Extension of the DFHRS Approach for Gravity Observations and Computation Design for a 1cm fitted DFHRS of Europe

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www.dfhbf.de

www.geozilla.de

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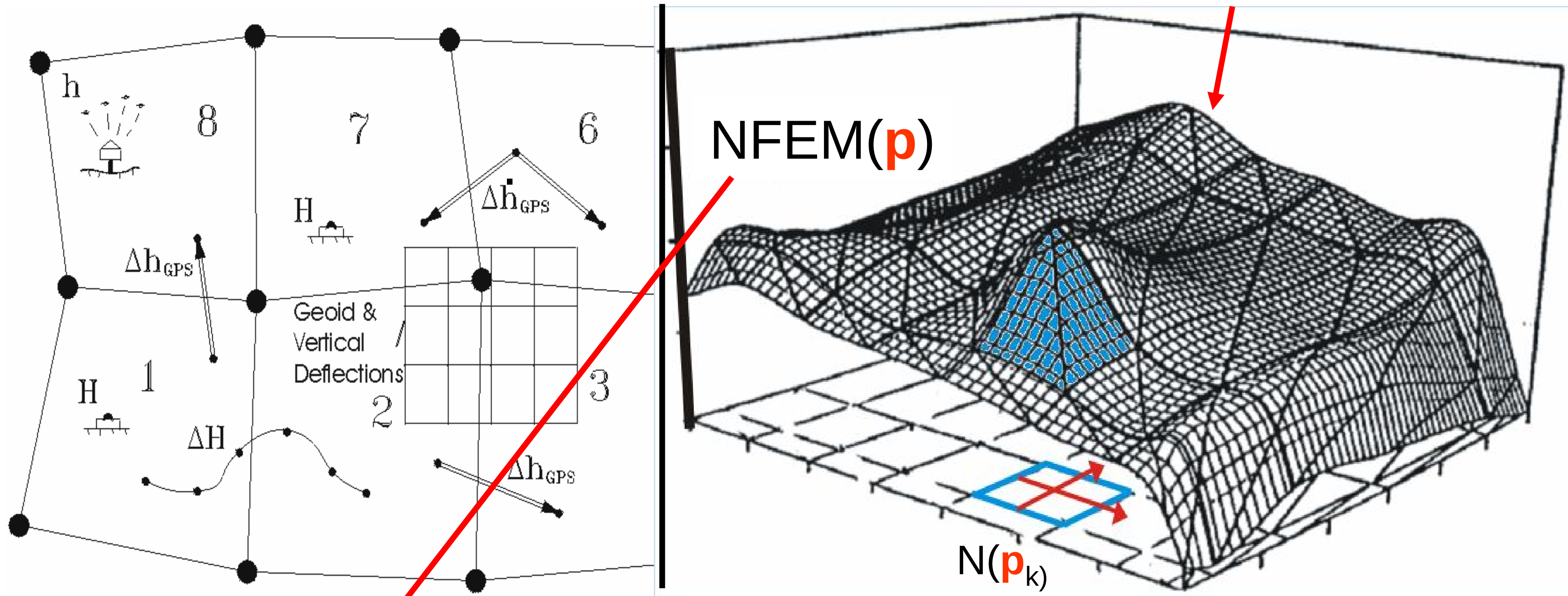
DFHRS-Concept

<u>Idea:</u> "All observations / data types to set in relation to the parameters p of a continuous „one layer" representation NFEM(p) of the HRS - Adjustment"



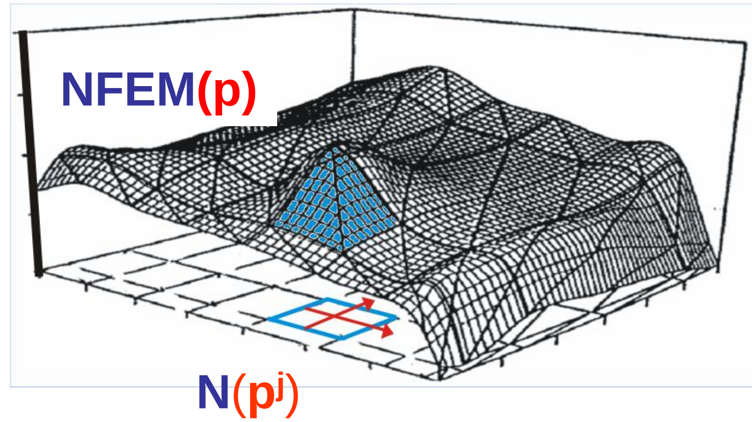
$$
\mathrm{NFEM}(\mathbf{p} \mid B, L) =: \begin{cases} N(p_k) = [1 \mid x\ y \mid x^2\ xy\ y^2 \mid x^3\ x^2y\ xy^2\ \ldots] \cdot \\ [p_{00} \mid p_{10}\ p_{01} \mid p_{20}\ p_{11}\ p_{02} \mid p_{30}\ \ldots]_k \\ = \mathbf{f}^T \cdot \mathbf{p}_k \\ + \text{Contunuity Conditions along the Borders} \end{cases}
$$

$N(p_k)$= Local Taylor-Series Expansion of the HRS

Digital FEM Height Reference Surface (DFHRS)- Concept

Complete New Computation of con- tinuous HRS (p and D m)!

DFHRS – Adjustment Approach State of the Art < 2005



$h_{GNSS} + v = H + \mathrm{NFEM}(p) - h_{GPS} \cdot D\, m$ <= GPS/Levelling Fitting Points

$H + v = H$

$N_G{'}^j + v^j = \mathrm{NFEM}(p) + \partial N_G(d^j)$ <= Any number Geoidmodels (Global, regional, local)

$\xi^j + v = -F_B / M(B) \cdot p + \partial \xi (d_{x,h})^j$ <= Sets of Deflections from Vertical

$h^j + v = -F_L/(N(B)\cdot\cos(B)) \cdot p + \partial \eta(d_{x,h})^j$ (Zenith Cameras or Geoidmodels)

$\frac{a}{4\pi\cdot\gamma(B)} \iint_\sigma Dg \cdot S(\psi) d\sigma + v = \mathrm{NFEM}(p)$ <= „Gravity“ by correlated Geoidmodels In the sense of an 2 step adjustment

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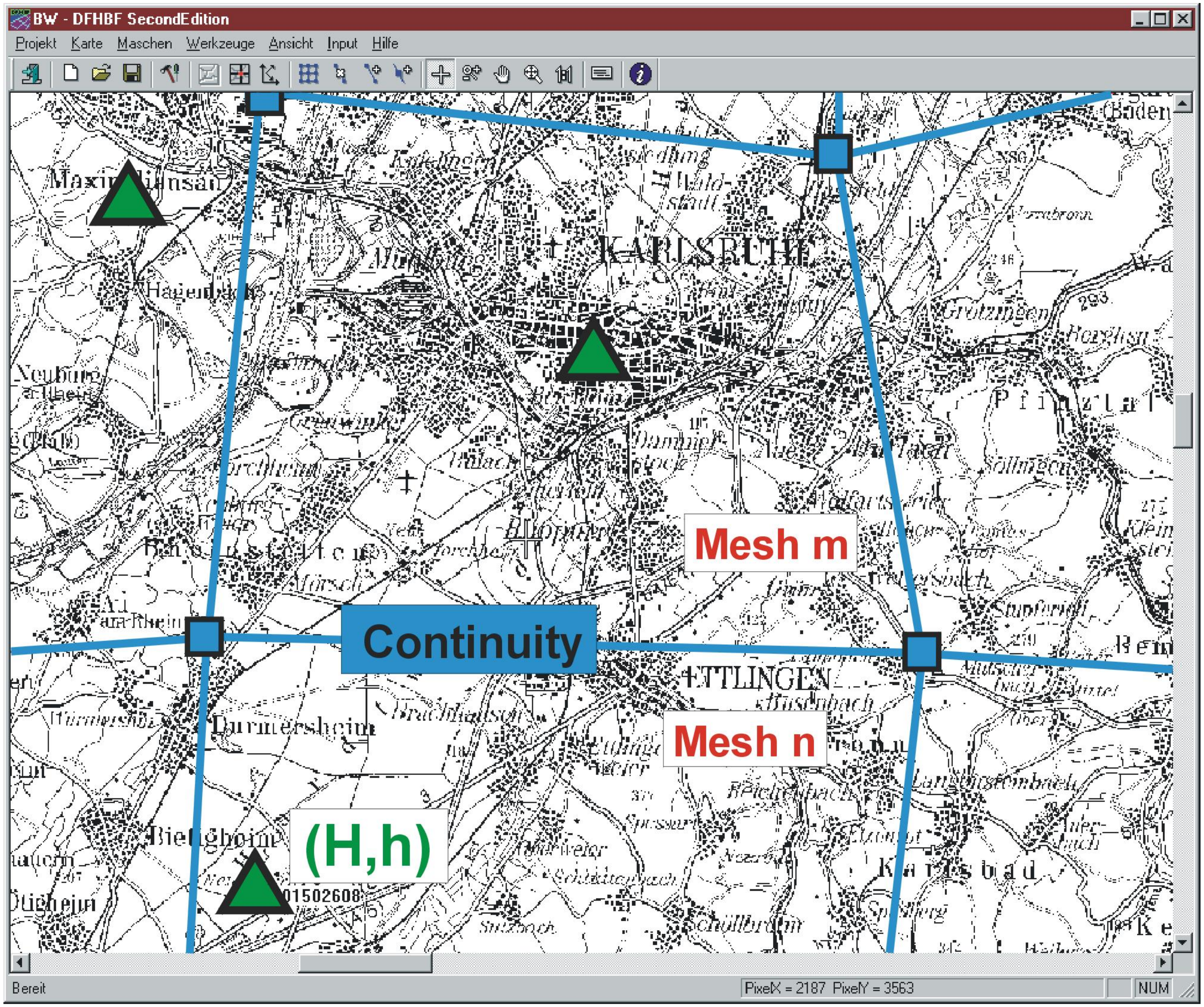
DFHRS Software

- Identical „Fitting“ Points (B,L,h;H)
- Meshes
- Patches



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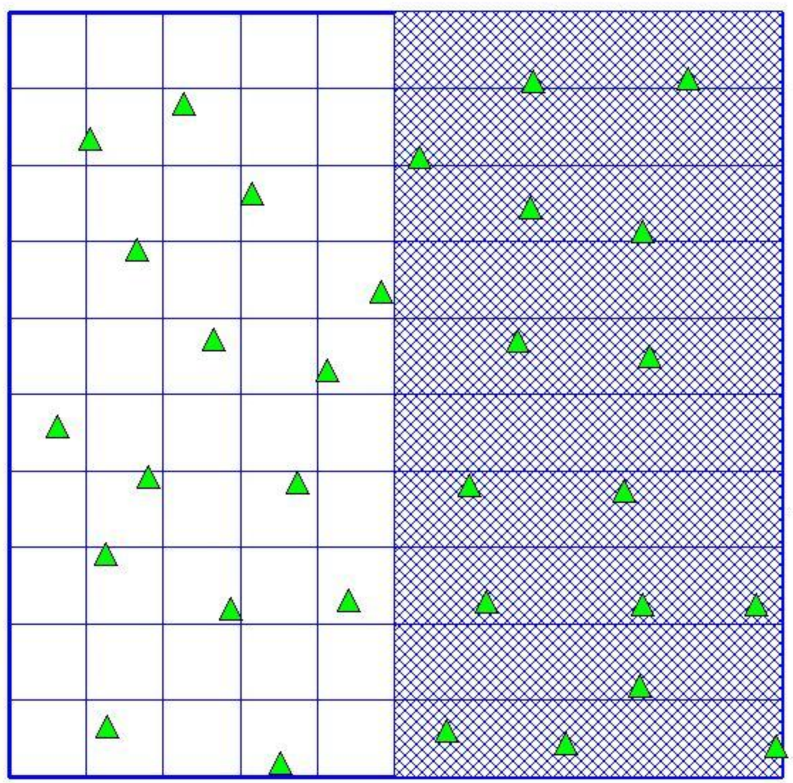
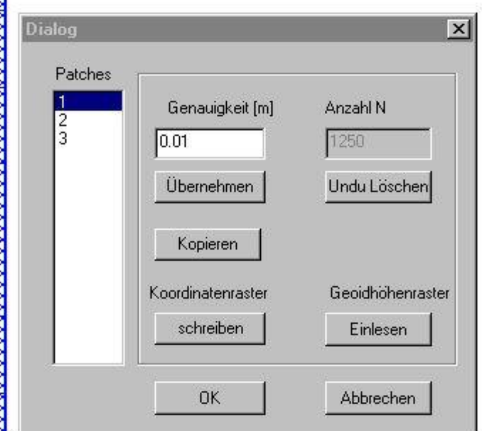
DFHRS Software

- *Hochschule für Technik und Wirtschaft*
  - University of Applied Sciences
- *LVA Baden-Württemberg*
- *LVA Hessen*
- *LVA Rheinland-Pfalz*
- *LVA Riga, Latvia*
- *University of Federal Forces Munich*
- *University Darmstadt*

Every geoid- and/or vertical deflection model N can be „patched“

¶$N_G(d_j)$





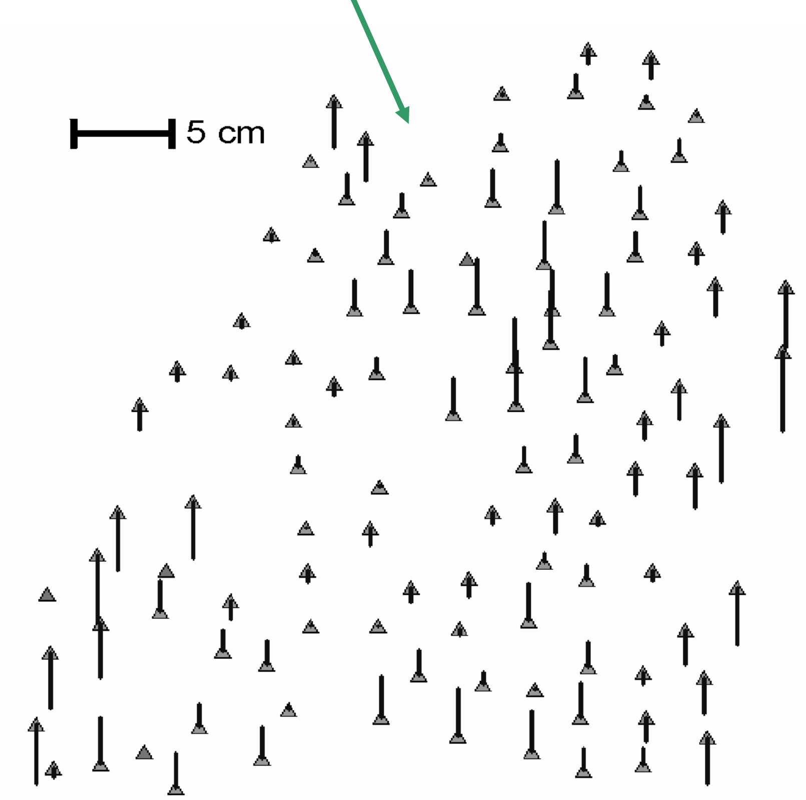
DFHRS Approach - „Patching“

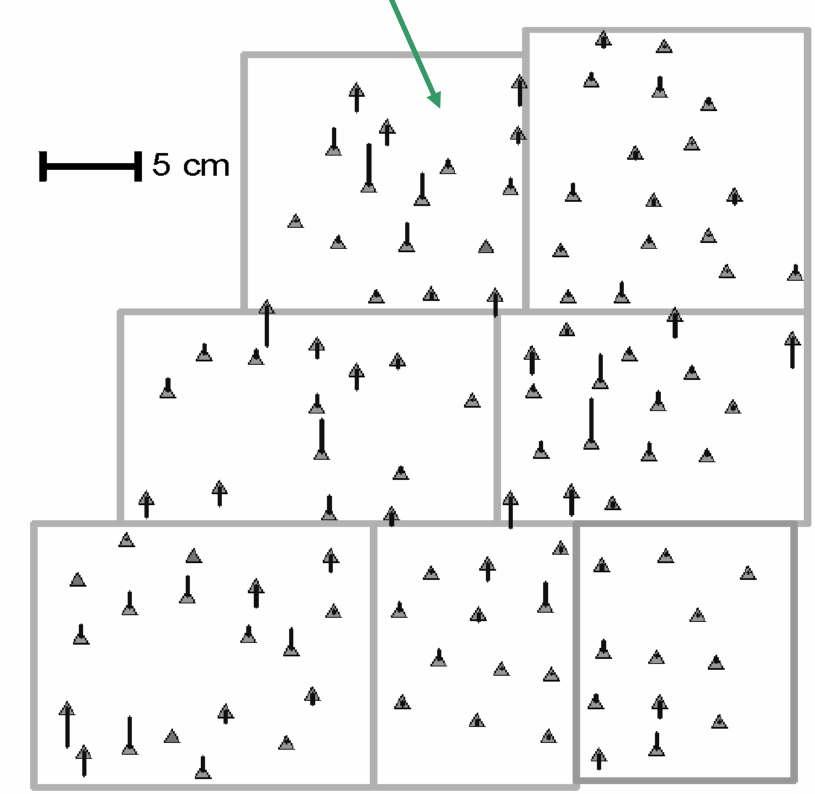
$NG'_j + v_j = NFEM(p) + \partial NG(d_j)$

Residuals of a Geoid Model **NG‘**

One Datum

7 „Patches“





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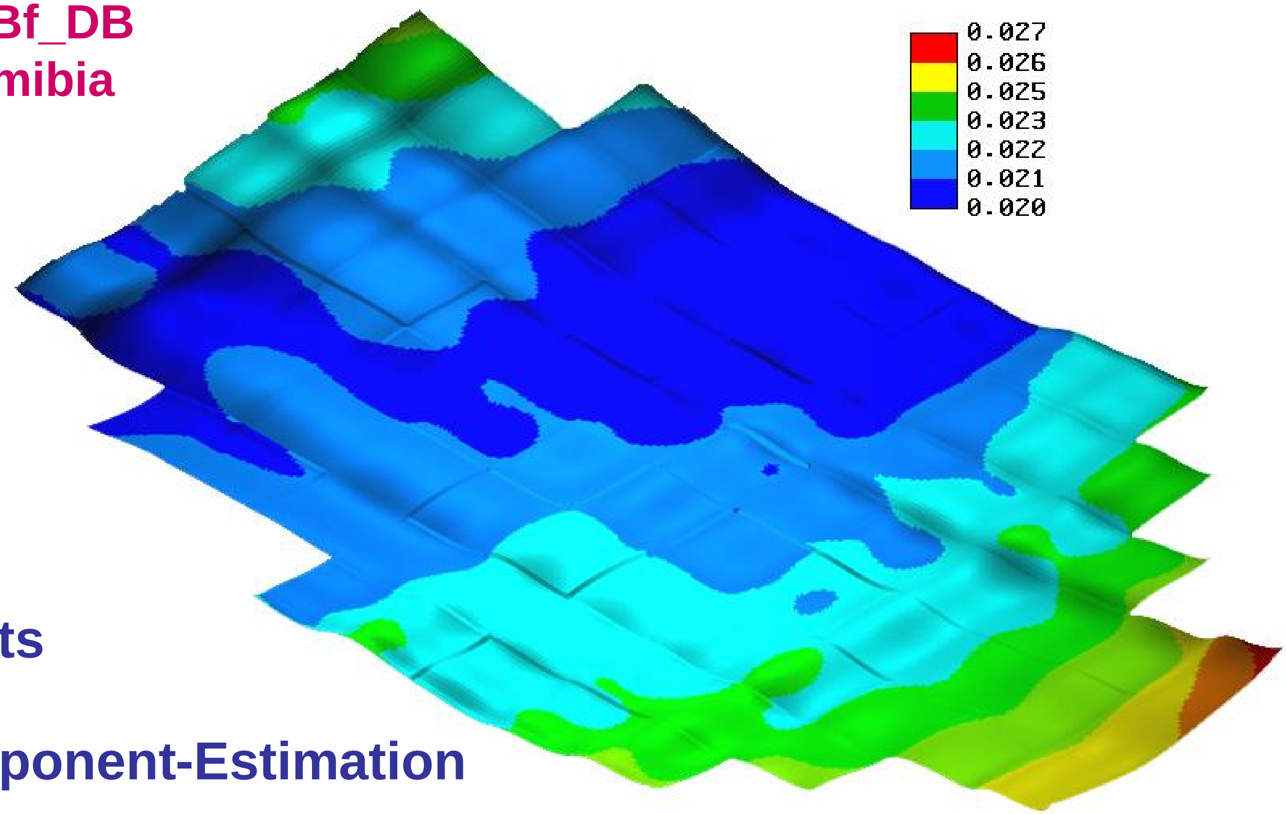
DFHRS_DB - Quality Proof

„Accuracy Surface“ based on

Kovariance Matrix of the DFHRS-Parameters (p,Δm)

<_3_cm DFHBf_DB
Windhuk, Namibia
EGM96

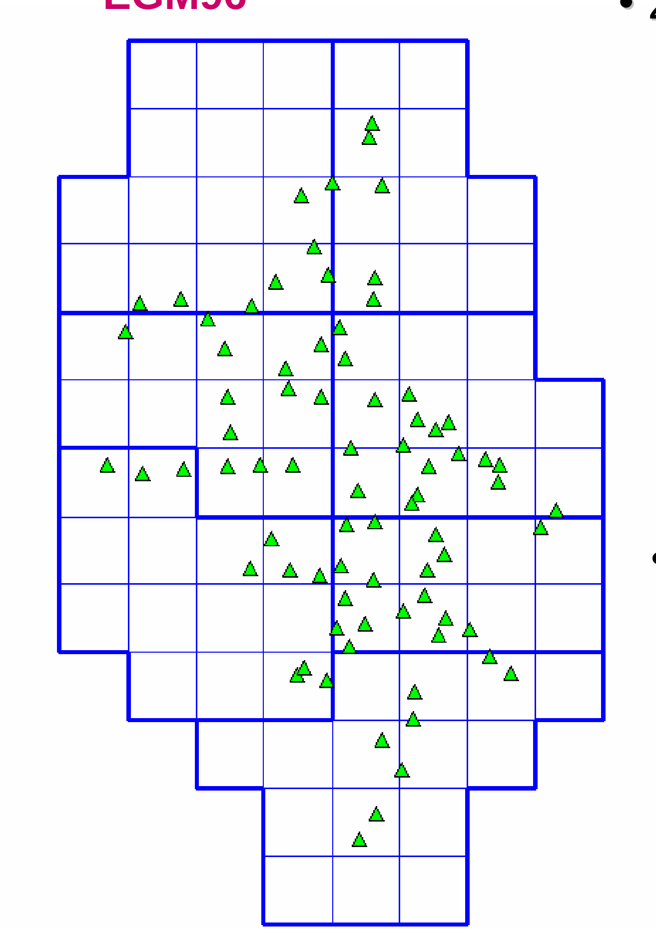


+ Statistical Tests

+ Variance-Component-Estimation

DFHRS_DB   Design Parameters

<_3_cm DFHRS_DB
Windhuk, Namibia
EGM96



Meshsize (p=3)

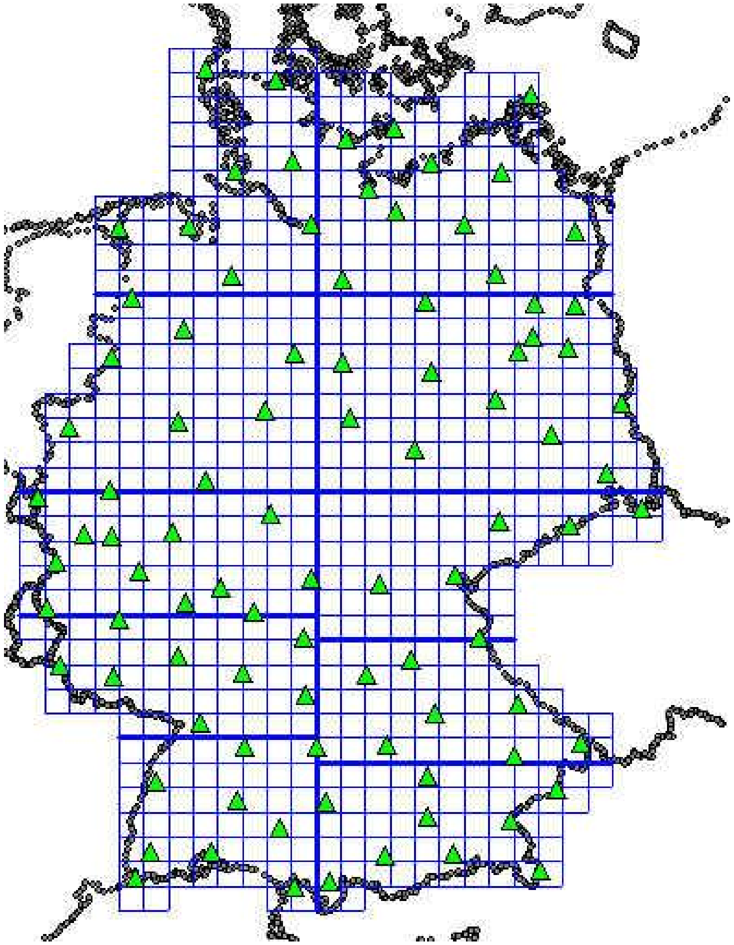
- 20-30 km : HRS approximation error < (5-10) cm
  - 10 km: HRS approximation error <1 cm
  - 5 km: HRS approximation error < 0.5 cm

Fitting Point Density (< 10 mm points, EGG97)

- 50 points per (100 km x 100 km): <_1_cm DFHRS_DB
- 10 points per (100 km x 100 km): < 3_cm DFHRS_DB
- 3-4 points per (100 km x100 km): < 5-10_cm DFHRS_DB

DFHRS_DB   Design Parameters

Design Studies < 5 - 10_cm DFHRS Germany



Patch-Size (EGG97)

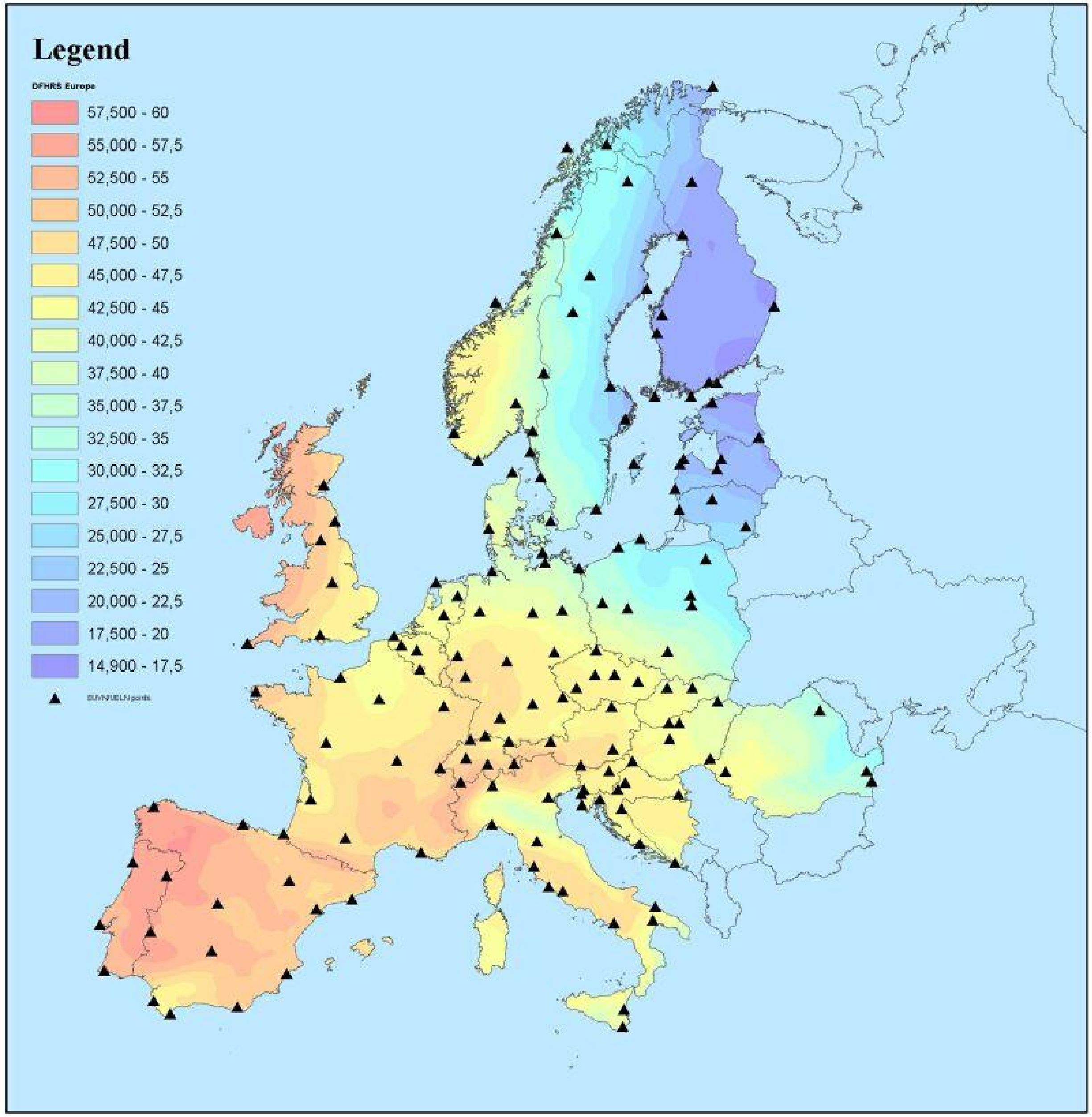
- 30 - 40 km for a < 1_cm DFHRS_DB
- 50 – 60 km for a < 3_cm DFHRS_DB
- 300 km for a < 10_cm DFHRS_DB

(3-5) points per patch

<_10_cm DFHBF_DB Europa

Isolines

30 km FEM Meshes



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< 10cm DFHRS Europe – „Fittingpoint-Design“

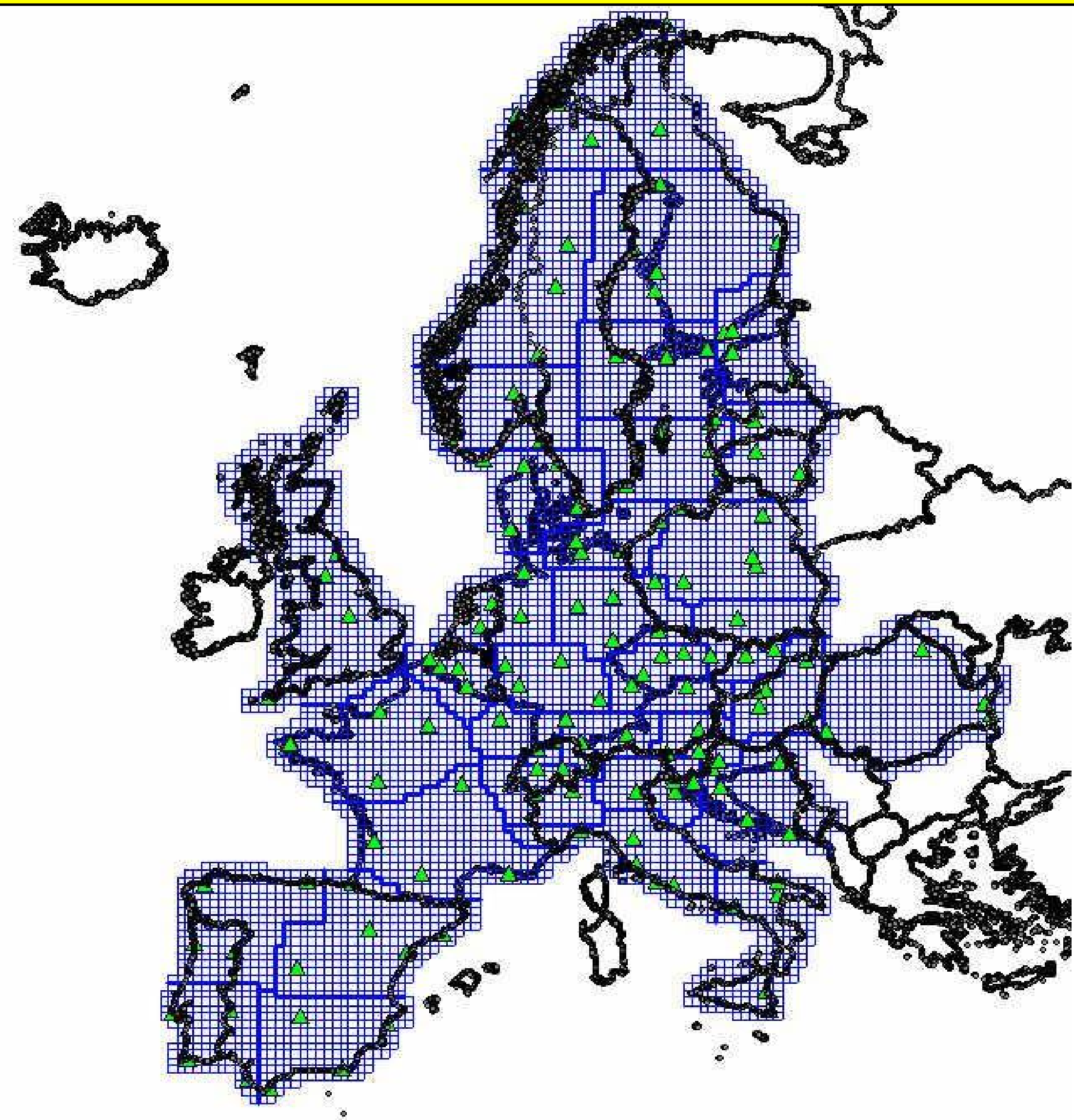
ETRS89/EVRS

„GPS-/Levelling-Points of EVN“

Fitting Points NFEM(p) =: h - H

Used for the 1st Version

< 10_cm DFHBFS Europe



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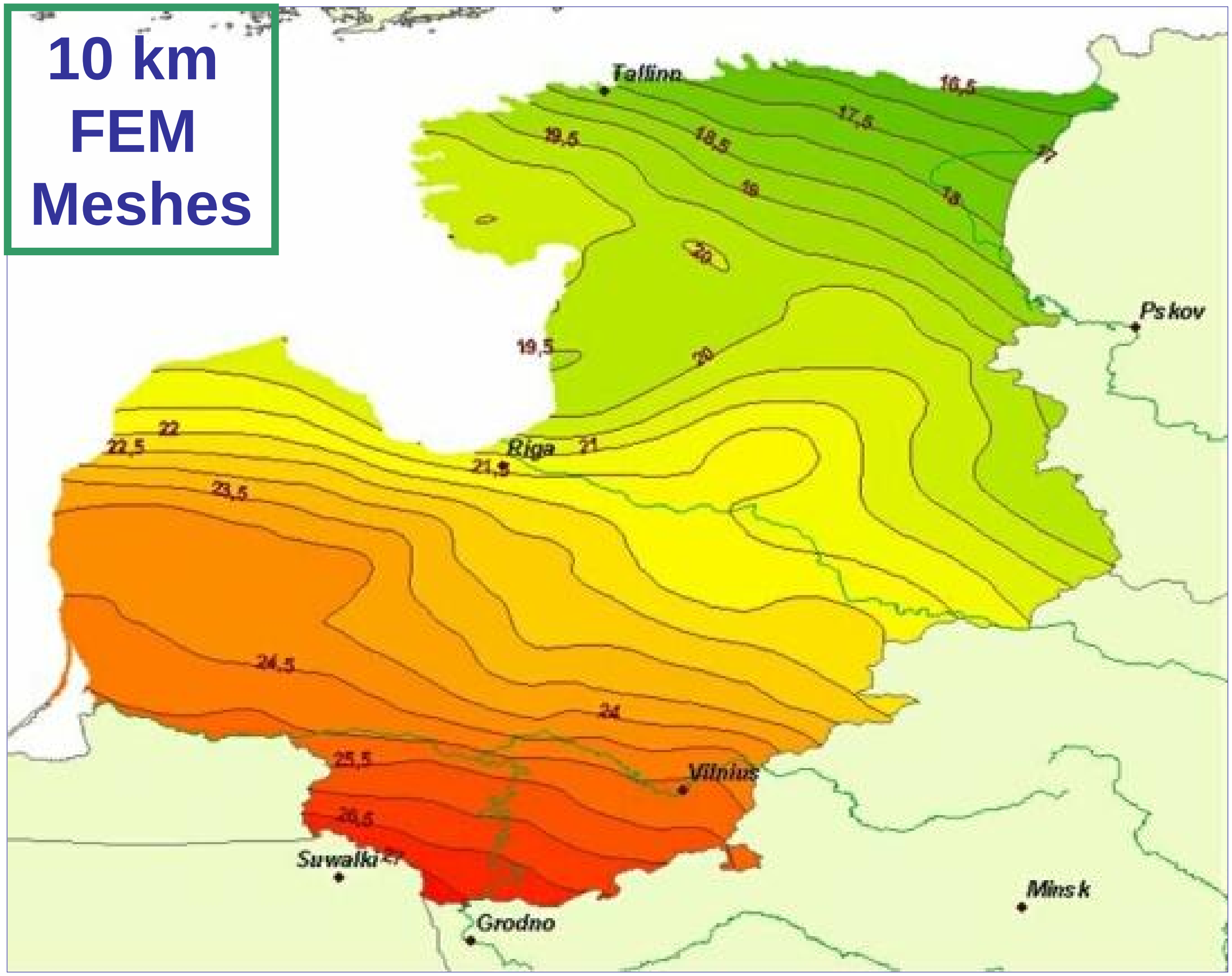
<_10_cm DFHRS_DB - Indepent Quality Control

<1_dm EVRF2004 (Present Version, 35 km meshes, 34 Patches)

	Austria	Germany	Estonia	Latvia	Lithuania	Switzerland
Number of unused control points	9	95	21	25	46	13
RMS [cm]	7.5	4.2	8.8	9.2	6.8	7.0

European HRS …. including <(1-3)cm_DFHRS Baltics (Latvia, Estonia, Lithuania)

10 km FEM Meshes



Master Thesis

of Mrs. Lauma Lace, Latvia

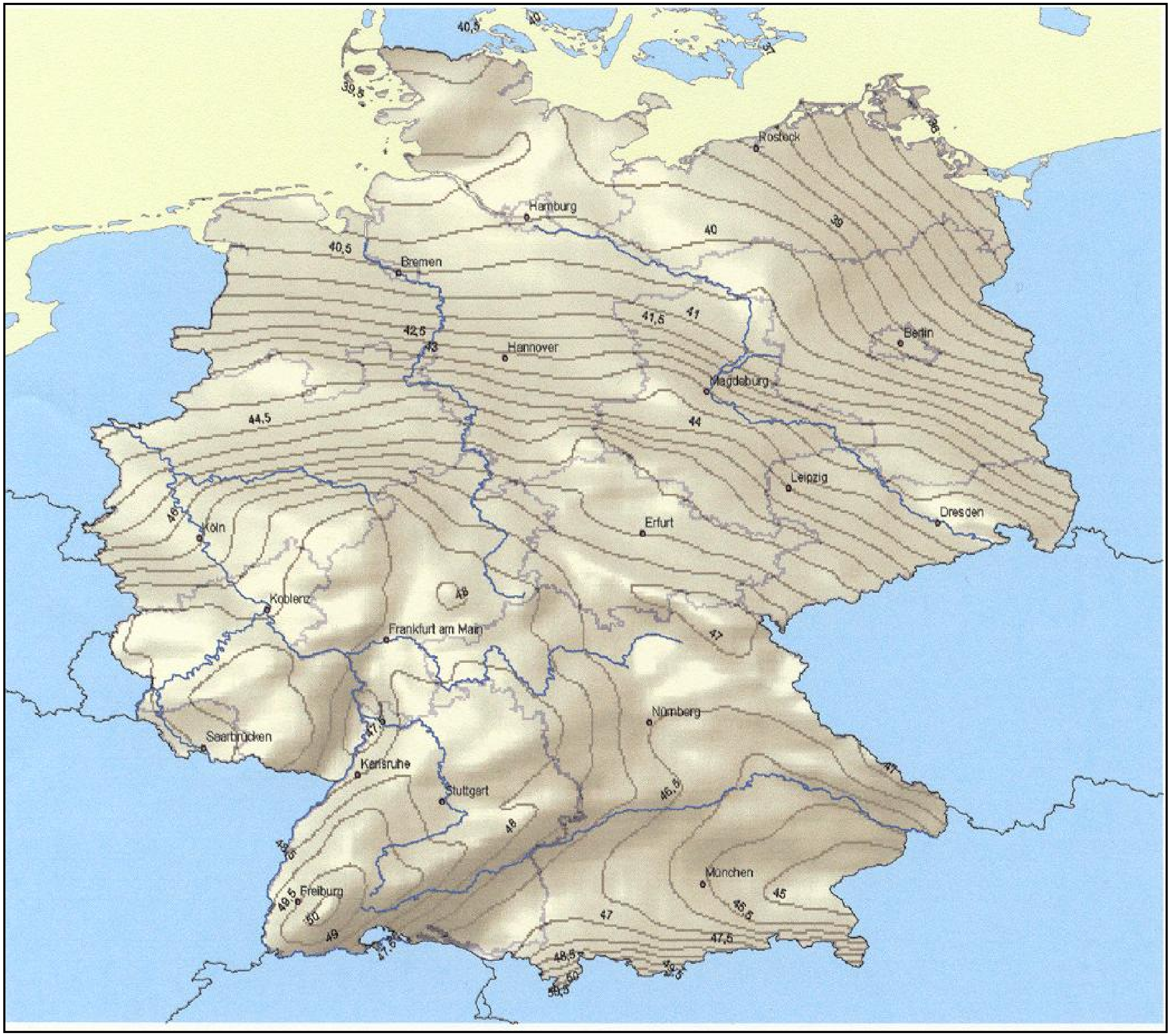
at

Karlsruhe University of Applied Sciences

European HRS… including

< 3cm DFHRS_DB Germany

10 km FEM Meshes



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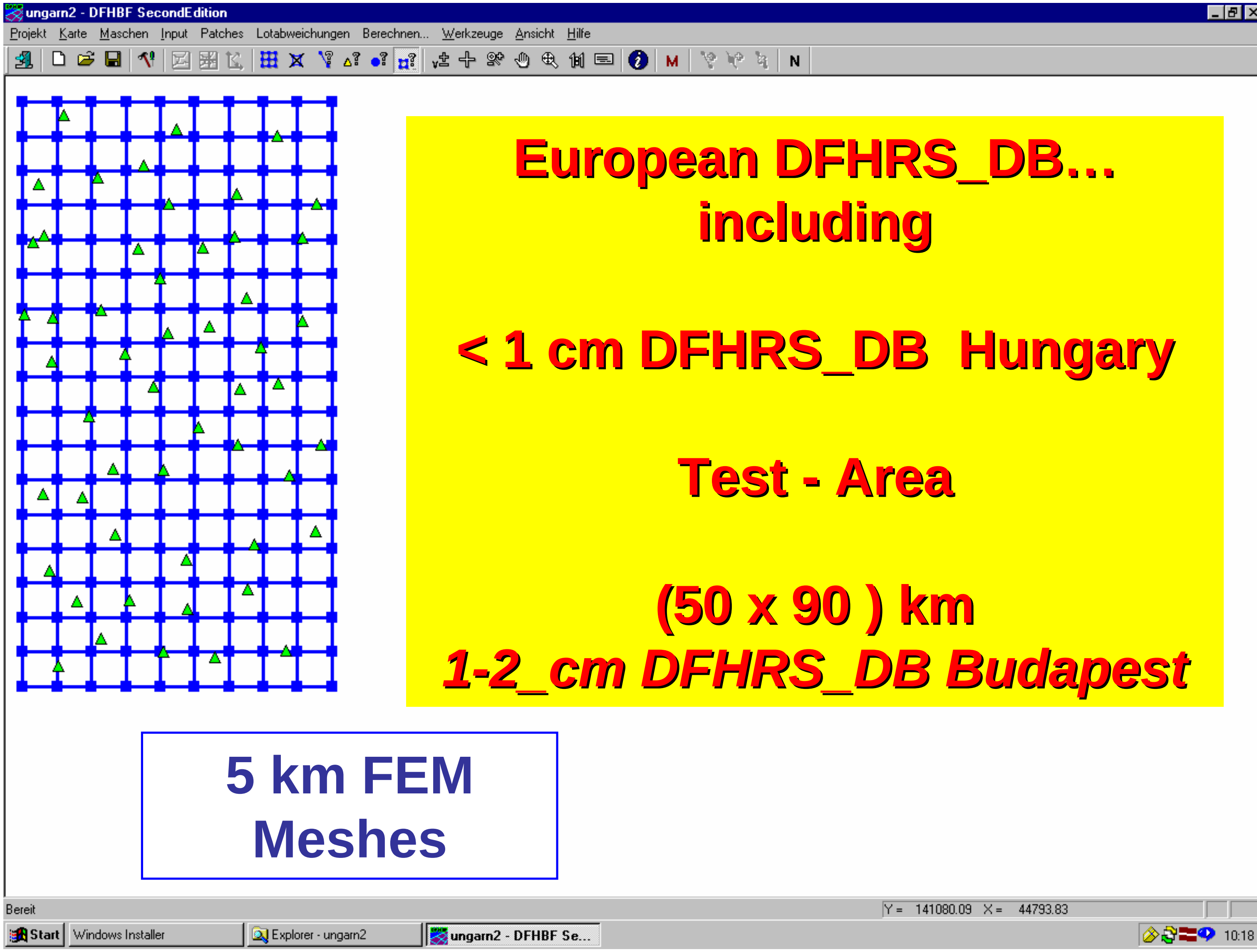

New 2005

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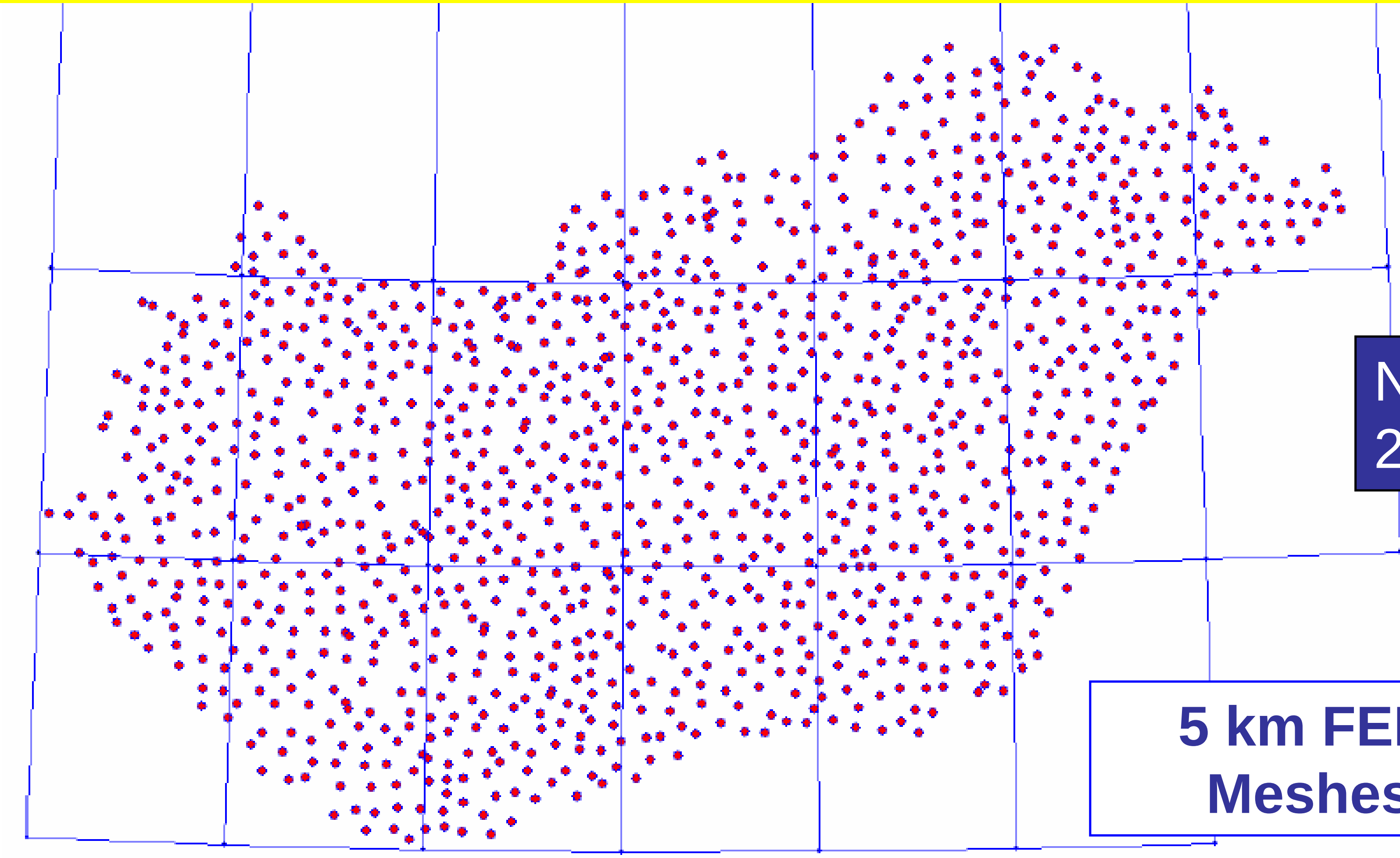
European HRS… including

< (1-3) cm DFHRS_DB Ungary - Masterthesis in DFHRS-Project & Cooperationproject with A.Kenyeres,Fömi, Hungary



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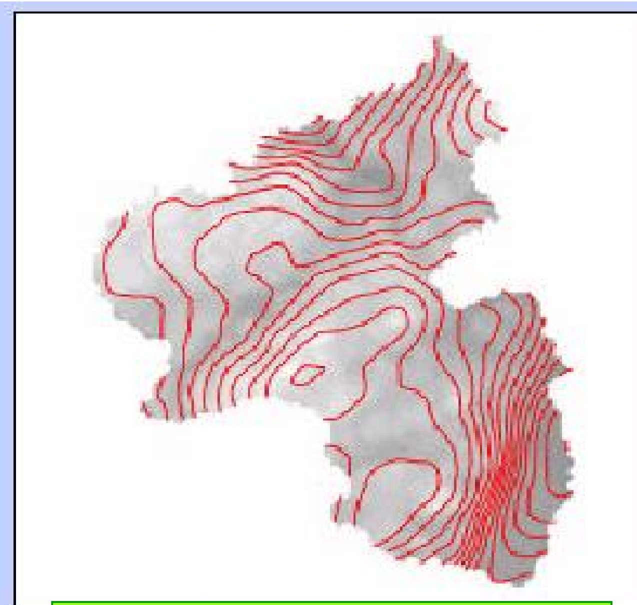
5 km FEM Meshes

… including
< 1cm DFHRS_DB Germany
< 1cm DFHRS_DB Luxembourg

5 km FEM Meshes

Official State Standard … over years



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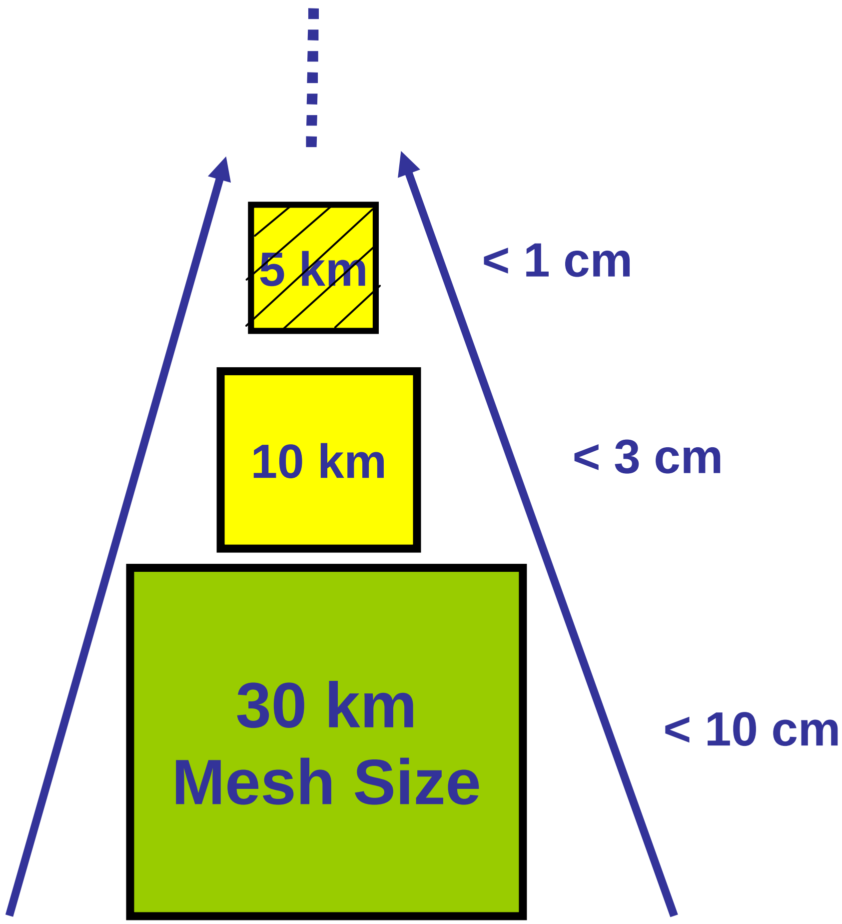
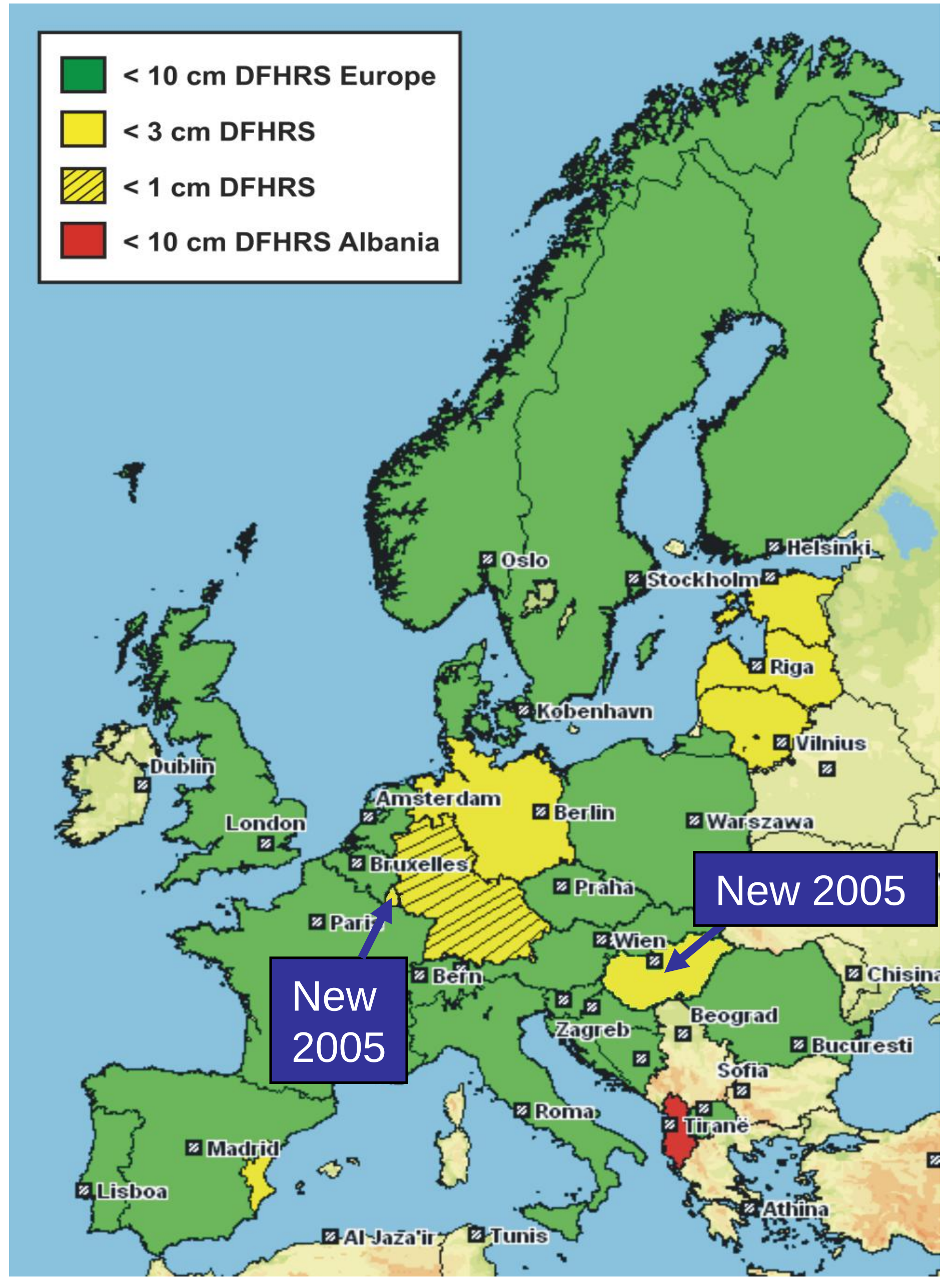
Herausgeber:

LVermGeo
Landesamt für Vermessung und Geobasisinformation Rheinland-Pfalz

Ferdinand-Sauerbruch-Straße 15
56073 Koblenz
http://www.lvermgeo.rlp.de

Overview about European DFHRS_DB





DFHRS_DB USA

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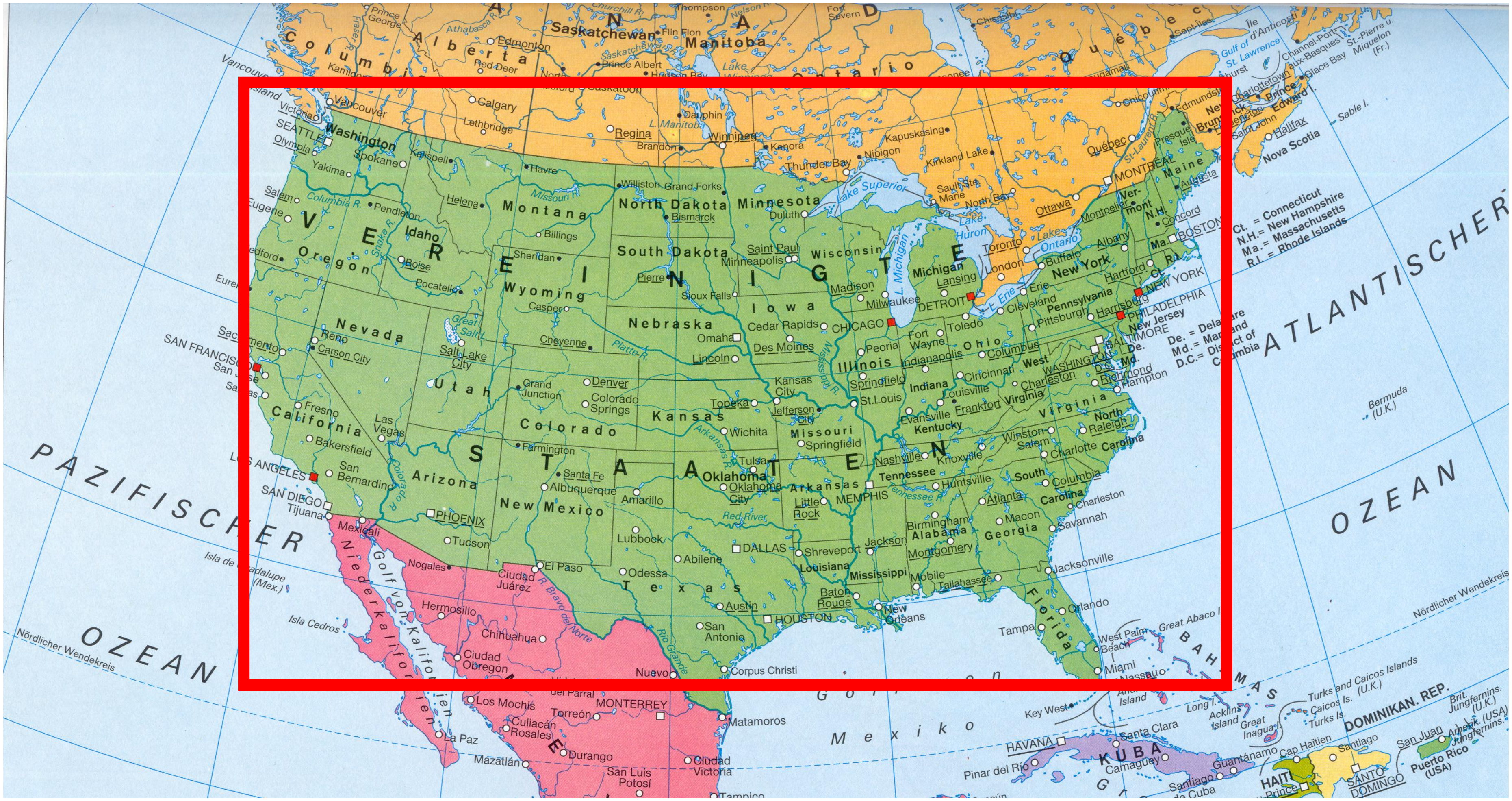

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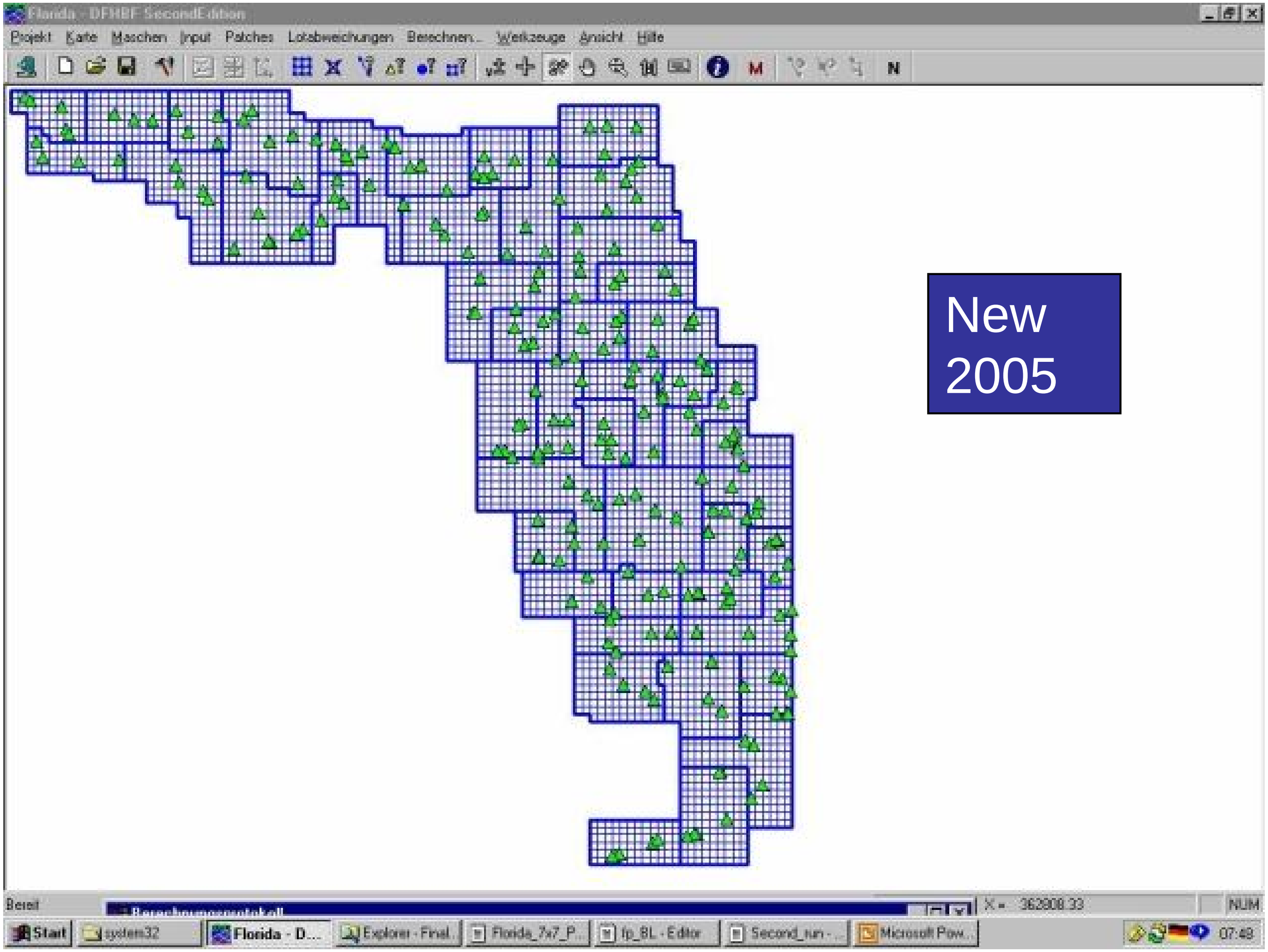
< 5 cm   DFHRS_DB Florida (… Masterthesis)



New
2005

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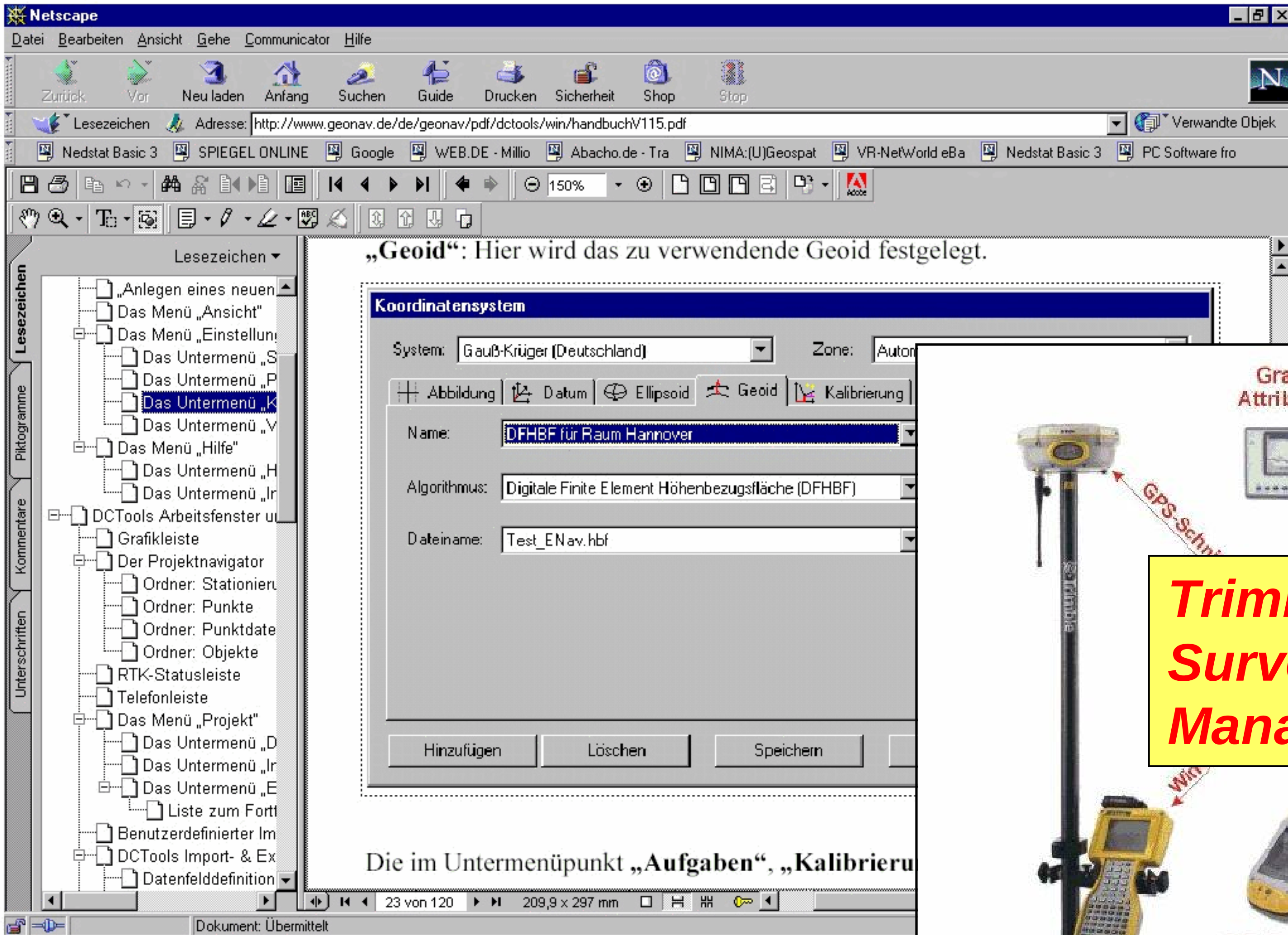
DFHRS in Practice



Trimble MAP500©

Trimble Survey Manager©

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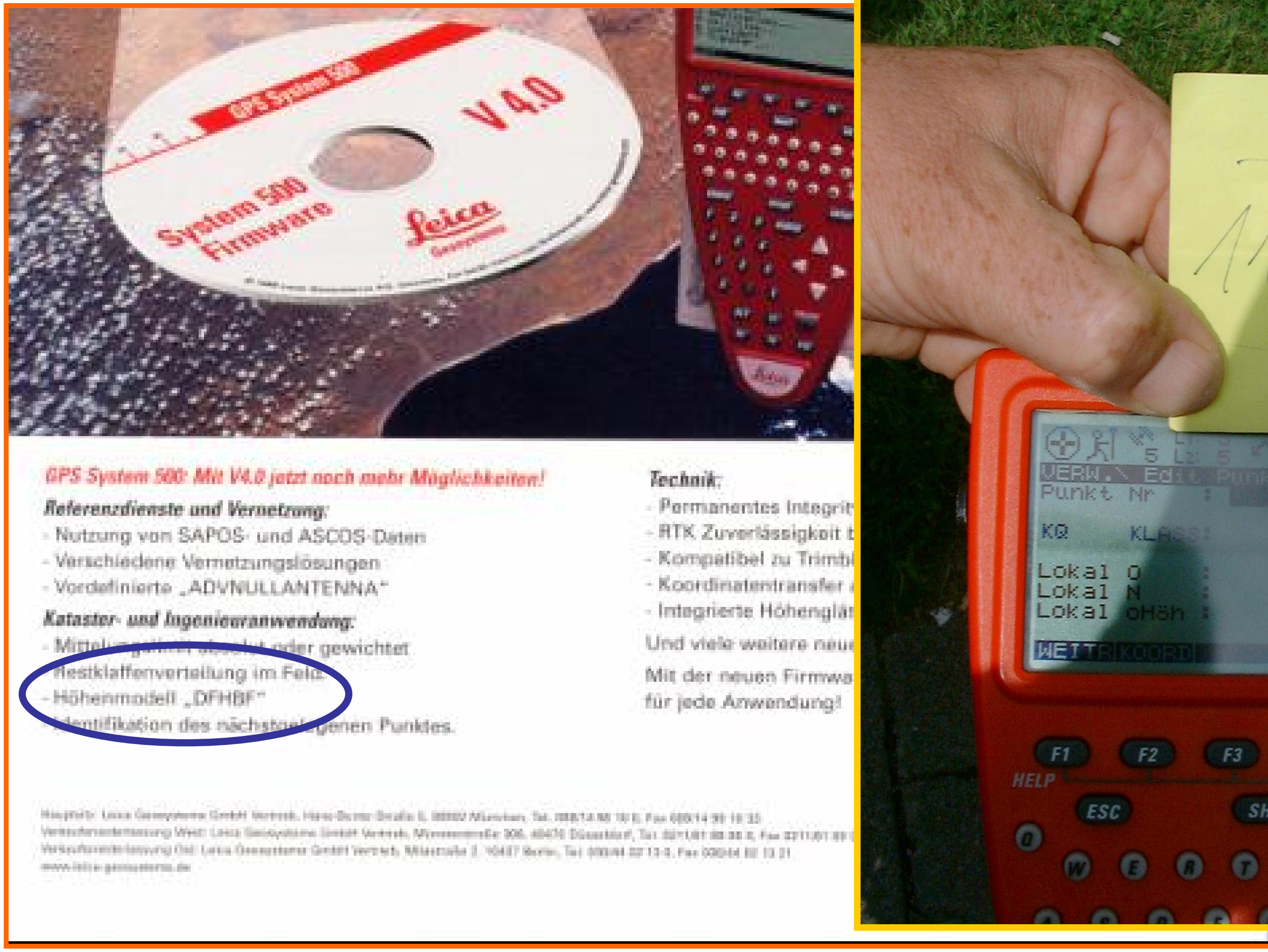
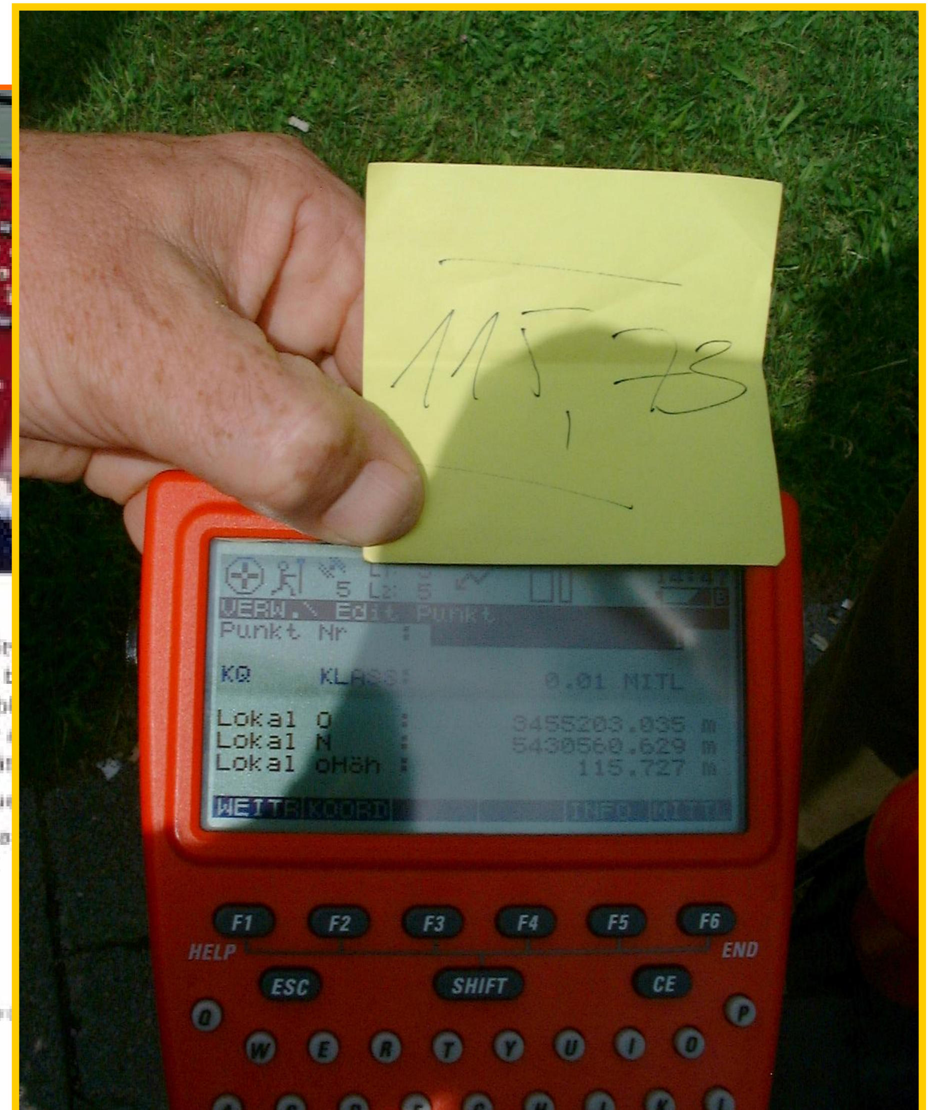
DFHRS in Practice





DFHRS in Practice

TOPSurv©

! New 2005 !

…… many other independent GNSS-Controller Software and GIS Packages

GART-2000®

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DFHRS - Extension to Gravity Observations

Gravity Potential of the Earth at position P : $W = V + Z$

$$V = \frac{G \cdot M}{r} \cdot (1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n \cdot (C_{nm} \cdot \cos m\lambda + S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\cos \vartheta) \quad \text{and} \quad Z = \frac{\omega^2}{2} \cdot r^2 \cdot \sin^2 \vartheta$$

Gravitational Potential. Coordinate System for Spherical Harmonices = „Geocentre“
Center of Masses → n=2

$$V = \frac{G \cdot M}{r} \cdot (1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n \cdot (C_{nm} \cdot \cos m\lambda + S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\cos \vartheta)$$

Observables at point P

$$\overset{\mathbf{r}}{g} = \begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix} \quad \text{mit} \quad g = \sqrt{W_x^2 + W_y^2 + W_z^2} \qquad \begin{pmatrix} \lambda = \arctan(\frac{W_x}{W_z}) \\ \varphi = \arctan(\frac{-W_z}{\sqrt{W_x^2 + W_y^2}}) \end{pmatrix} \quad \text{und} \quad g \cdot \begin{pmatrix} \cos\varphi \cdot \cos\lambda \\ \cos\varphi \cdot \sin\lambda \\ \sin\varphi \end{pmatrix}$$

As observables in an adjustment ?
ð „Integrated 3D network adjustment“ (Krarup 1980, Hein 1980)
ð … now again in the DFHRS concept

DFHRS - Extension to Gravity Observations

Gravity Observations and Anomalous Potential

1. Gravity disturbances from Gravity Observations:

Gravimeter for Gravity Measurements.
GPS for Georeferencing (….. ellipsoidal height h is needed)

Observations l : $\delta g_P = g_P - \gamma_P$

Relation of l to the Functional Model of the Anomalous Potential T_P

$$\delta g_P = -\left(\frac{\partial T}{\partial n}\right)_P \quad => \text{ Spherical Approximation } \delta g_P \cong -\left(\frac{\partial T}{\partial r}\right)_P .$$

It holds

$$dg_P = \frac{G \cdot M}{r^2} \cdot \sum_{n=2}^{\infty} (n+1) \cdot \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (\delta C_{nm} \cdot \cos m\lambda + \delta S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\cos \vartheta)$$

= DFHRS Concept

Georeferencing in Spherical Harmonics ?
=> Position P in $P(r, \lambda, \vartheta)$ <=> $P(x, y, z)$ <=> $P(B, L, h_P)$ at Earth Surface

DFHRS - Extension of Gravity Observations

Gravity Observations and Anomalous Potential

2. Gravity Anomalies from Gravity Observations:

Instrumentation: Gravitymeter and Levelling. Nivelling for Georeferencing (… Normal Heights H_N are needed).

Observations l: $\Delta g_P = g_P - \gamma_Q$

$$\gamma_Q = \gamma_0(B) \cdot [1 - 2(1 + f + m - 2f \cdot \sin^2 B) \cdot \frac{H_N}{a} + 3\left(\frac{H_N}{a}\right)^2]$$

Relations of l to the functional Model of Anomalous Potential T_P

$$\Delta g_P = -\left(\frac{\partial T}{\partial n}\right)_P + \frac{1}{\gamma_Q} \cdot \left(\frac{\partial \gamma}{\partial \bar{n}}\right)_Q \cdot T_P$$

$$\Delta g_P \cong -\left(\frac{\partial T}{\partial r}\right)_P - \frac{2}{r} \cdot T_P$$ - „*Fundamental Equation of Physicl Geodesy*"

…… „Solution of the above Differential Equation " leads to (Stokes, modified by Pizetti) :

$$N_{QG} = \zeta(r, \vartheta, \lambda) = \frac{a}{4\pi\gamma} \iint_\sigma S(r, \psi) \cdot \Delta g_P \cdot d\sigma$$

DFHRS - Extension to Gravity Observations

$$\Delta g_P \cong -\left(\frac{\partial T}{\partial r}\right)_P - \frac{2}{r}\cdot T_P$$ - *„Fundamental Equation of Physicl Geodesy"*

…… „Solution of the above differential equation " leads to (Stokes, modified by Pizetti)

$$N_{QG} = \zeta(r,\vartheta,\lambda) = \frac{a}{4\pi\gamma}\iint_\sigma S(r,\psi)\cdot\Delta g_P\cdot d\sigma$$

….. „Setting up the Derivatives" leads to **= DFHRS Concept**

$$\Delta g_P = \frac{G\cdot M}{r^2}\cdot\sum_{n=2}^{\infty}(n-1)\cdot\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\delta C_{nm}\cdot\cos m\lambda + \delta S_{nm}\cdot\sin m\lambda)\cdot P_{nm}(\cos\vartheta)$$

Georeferencing within the Spherical Harmonics ?

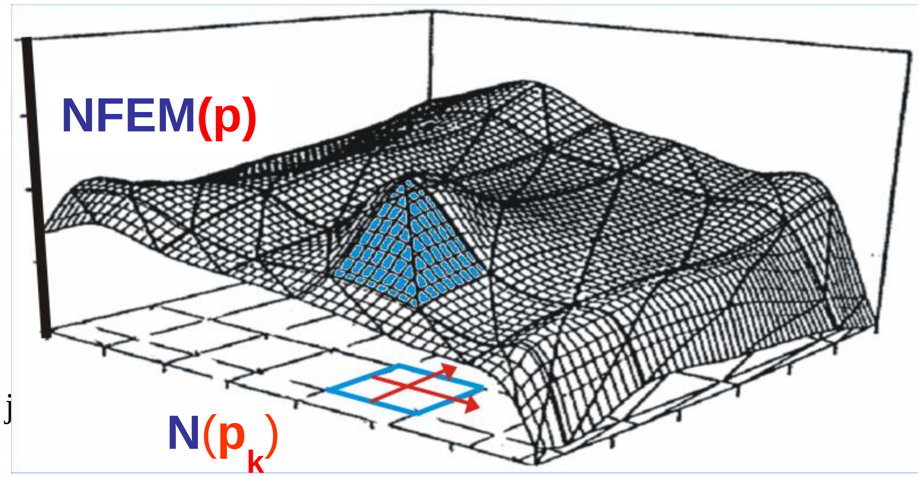
Position of Point P $P(r,\lambda,\vartheta)$ <=> $P(x,y,z)$ <=> $P(B,L,h_P)$ at the Earth Surface.

⇨ **Adjustment Approaches for Gravity-Anomalies and -Disturbances**

DFHRS – Extended Observation Equations

$h_{GNSS} + v = H + f^T \cdot p - h_{GPS} \cdot D\, m$

$H + v = H$



$N_G{}'^{j} + v^j = f^T \cdot p + \partial N_G(d^j)$

$\xi^j + v = -f_B^T / M(B) \cdot p + \partial\, \xi\, (d_{x,h})^j$

$\eta^j + v = -f_L^T / (N(B) \cdot \cos(B)) \cdot p + \partial\, \eta(d_{x,h})^j$

$\frac{a}{4\pi \cdot \gamma(B)} \iint_\sigma \Delta g \cdot S(\psi) d\sigma + v = \mathrm{NFEM}(p) = f^T \cdot p$

$$\Delta g_P + v = \frac{G \cdot M}{r^2} \cdot \sum_{n=2}^{\infty} (n-1) \cdot \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\delta C_{nm} \cdot \cos m\lambda + \delta S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\cos\vartheta)$$

$$0 + v = f^T \cdot p - \frac{G \cdot M}{a \cdot \gamma_{(Q)}} \cdot \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (\delta C_{nm} \cdot \cos m\lambda + \delta S_{nm} \cdot \sin m\lambda) \cdot P_{nm}(\cos\vartheta)$$

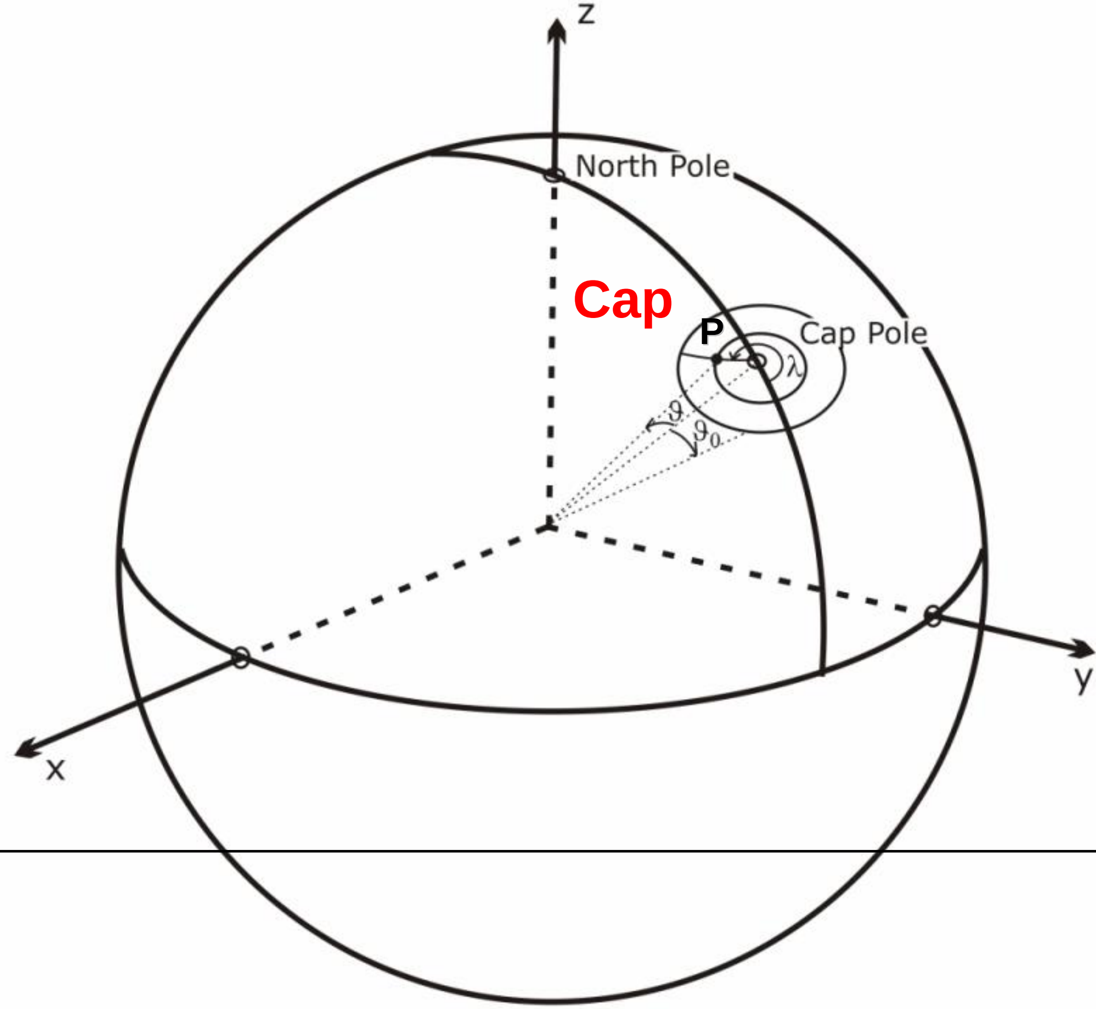
2005

DFHRS – Extension to Gravity Observations

Spherical

Cap

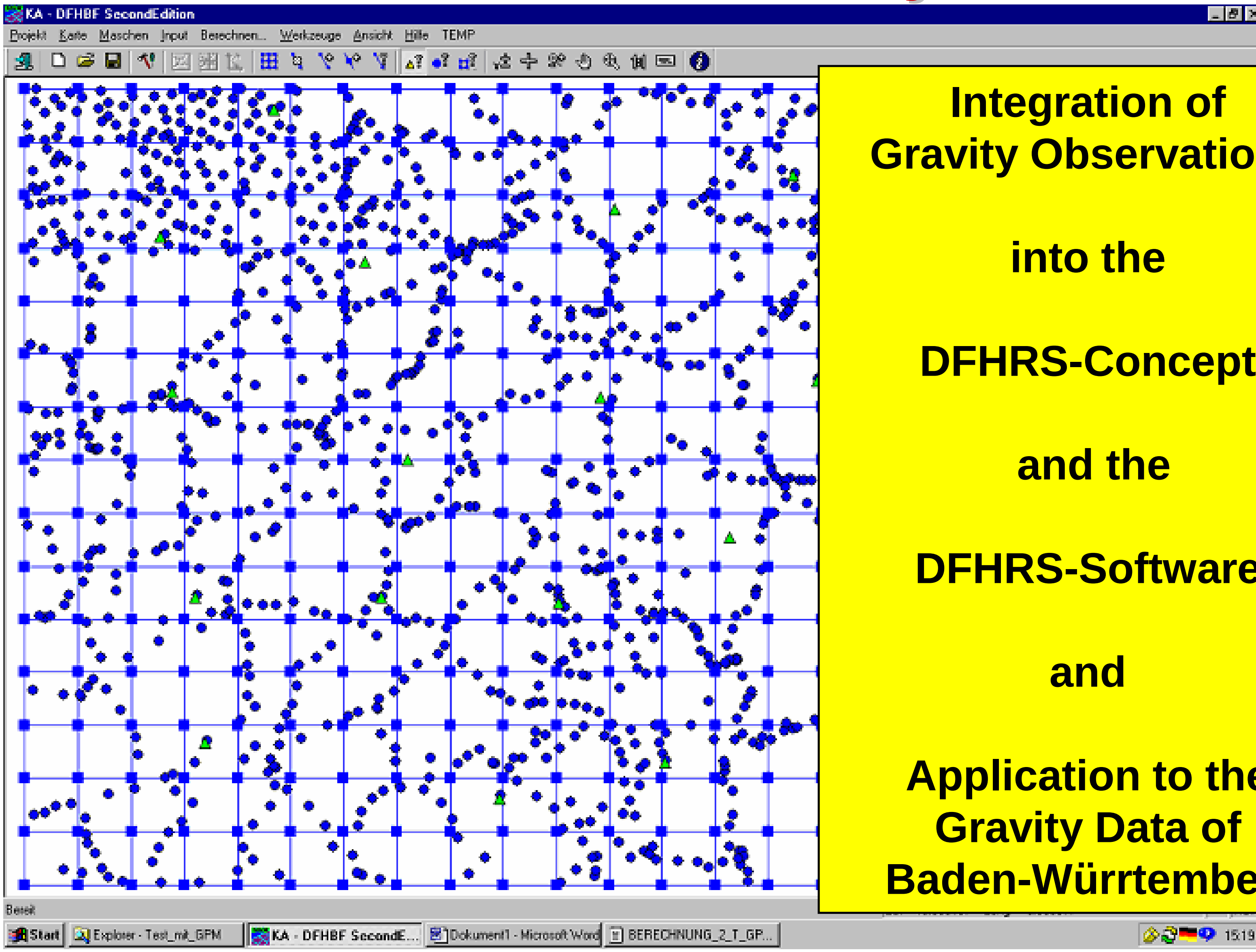
Harmonics



$$\Delta g_P(r,\eta,\vartheta) =$$

$$= \frac{G \cdot M}{r_{Q_0}^2} \cdot \sum_{k=2}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r_{Q_0}} \right)^{n_k(m)} (n_k(m)-1) \cdot (\overline{C}_{n_k(m),m} \cos m\overline{\lambda} + \overline{S}_{n_k(m),m} \sin m\overline{\lambda}) \cdot P_{n_k(m),m}(\cos(\overline{\vartheta}))$$

DFRHS – Extension to Gravity Observations



Integration of Gravity Observations

into the

DFHRS-Concept

and the

DFHRS-Software

and

Application to the Gravity Data of Baden-Württemberg

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DFRHS – Extension to Gravity Observations

Protocol Extract DFRHS Software - Identical Points Normal Heights H

Punktnummer	Höhe/Zielsys. [m]	Verb. [m]	Std.abw. [m]
721700108	691.458	-0.00028	0.00073
731601108	758.703	0.00057	0.00073
732000108	490.124	0.00055	0.00073
732100208	392.555	-0.00013	0.00073
741800208	616.968	0.00007	0.00073
751700408	727.193	-0.00064	0.00072
751810608	518.835	-0.00002	0.00071
752000208	463.330	-0.00098	0.00072
752121708	754.845	0.00006	0.00073
761700108	681.846	0.00100	0.00072
761800108	610.718	-0.00112	0.00072
761900108	921.755	0.00138	0.00072
762100208	809.084	0.00002	0.00073
781702408	704.955	-0.00025	0.00073
781900108	938.387	0.00023	0.00073
782022208	798.693	-0.00047	0.00073
:	:	:	:

DFRHS – Extension to Gravity Observations

Protocol Extract DFRHS Software
- Gravity Anomaly Observations

Nr	B	L	H	Dg	v	r(%)	NV
7217812800	48.70455	8.65901	642.66	47.35	-1.41	90.01	0.99
7217812900	48.71980	8.66704	670.25	47.81	-0.56	86.14	0.41
7217813000	48.72404	8.64581	700.85	49.63	0.15	86.97	0.11
7217813100	48.72762	8.55931	680.19	48.44	-1.43	90.00	1.00
7217813400	48.70274	8.62133	720.77	54.87	-0.74	92.15	0.51
7217813500	48.70714	8.58010	651.03	44.64	1.22	91.99	0.85
7217813600	48.73443	8.50825	798.13	61.70	-0.24	77.71	0.18
7217813800	48.72121	8.51805	575.79	36.44	0.94	87.63	0.67
7218802100	48.71432	8.74292	333.23	11.76	0.62	84.95	0.45
7218802107	48.70895	8.74090	348.02	12.99	1.11	88.94	0.79
7218803304	48.73078	8.73333	329.78	9.37	0.14	45.93	0.14
7218803404	48.70407	8.73647	337.49	11.75	1.41	89.95	0.99
7218803504	48.70271	8.74608	338.52	11.71	1.26	90.61	0.88
7218804204	48.70136	8.80471	547.03	33.85	-0.62	88.68	0.44
7218804304	48.70358	8.79797	505.22	29.32	-0.21	88.57	0.15
7218804404	48.71648	8.74922	379.82	16.56	0.26	83.66	0.19
7218804504	48.70953	8.76052	433.83	23.20	-0.79	89.04	0.56
7219803000	48.71078	8.91878	406.17	14.02	0.30	74.21	0.23
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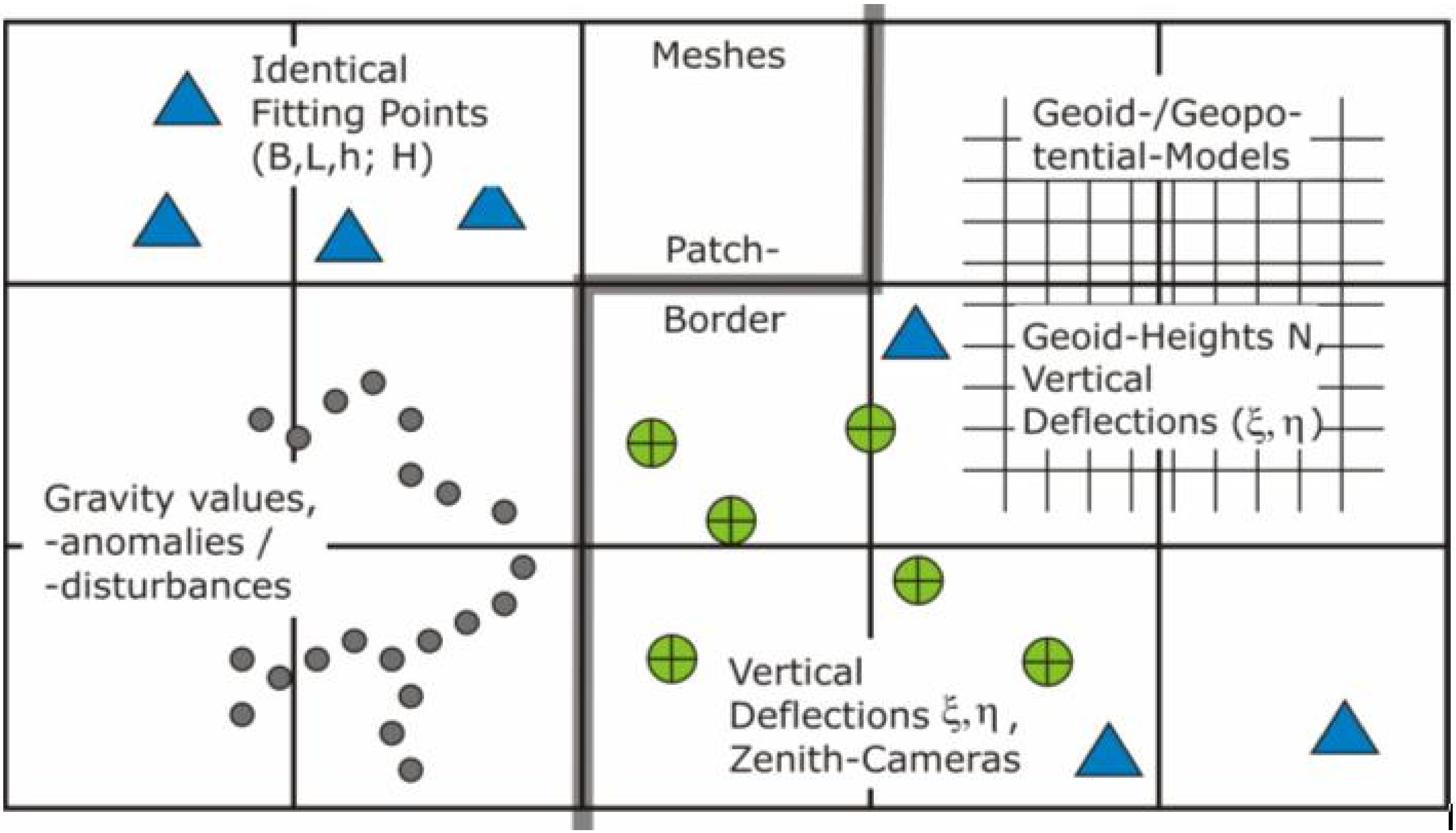
DFRHS – Extension to Gravity Observations

Geodetic Network Optimization - 1st/2nd/3rd Order Design: A,P =>C_p



Summary and Conlusions

- *GNSS-Age*
- *GNSS-Positioning*
- *Fundamental Transformation / Transition Problems*

Solution Concept for Heighting

www.dfhbf.de
www.geozilla.de

- Strict mathematical base for continuous FEM_HRS & **DFHRS-Software**
- New concept for an overdetermined BVP =>parametric HRS determination
- Mesh and patch-design => Any accuracy and any! area size (<= FEM)
- Open for all geometrical & physical (e.g. gravity) observations!
- DFHRS = (Leading) Geoidfitting Concept
- Ready for 1 cm EVRS using existing data +EPN densification fitting-points!
- High practical relevance for GNSS services and GIS
- Industrial Standard in GNSS-Equipment and GIS
- DFHRS_DB => RTCM 3.0 Message used in GNSS-Services, NTRIP etc.
- High Capacities for International Co-operations