Review of the ETRS89 Realisation

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Current approach of a TRS Definition

Assume a 7-parameter model with linear time evolution:

- Definition at a chosen epoch, by selecting 7 transformation parameters,

and

- A law of time evolution, by selecting 7 rates of the 7 transformation parameters,

tending to satisfy the theoretical definition of the corresponding TRS

ETRS89 Definition

- Coincides with ITRS at epoch 1989.0:
  - Definition at a reference epoch (1989.0)
  - The 7 parameters between ITRS and ETRS89 are zero at 1989.0
- Fixed to the stable part of the Eurasian plate
  - Co-moving with the plate: law of time evolution
  - Time derivatives of the transformation parameters are zero except the 3 rotation rates

ETRS89 Realisation

- **Expression in $ITRF_{YY}$ at central epoch (t_c) of the implied observations**
- **Expression in ETRS89 using 14 transformation parameters some of them are zeros**

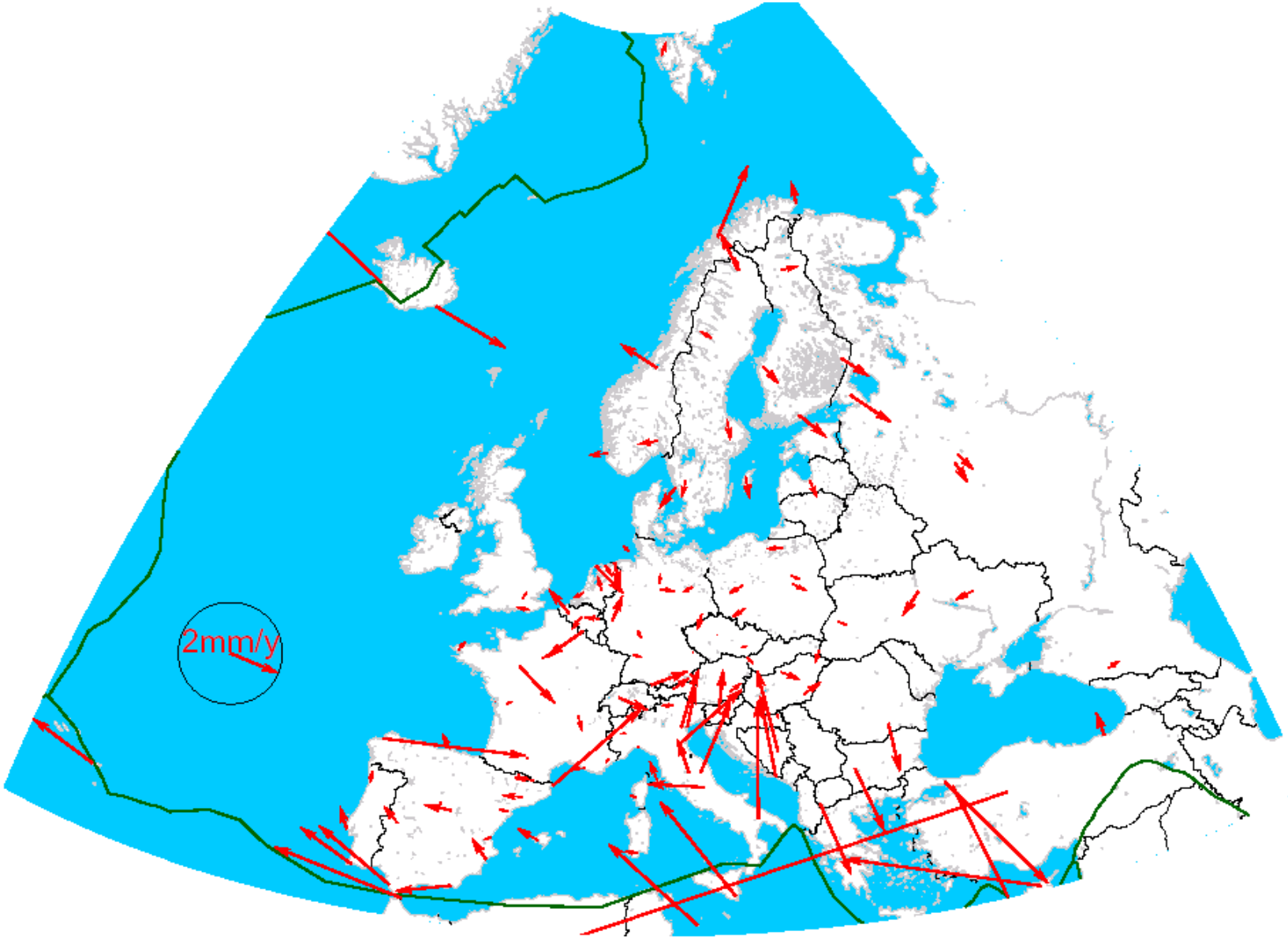
Positions

$$X^E(t_c) = X^I_{YY}(t_c) + T_{YY} + \begin{pmatrix} 0 & -\dot{R}3_{YY} & \dot{R}2_{YY} \\ \dot{R}3_{YY} & 0 & -\dot{R}1_{YY} \\ -\dot{R}2_{YY} & \dot{R}1_{YY} & 0 \end{pmatrix} \times X^I_{YY}(t_c).(t_c - 1989.0)$$

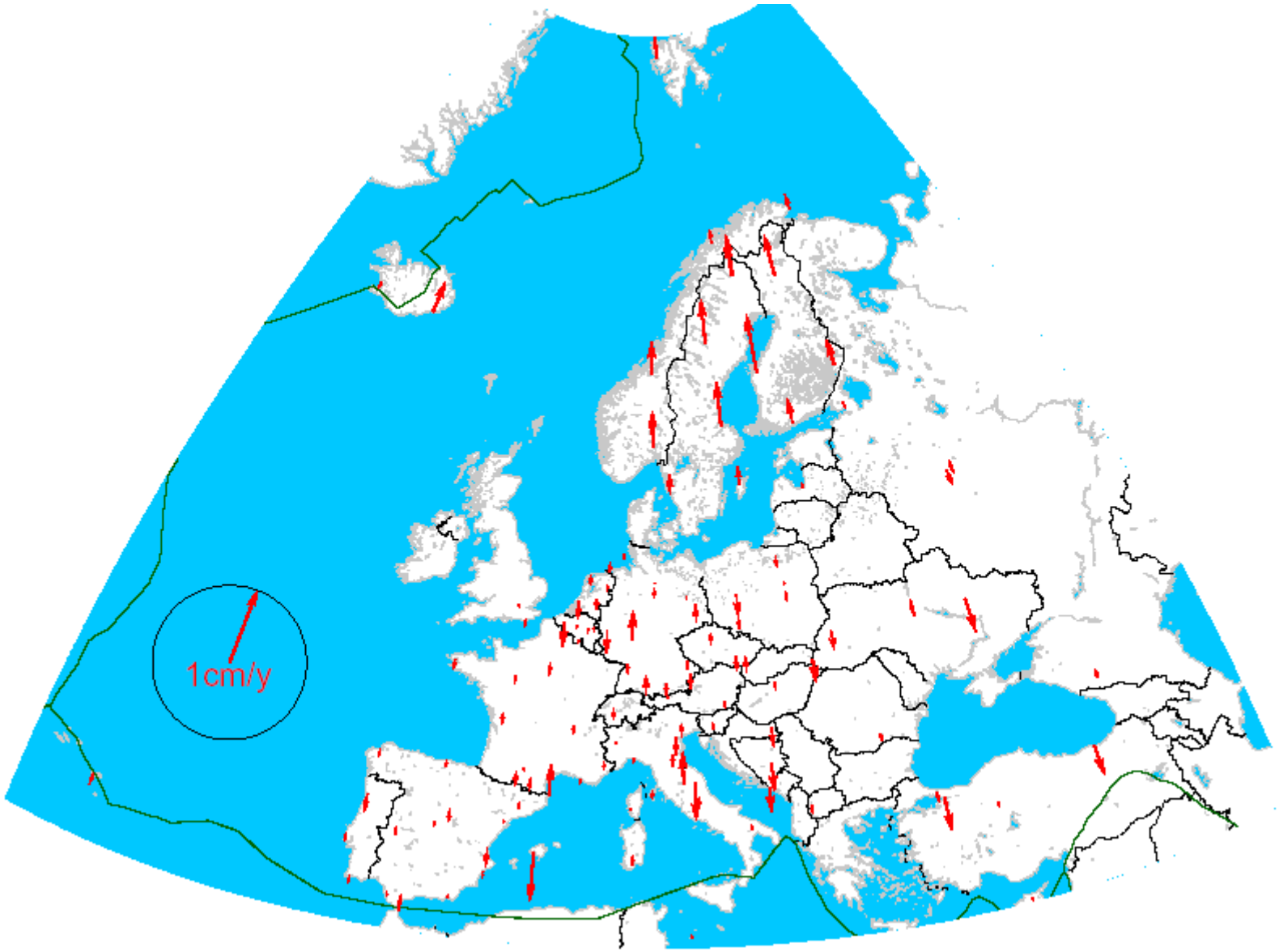
Velocities

$$\begin{pmatrix} \dot{X}^E_{YY} \\ \dot{Y}^E_{YY} \\ \dot{Z}^E_{YY} \end{pmatrix} = \begin{pmatrix} \dot{X}^I_{YY} \\ \dot{Y}^I_{YY} \\ \dot{Z}^I_{YY} \end{pmatrix} + \begin{pmatrix} 0 & -\dot{R}3_{YY} & \dot{R}2_{YY} \\ \dot{R}3_{YY} & 0 & -\dot{R}1_{YY} \\ -\dot{R}2_{YY} & \dot{R}1_{YY} & 0 \end{pmatrix} \times \begin{pmatrix} X^I_{YY} \\ Y^I_{YY} \\ Z^I_{YY} \end{pmatrix}$$

EPN ETRS89 Horizontal Velocities



EPN ETRS89 Vertical Velocities



How to realise the ETRS89 ?

- **ITRFyy ==> ETRFyy**

  Straightforward: clear transformation formula

- **GPS campaign, weekly solution, others…**

  All the problem is how to express first the solution in the ITRF ?

  1. **Fixing (constraining) some points to ITRF values**
  2. **Using 7-parameter transformation**
  3. **Using Minimum constraint approach(*)**

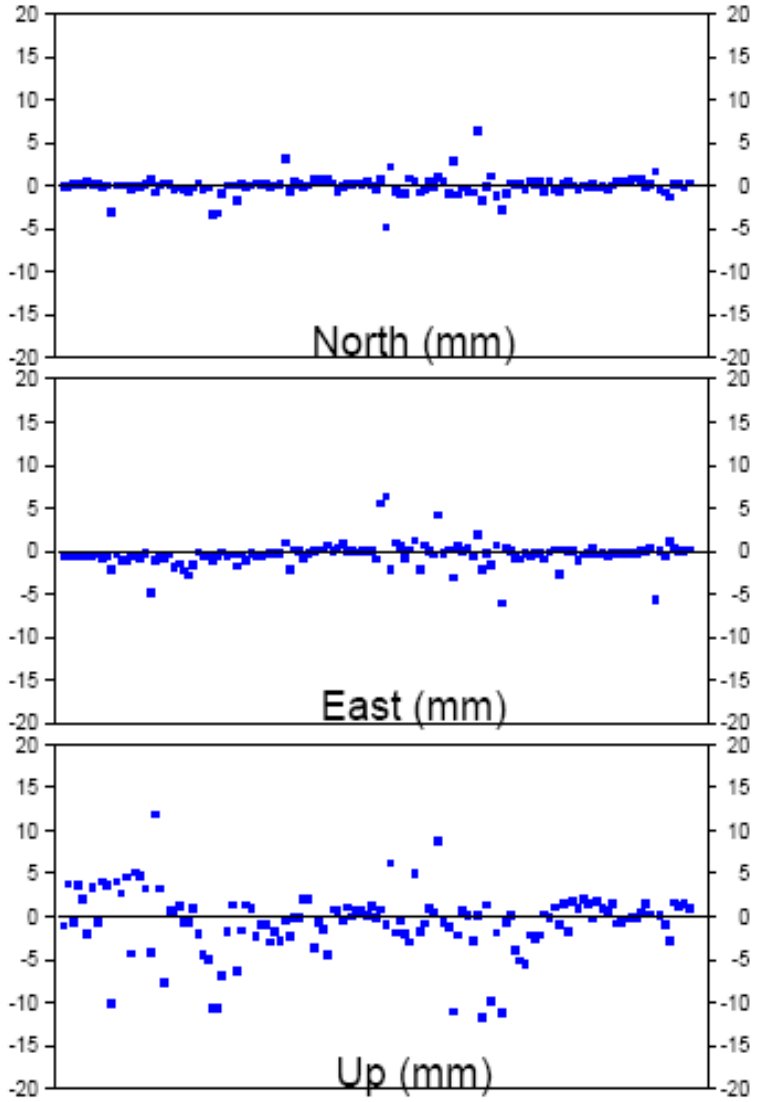
  In all cases a reference set of stations is needed

(*) **See e.g. (Altamimi, 2002, Proceedings of Ponta Delgada)**

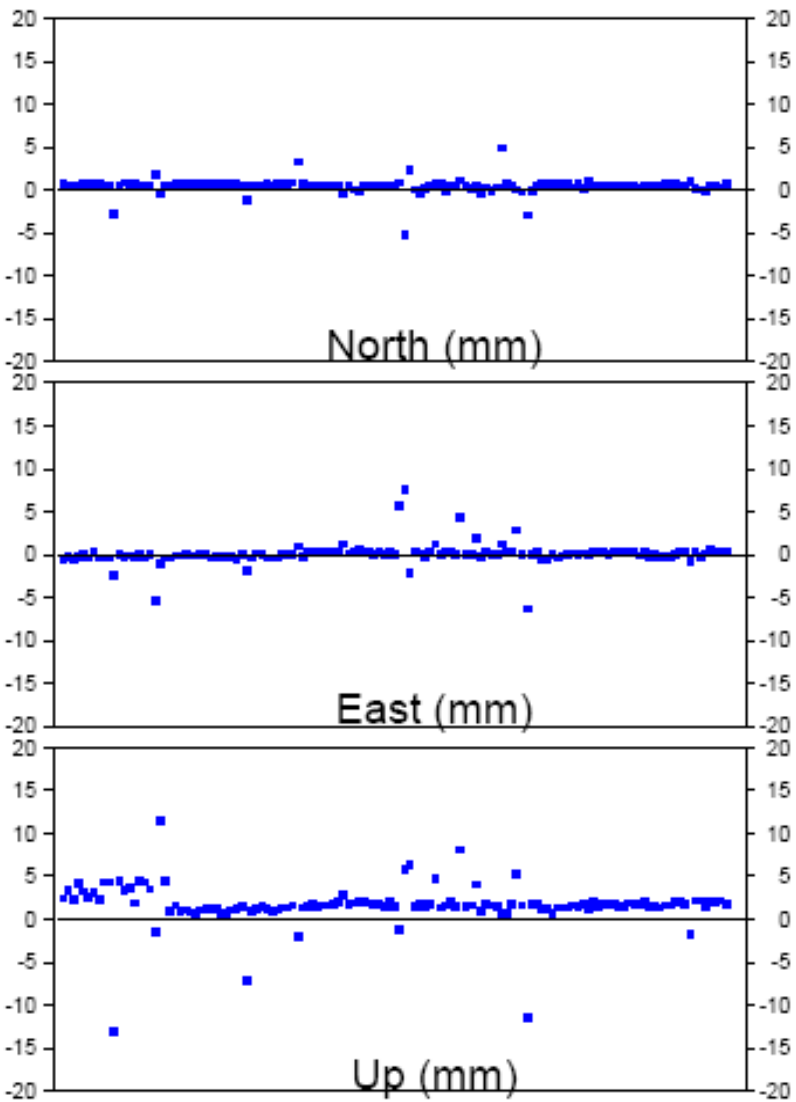
Numerical Example: EPN Week 1200

- **Expression in ITRF2000:**
  - **Fixing 12 stations**
  - **Transformation over the 12 stations**
  - **Minimum constraints over the 12 stations**

Constrained - Transformed



Constrained – MC'd



ETRS89 – Epoch of station positions

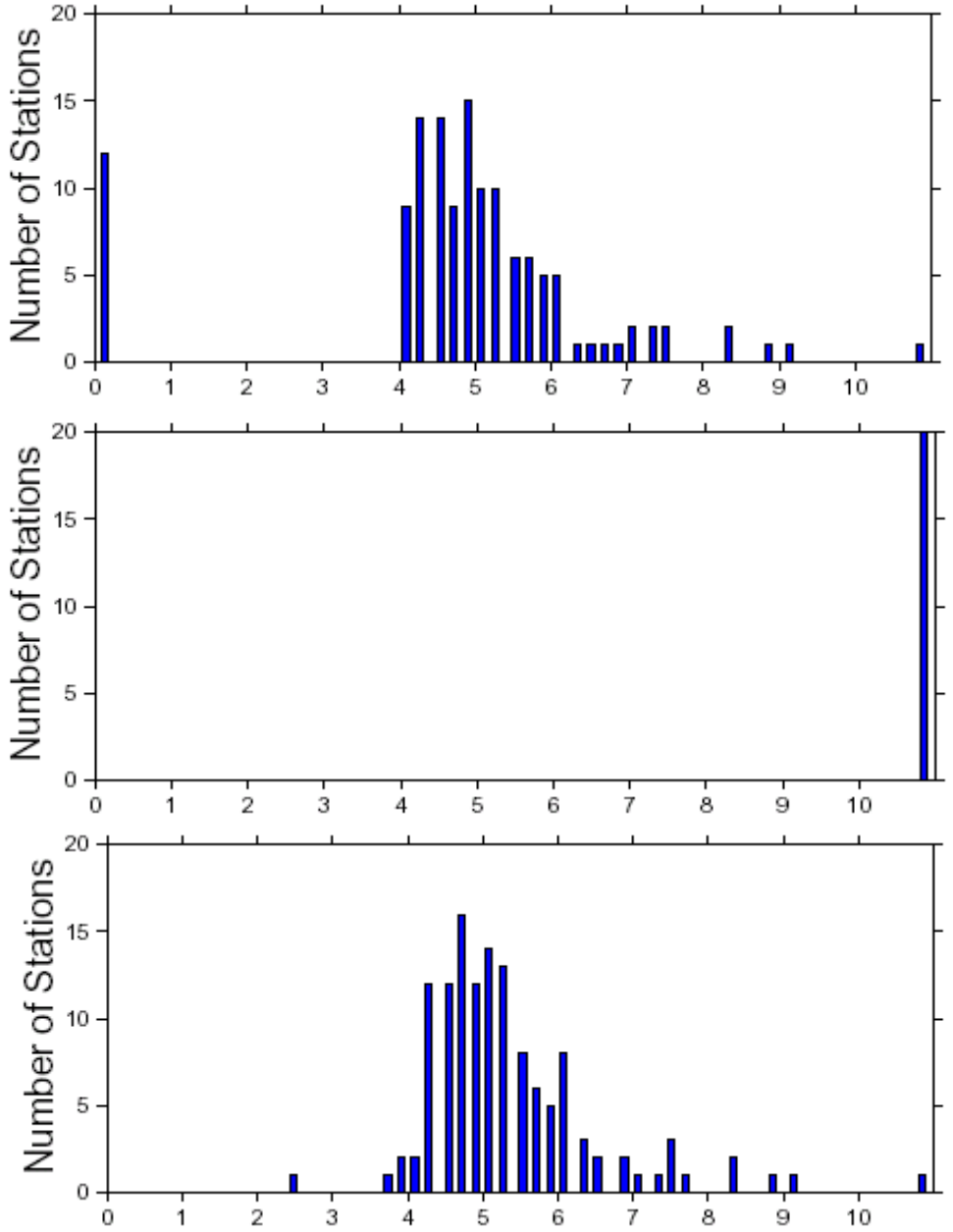
- **GPS campaign, Weekly solution:**
  - **Central Epoch of the used observations**
  - **Positions should'nt be mapped to « 1989.0 » by any kind of velocities, otherwise positions will be degraded by**

$$\mathrm{Var}(X(t)) = \mathrm{Var}(X(t_s)) + 2(t - t_s)\,\mathrm{Cov}\left(X, \dot{X}\right) + (t - t_s)^2 \mathrm{Var}\left(\dot{X}\right).$$

- **Multi-year solution (Positions & Velocities)**
  - **Central Epoch of the used observations**
  - **Epochs of minimum variance per station**

$$t = t_s - \frac{[\mathrm{Cov}(x, \dot{x}) + \mathrm{Cov}(y, \dot{y}) + \mathrm{Cov}(z, \dot{z})]}{[\mathrm{Var}(\dot{x}) + \mathrm{Var}(\dot{y}) + \mathrm{Var}(\dot{z})]}.$$

Formal Errors (mm): EPN Week 1200

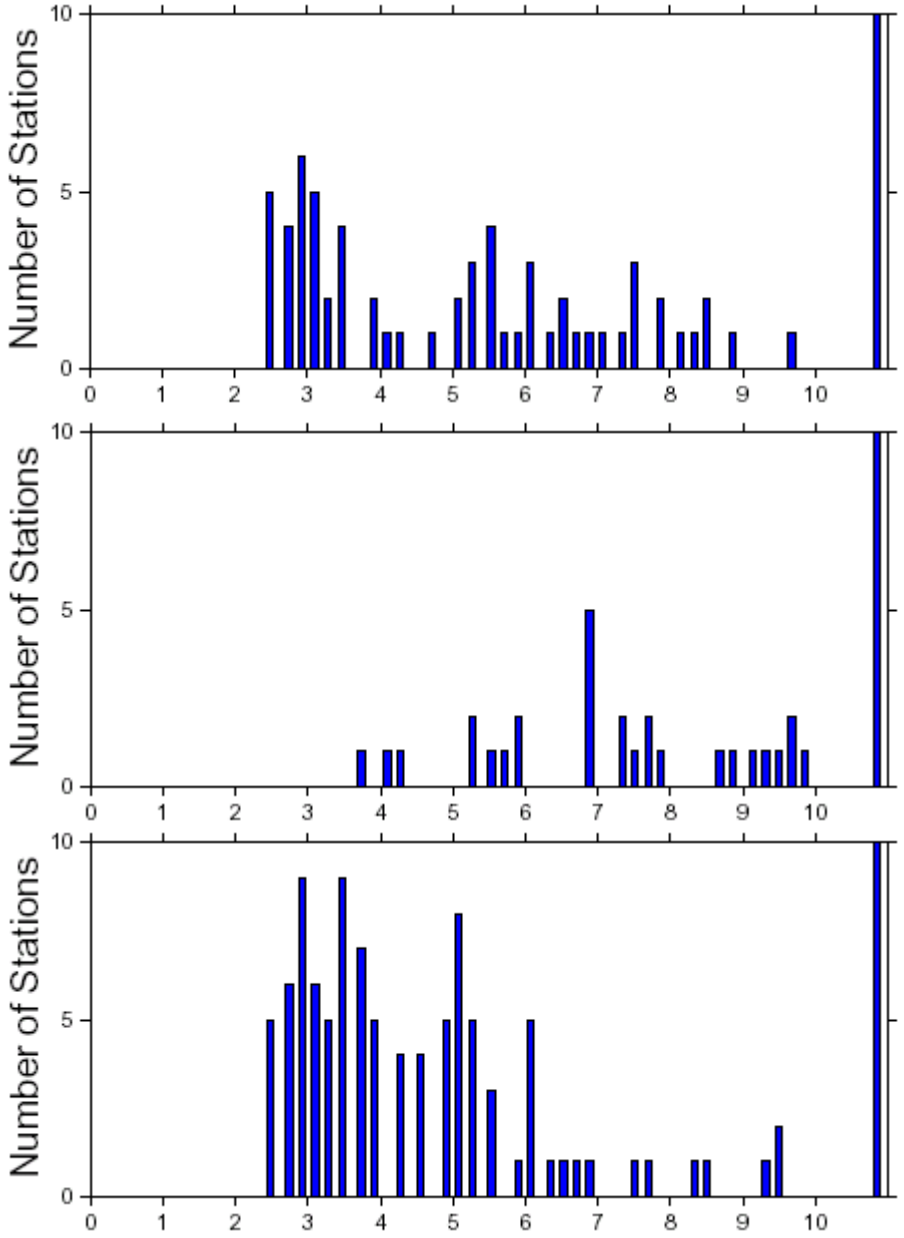


Constrained

Free

MC

Formal errors(mm)
ITRF2000 ==> ETRF2000



Epoch 1997.0

Epoch 1989.0

Epoch of Minimum Variances

Conclusions

- **ETRS89 is fully linked to ITRS**
- **Clear/rigorous mathematical transformation formula btw ITRS and ETRS89 realisations invloving both positions and velocities**
- **Differences may occur btw different realisations:**
  - **ITRF updates**
  - **The used strategy**
  - **The network effect**
  - **…**
- **Epoch of GPS campaign should not be mapped to epoch « 1989.0 »**