ACCURACY OF THE LAST PRECISE LEVELLING CAMPAIGN IN POLAND[1]

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Abstract. The paper describes very shortly first, second and third levelling campaign in Poland and gives accuracy of these networks estimated by Lallemand's formula. Next the fourth precise levelling network measurements from 1999 to 2003 are analysing using Lallemand's and Vignal's formulas. The obtained results are compared with the values taken from the previous campaigns. It is seen that in Polish successive campaigns the random errors decreased radically, whereas the systematic errors remained almost the same.

As an alternative the fourth levelling campaign was estimated by the methods of variance and covariance analysis, which shows that the lines are contaminated by systematic errors.

1 Introduction

Before the electronic era, levelling looked to be one of the most accurate techniques in geodesy. It is unrivalled in the determination of levelling networks covering large area. However, errors originating from instruments, ambient circumstances and observer, have such character that it is very difficult remove them from observations, also assessment of leveling accuracy is not easy task.

During the last decades, several methods for accuracy estimation have been developed. A detail discussion of these methods is presented in (Jordan at al.,1956, pp.223-255). The requirement, which a precise levelling must fulfil were defined for the first time at the Second Surveying Conference in Berlin in 1867, and were revised in 1871. As the result, the so called old error formulas were developed.

Next they were again revised at the General Conference of International Surveying in Hamburg in 1912 and so call new formulae or Lallemand's formulae for random and systematic errors in levelling networks were developed. At the Oslo Assembly of the International Association of Geodesy in 1948, the levelling error formulae were again reviewed and the resolution for the so called Vignal method of estimation a levelling accuracy was adopted.

Since 1955 A.M. Wassef and Messh e.g. in papers (Wassef, 1955), (Wassef and Messh, 1960), (Wassef, 1962), have demonstrated the application of mathematical statistics specifically to study levelling error and levelling networks. In (Wassef, 1974) has been proposed the application of the analysis of variance to study levelling discrepancies. The proposed method based on testing for the significance of means by the Fisher or F test. In paper (Ebong,1985) M.B. Ebong extended method proposed by Wassef, using a multiple comparison F test.

Unmodelled systematic effects in levelling may be revealed through autocorrelation function of discrepancies (Vanicek and Craymer, 1983) between the forward and backward running of levelling sections. Test results, conducted with simulated data indicate that autocorrelation function can be used as a diagnostic tool to detect systematic effects (ibid.).

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The aim of the study is accuracy estimation of the fourth Polish levelling campaign by the Lallemand's and Vignal's as well as test for significant differences between lines caused by different sources of random and systematic errors.

First the fourth Polish levelling campaign is described. Then computation of section and line discrepancies is explained. The random and systematic error computed by the Lallemand's and Vignal's formulas is portrayed. In the next paragraph the theory of analysis of variance is given in outline and practical computations are demonstrated. The last paragraph comprises conclusions.

2 The fourth levelling campaign

In Poland the fourth precise levelling campaign started in 1999 and was finished in 2003 (Paczus,2001). The measurements have been done using Zeiss Ni 002 (66% of the network), Zeiss DiNi 11 (31% of the network) and Topcon NJ (3% of the network) levels (ibid.).

The network consist of 16 150 sections with average length 1.1 km, 382 line with average length about 46 km, 135 loops, and 245 nodal points. Total length of levelling lines is 17 516 km (see Fig. 2-1).

Generally, the levelling sections have been measured forward and backward. Length of sight was up to 40 m, sequence of reading "backward – forward - forward - backward" and then "forward-backward-backward-forward" or "backward – backward - forward - forward". Each station observations were corrected for scale, temperature and earth tide. Before and after every field season the rods were calibrated (ibid.).

The measurement results used in this study were corrected due to rod scale, temperature and earth tides (ibid.).

Fig. 2-1 Fourth precise levelling network in Poland (1999-2003)

2.1 Lallemand's formulas

If all errors are random the standard deviation σ_L of a line L km long, could be express in the form $\eta\sqrt{L}$. Other formulae have been proposed in order to express the existence of systematic errors (Bomford, 1971, p.244)

$$\sigma_L = \sqrt{\eta^2 L + s^2 L^2} \tag{2-1}$$

where η is a random error in $mm/\sqrt{km}$ accumulating as $\sqrt{L}$, while s is a systematic error, accumulating in proportion to L.

The so call new formulas or Lallemand's formulas for random and systematic errors in levelling networks were approved at the General Conference of International Surveying in Hamburg in 1912 (Jordan at al.,1956, pp.223-255).

According to the recommendations of this conference, the random mean error is computed from the formula

$$\eta^2 = \frac{1}{4}\left[\frac{\sum \Delta^2}{\sum L} - \frac{\sum r^2}{(\sum L)^2}\sum\frac{S^2}{L}\right] \tag{2-2}$$

and the systematic mean error by the formula

$$s^2 = \frac{1}{4\sum L}\sum\frac{S^2}{L} \tag{2-3}$$

or using the loop misclosures φ by the formula

$$s^2 = \frac{1}{\sum L^2}\left[\frac{1}{2}\sum\varphi^2 - \eta^2\sum L\right] \tag{2-4}$$

where L is length of the levelling loop.

In order to estimate the accuracy of observed height differences, first the discrepancies Δ from forward and return sections levelling were computed. After division them by length of a section the discrepancies per 1 kilometre were computed (Waasef,1955). The total number of standardized discrepancies in the fourth Polish network is 15 809, but some were detected as outliers, and were rejected. Finally only 15 785 discrepancies were used in further computations. The statistics of these discrepancies are given in Table 2-1.

Table 2-1 Statistics of the standardized discrepancies Δ/r for the complete levelling network (in mm)

Number of discrepancies	Mean	Standard deviation	Min value	Max value
15 785	+0.07	±0.71	-4.40	4.80

Histogram of discrepancies Δ/r for the whole levelling network (Fig. 2-2) shows a quite good agreement with normal distribution with the small skewness (0.095) and kurtosis (3.532).

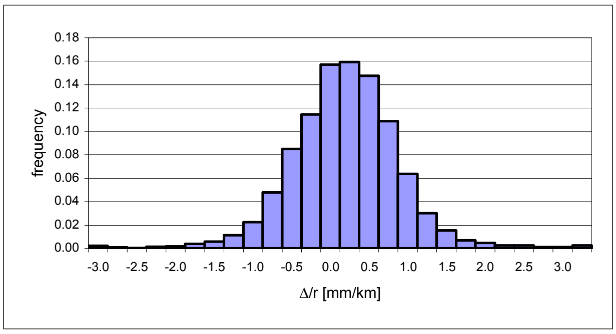

Fig. 2-2 Histogram of discrepancies Δ/r for the complete levelling network

First, the random and systematic errors were computed for each levelling line, which enable us to illustrate the spatial distribution of these errors in the network on Fig. 2-3. Random errors computed for each line were arranged into four groups, and from the picture (Fig. 2-3a) we can see that there is lack of small errors (blue arrows) there are some errors in the class 0.1 – 0.2 $mm/\sqrt{km}$, the main amount of errors is in the class 0.2 – 0.3 $mm/\sqrt{km}$, numerous errors are in lass 03. – 0.4 $mm/\sqrt{km}$ and few errors are bigger than 0.4 $mm/\sqrt{km}$.

In the case of systematic errors (Fig. 2-3b) the main amount of the errors is in the first class, from 0 to 0.1 mm, some errors in the second class 0.1 – 0.2 mm, and single values in the next two classes.

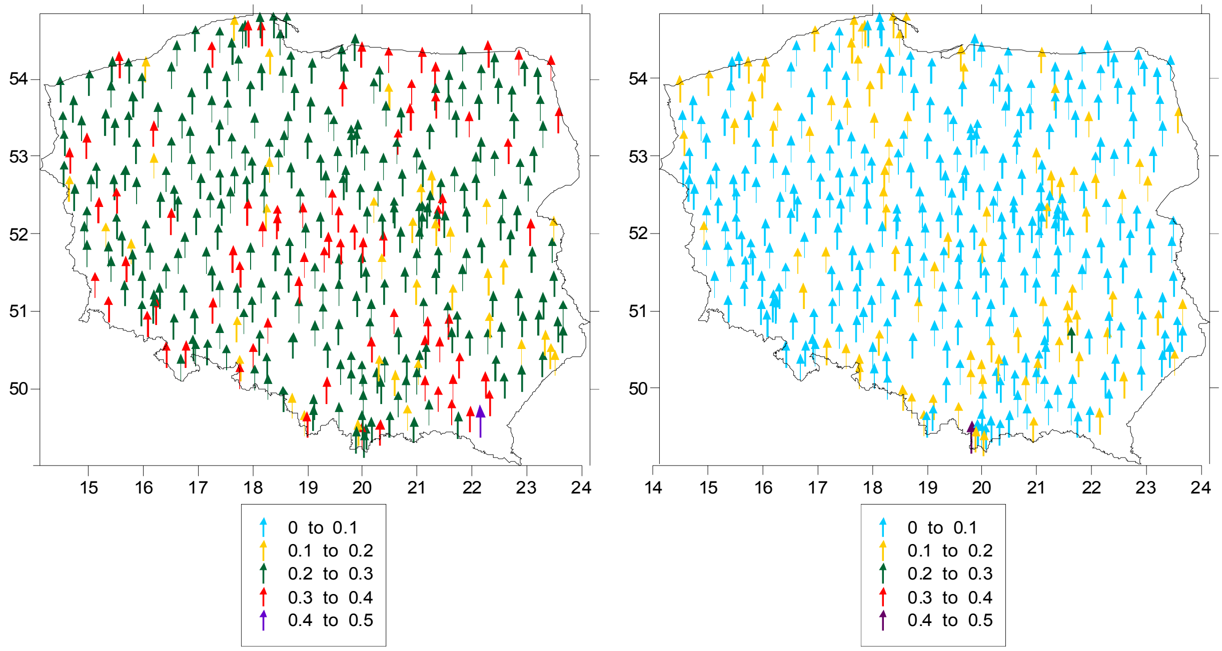

Fig. 2-3 Distribution of random and systematic errors (in millimetres) on the territory of Poland

In the next step, according to the Lallemand’s formulas, equations (2-2) and), the mean random error η is ± 0.265 $mm/\sqrt{km}$ and the mean systematic error ± 0.077 mm/km from the line discrepancies was computed for the entire network. Second time the systematic error ± 0.097 mm/km was computed from the loop misclosures. Therefore the total error, which is combination of both errors, is from ±0.276 to ±0.282 $mm/\sqrt{km}$.

Comparison of computed values with the results obtained in the previous campaigns (Wyrzykowski, 1988) shows decreasing tendency of random error, while the systematic error remain almost the same (see Fig. 2-4).

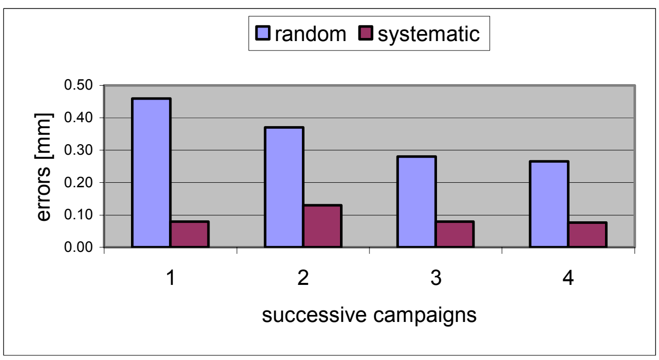

Fig. 2-4 Random and systematic errors in successive levelling campaigns

2.2 Vignal formulas

As was mentioned in par 2.1 at the Oslo Assembly of the International Association of Geodesy in 1948, the levelling error formulas were again reviewed and the new resolution for the method of estimation a levelling accuracy was adopted. The errors were divided into two groups, random and systematic group, which were assumed independent of each others. The random errors are caused by sources which are independent in all successive observations and obey Gauss's law of error distribution. The systematic errors are due to factors acting in the similar way on the successive or neighbouring levelling observations. They do not obey Gauss's law. They become random only for distance exceeding a certain limit distance Z, which is few tens of kilometres.

According to these formulas the total error in the fourth levelling was computed in the following way. First the mean accidental limiting value of the total error was computed from

$$u_L^2 = \frac{1}{4n_L}\Sigma\frac{S^2}{L} \qquad (2\text{-}5)$$

where S is line misclosures, L is length of a line and n_L is number of line in the network, or from

$$u_F^2 = \frac{1}{n_F + 1}\left(\Sigma\frac{\varphi^2}{F} + \frac{\varphi_e^2}{F_e}\right) \qquad (2\text{-}6)$$

where n_F is number of the loops, φ is loop misclosures, φ_e misclosures of the circumference loop, F is length of a loop, F_e is the length of circumference loop.

The limit distance Z is reached when u_L is no longer increase significantly with the mean value of L or F.

The mean random error is computed from

$$\eta^2 = u_r^2 - \xi^2 \times j^2 \qquad (2\text{-}7)$$

where u_r and j are computed from the formulas

$$u_r = \frac{1}{4n_r}\Sigma\frac{\Delta^2}{r} \quad \text{and} \quad j^2 = \frac{K}{Z}\times r_m \qquad (2\text{-}8)$$

The value K=2 and Z=50 km, which were used here have no essential significance in this connection.

After that the systematic error is consequently equal

$$\zeta^2 = u_L^2 - \eta^2 \qquad (2\text{-}9)$$

Using computed for each section discrepancies Δ and applying formulas (2-7) - (2-9) by iteration, the value $\pm 0.26\ mm/\sqrt{km}$ for the random and $0.45\ mm/km$ for systematic error was obtained.

These values were compared with previous computations (Fig. 2-5). One may observed that random errors estimated in two different ways are almost the same, while the estimation of systematic errors significantly differ each other. Since we had access to data from first Polish campaign then we could estimate random and systematic errors in that network by Vignal's formulas. The results are very similar (Fig. 2-6). Random errors are more or less the same, but systematic errors differ drastically.

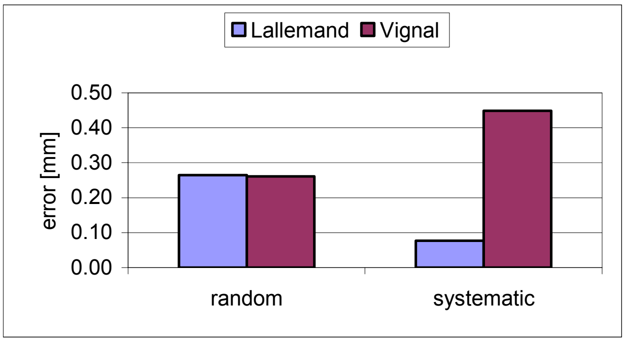

Fig. 2-5 Comparison of random and systematic errors computed from Lallemand's and Vignal's formulas for the fourth campaign

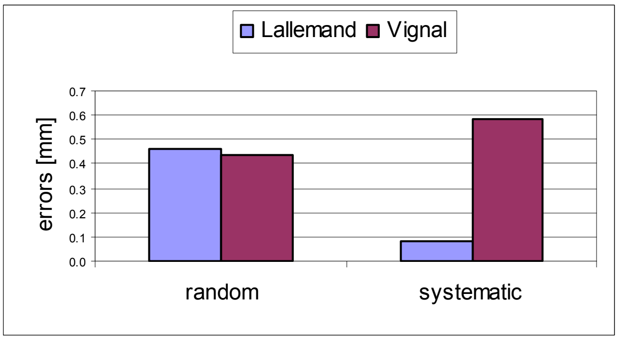

Fig. 2-6 Comparison of random and systematic errors computed from Lallemand's and Vignal's formulas for the first campaign

2.3 Analysis of variance

The purpose of analysis of variance (*ANOVA*) is to test for significant differences between means by comparing (i.e., analyzing) variances. More specifically, by partitioning the total variation into different sources e.g. into levelling lines, we are able to compare the variance due to the between-groups (or treatments) variability with that due to the within-group (treatment) variability. In that paragraph some of the classical formulas are shown, and the method is applied to the data of fourth levelling campaign.

Let us suppose that the levelling net consists of m lines $(L_1, L_2 .... L_m)$ and that on the line L_i there are $n_j + 1$ bench marks dividing the line into n_j sections. Total number of sections in the network is,

$$n = \sum_{i=1}^{m} n_j \tag{2-10}$$

If we consider line *i* and the section *j* into this line, then the discrepancies Δ_{ij} means the difference between forward and backward measurements of the section *ij*, and

$$w_{ij} = \frac{\Delta_{ij}}{r_{ij}} \tag{2-11}$$

means the discrepancies per kilometre along the section *ij*, where r_{ij} is length of the section in km. We assume that w_{ij} were taken from normal population.

Mean value for the line *i* is equal

$$\overline{w}_i = \frac{1}{n_j}\sum_{j=1}^{n_j} w_{ij} \tag{2-12}$$

The general mean of all discrepancies w_{ij} into network is

$$\overline{w} = \frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n_j} w_{ij} = \frac{1}{n}\sum_{i=1}^{m} n_j \overline{w}_i \qquad \text{(2-13}$$

The sum of squares between w_{ij} and $\overline{w}$ is (Brandt,2002)

$$\begin{aligned} Q &= \sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(w_{ij} - \overline{w}\right)^2 \\ Q &= \sum_{i=1}^{m} n_i\left(\overline{w}_i - \overline{w}\right)^2 + \sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(w_{ij} - \overline{w}_i\right)^2 = Q_A + Q_W \end{aligned} \tag{2-14}$$

Factor Q_A is the sum of squares between the levelling lines and is due to two different sources of errors in a line, while Q_W is the sum of squares between discrepancies w_{ij} and the mean values $\overline{w}_i$ of a line.

An expressions

$$s_A^2 = \frac{Q_A}{m-1} \qquad s_W^2 = \frac{Q_W}{n-m} \tag{2-15}$$

are unbiased estimators of empirical variance, and quotient

$$F = {s_A^2}\big/{s_W^2} \tag{2-16}$$

can be used to test a null hypothesis

Under the null hypothesis (that there are no mean differences between groups or treatments in the population), the variance estimated from the within-group (treatment) variability should be about the same as the variance estimated from between-groups (treatments) variability. Finally we have

$$F = {s_A^2}\big/{s_W^2} > F_{1-\alpha}\left(m-1, n-m\right) \tag{2-17}$$

The analysis is usually tabulated as follows:

Table 2-2 Scheme for the analysis of variation of the levelling lines of the fourth campaign

Source of variationi	SS (Sum of squares)	DF (dgree of fredom)	MS (mean squres)	F
Between lines means	QA	m-1	s_A^2	$F = {s_A^2}\big/{s_W^2}$
Within lines	QW	n-m	s_W^2	
Total	Q	n-1	s^2	

The method of analysis outline above was applied to the fourth levelling campaign in Poland, which was briefly described in the introduction of sec. 2. By following the standard computing procedure we obtain the following table of the analysis of variance:

Table 2-3 Analysis of variance of the fourth levelling campaign in Poland

Source of variationi	SS (Sum of squares)	DF (dgree of fredom)	MS (mean squres)	F	F_{theo}	p
Between lines means	389.708	376	1.036	2.304	1.125	0.95
Within lines	6866.608	15 251	0.450			
Total	7256.316	15 635	0.464			

The ratio of the MS between lines means to the MS within the lines is 2.30, whereas on the 0.95 per cent confidential level, the theoretical value gives for n_1 = 377, and for n_2 = 15251 the value F=1.13. It means that deviation from homogeneity is therefore very highly significant.

3 Conclusions

Accuracy of the network estimated using Lallemand's formulas give us for the random error value ±0.265 $mm/\sqrt{km}$, while for systematic error value ±0.077 mm/km. Comparison with values obtained in previous campaigns, shows that random error decrease significantly, while systematic error remains more or less the same.

Accuracy estimation using Vignal's formulas gives almost the same value for random error, whereas systematic errors differ considerably.

Variance analysis shows that levelling lines are affected by systematic errors.

It is hoped that the present study will give geodetic community some valuable information about accuracy of fourth levelling campaign in Poland.

4 Acknowledgments

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