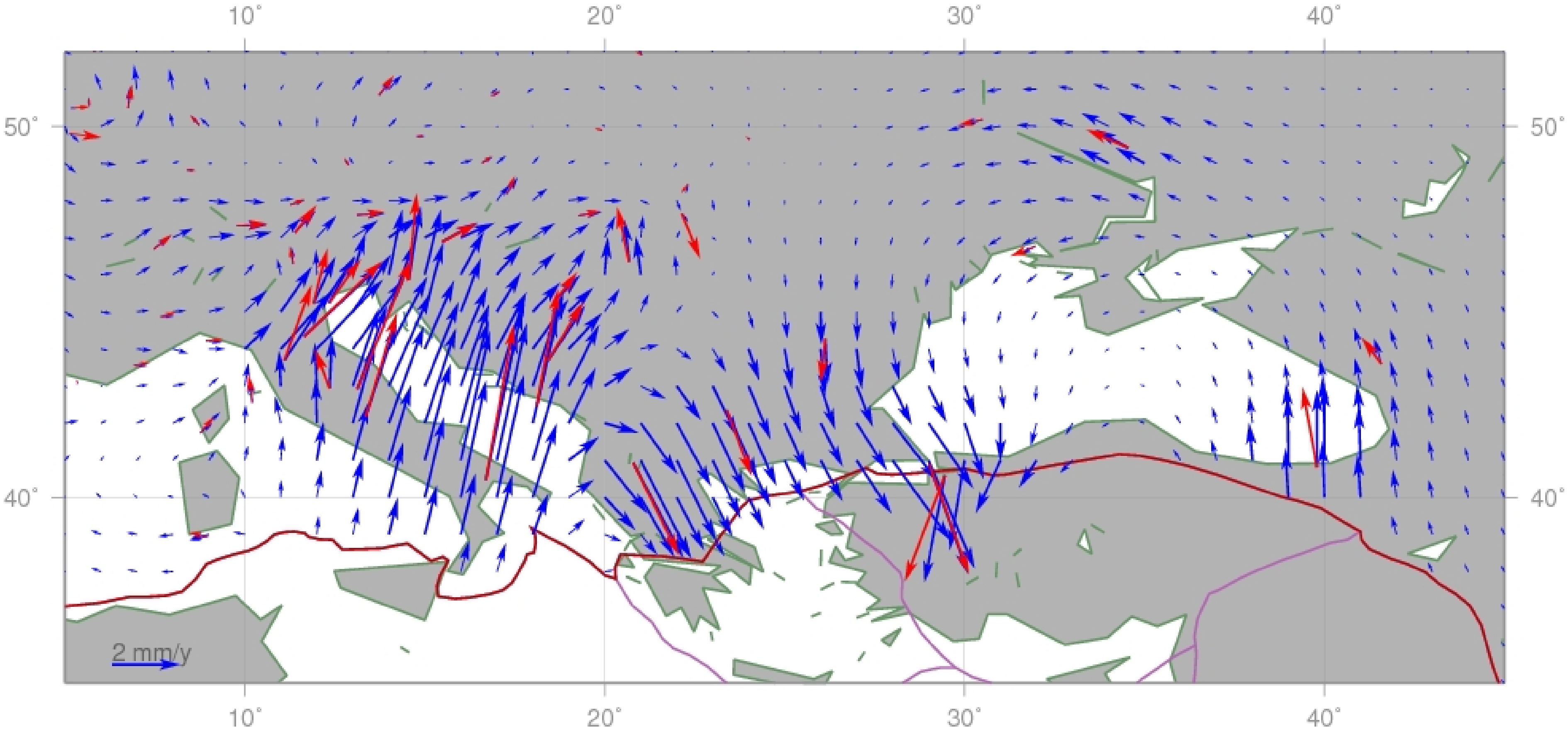

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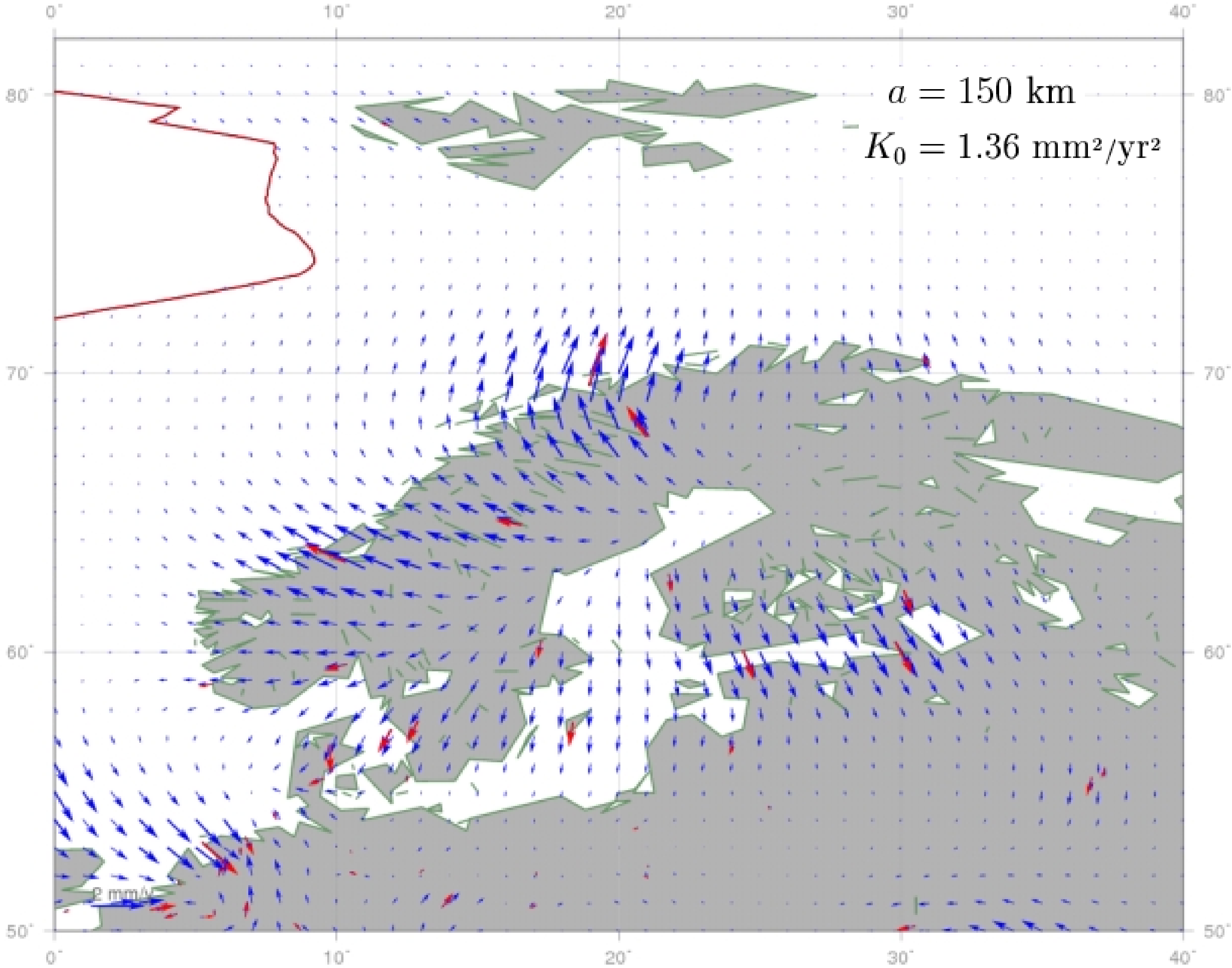

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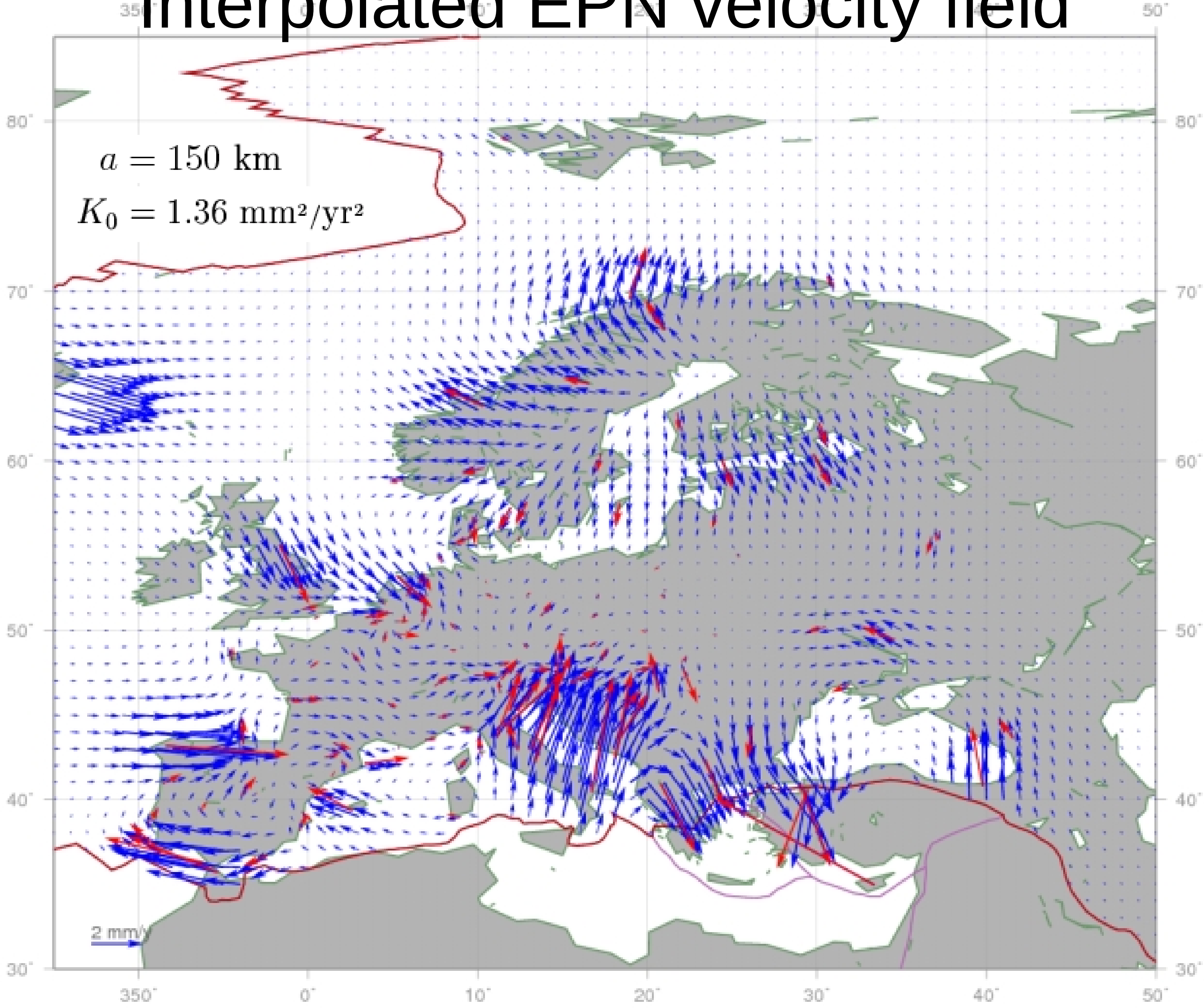

Conclusion

- A new method to interpolate a horizontal velocity field
- Contribution to the DEVF
  - Applied to the EPN horizontal velocity field
  - First test model to predict velocities over Western Eurasia
- More refinement and validation still to be done
- Confrontation with geophysical models still to be done
- Extension to vertical velocities