Conventions of the European Vertical Reference System 2007

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II. European Vertical Reference System 2000 (EVRS 2000)

III. Definition and Realization of a GVRS

IV. EVRS Realization – Principles

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VI. EVRS 2007 Conventions - Realization

I. Objectives of EVRS 2007

- Request of EC, Consideration of user requirements in Europe
- Realization of an up-to-date European Height Reference Frame
- Continuation of the previous development of European Vertical Reference System
- Guarantee of a 1 cm accuracy level for datum and network realization
- Alignment to IVRS/WHS

II. European Vertical Reference System 2000 (EVRS 2000)

- It is prepared by the IAG Subcommissin for Europe for adoption by the European Commission to promote widespread use as a de-facto standard for future pan–European GIS data products and services.

- The EVRS is defined as a World Height System (WHS) and realized for practical use as a static system under the name EVRF2000 by
  - the datum of 'Normaal Amsterdams Peil' (NAP)
  - gravity potential differences with respect to NAP or equivalent normal heights.

- The EVRS and the European Terrestrial Reference System 1989 (ETRS89) form together a European Spatial Reference System as an integrated reference.

EVRS 2000 definition (Tromsø)

The European Vertical Reference System (EVRS) is a gravity-related height reference system. It is defined by the following conventions:

a) **The vertical datum is the zero level of which the Earth gravity field potential W_0 is equal to the normal potential of the mean Earth ellipsoid U_0:**

$$W_0 = U_0.$$

b) **The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points P and the potential of the EVRS zero level W_0. The potential difference $-\Delta W_P$ is also designated as geopotential number c_P:**

$$-\Delta W_P = W_0 - W_P = c_P.$$

Normal heights are equivalent to geopotential numbers.

c) **The EVRS is a zero tidal system[1], conforming to the IAG Resolutions No 16 adopted in Hamburg in 1983**

datum

geocentric, including oceans and atmosphere

W_0 independent from the tidal system (Bursa)

coordinate system

SI units
$m^2 \cdot s^{-2}$

$W_p = U_p + T_p$ *(BVP)*

$W_p = W_0 - c_p$ *(levelling)*

$$H_n = \frac{c_p}{\bar{\gamma}}$$

frame

1) In a) and b) the potential of the Earth includes the potential of the permanent tidal deformation but excludes the permanent tidal potential itself.

Realization of the EVRS 2000 (EVRF 2000)

Datum

a) The vertical datum of the EVRS is realized by the zero level through the Normaal Amsterdams Peil (NAP). Following this, the geopotential number in the NAP is zero:

$$c_{NAP} = 0.$$

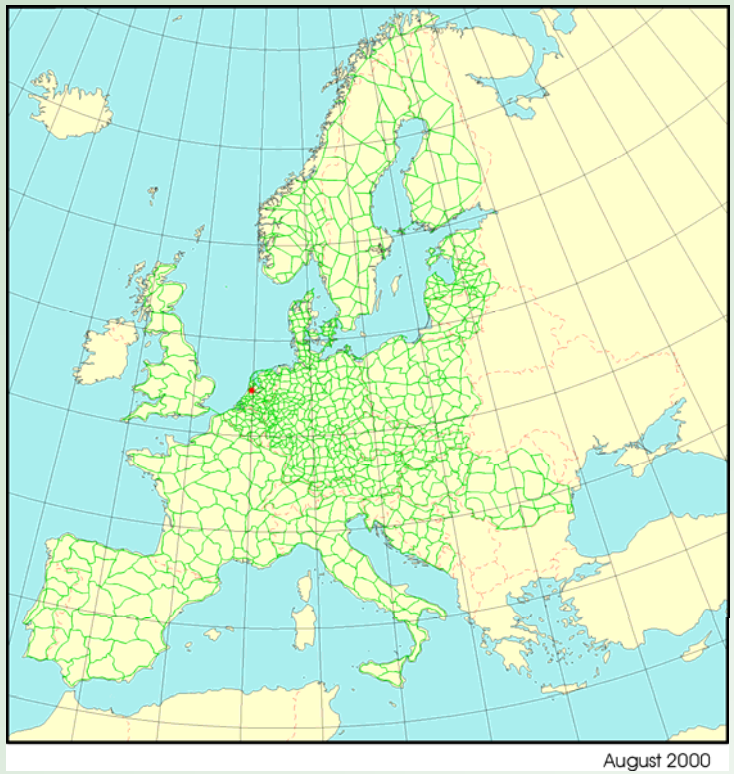
b) For related parameters and constants the Geodetic Reference System 1980 (GRS80) is used. Following this the Earth gravity field potential through NAP W_{NAP} is seed the normal potential of the GRS80

c) The EVRS2000 datum is fixed by the geopotential number 7.0259 $m^2 s^{-2}$ and the equivalent normal height 0.71599 m^2 of the reference point of the UELN No. 000A2530/13600.

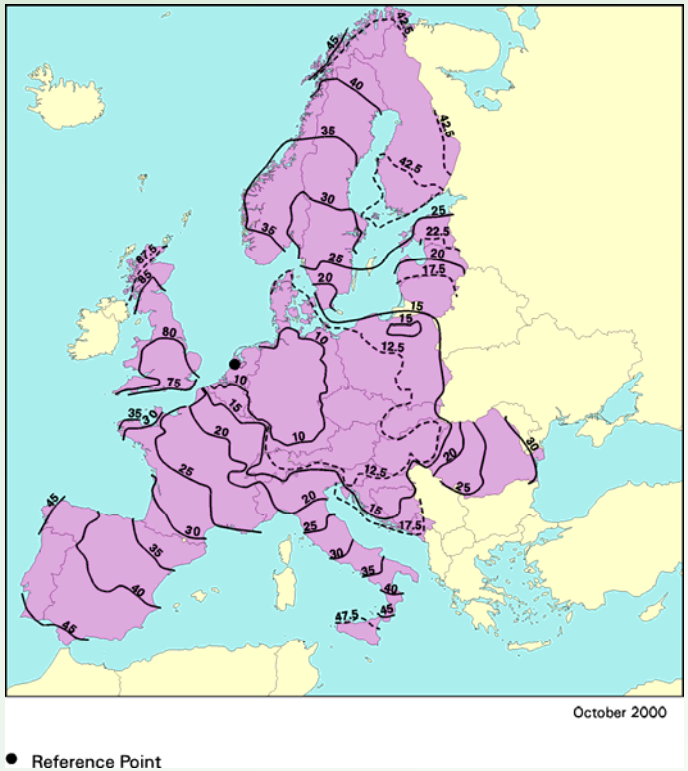
$$W_{NAP} = U_{0GRS80}$$

Frame

The adjustment of geopotential numbers was performed as an unconstrained adjustment linked to the reference point of UELN 73 (in NAP). In January 1999, the adjustment version UELN 95/13 was handed over to the participating countries as the UELN 95/98 solution.



United European Levelling Network 1995 (UELN-95/98)



UELN 95/98 – Isolines of Precision [kgal · mm]

IAG Inter-commission Project (ICP) 1.2

Vertical Reference Frames (2003 – 2007)

Objective:

Provide the fundamentals for the installation of a unified global vertical reference frame.

Tasks:

To elaborate a proposal for the definition and realization of a global vertical reference system (World Height System – WHS)

To derive transformation parameters between regional vertical reference frames

To establish an information system describing the various regional vertical reference frames and their relation to a world height frame (WHF).

III. Definition and Realization of a GVRS

In alignment to the Conventions of the International Earth Rotation and Reference System Service (IERS) from 2003 we have to consider Vertical Reference System and Frame (GVRS, GVRF)

- **Definition of GVRS: Conventions for datum, codes, time dependent variations, parameters**
- **Realization of GVRS (GVRF): Conventions and specification for procedures of computations (data reductions, selections of alternative procedures), selection of data, station distribution.**

GVRS Conventions

under discussion in ICP1.2

aligned to IERS 2003 Conventions

The Global Vertical Reference System (GVRS) definition fulfils the following four conventions:

1. **The vertical datum is defined as the equipotential surface for which the Earth gravity field potential is constant:**

$$W_0 = const.$$

Note: The vertical datum defines the relationship of the physical heights to the Earth body. W_0 shall be conventional and reproducible.

2. **The unit of length is the meter (SI). The unit of time is second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modelling;**

3. **The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points P and the potential of the GVRS conventional zero level W_0. The potential difference ΔW_P is also designated as geopotential number c_P:**

$$-\Delta W_P = c_P = W_0 - W_P \,.$$

4. **The GVRS is a zero tidal system, in agreement with the IAG Resolution No 16 adopted in Hamburg in 1983.**

VRS Realization and Unification

Principles

a) Levelling

$W_p = W_0 - c_p$ by adjustment of levelling networks,

$$H_n = \frac{c_P}{\bar{\gamma}}$$

related to long term tide gauge observations, normally W_0 of the zero level of the levelling network not used if not known

b) BVP/GGM and GNSS

$W_p = U_p + T_p$ by combination of a

$$\zeta = \frac{T_p}{\gamma_Q} = \frac{W_P - U_P}{\gamma_Q}$$

conventional GGM

(EGM06 or a combined CHAMP/GRACE model EIGEN-CG03C, …)

and ellipsoidal heights h_p

$$H_n = h_P - \zeta$$

(ITRF2005)

Practical possibilities for VRS realization

i. On continents: by common adjustments of existing levelling networks c_{P_k}

ii. General case for realization and unification: combination of GNSS or GNSS/levelling with GGM

iii. Over oceans: using a model of mean sea surface topography and tide gauge observations (unification only)

IV. EVRS Realization – Principles

(1) New adjustment of the UELN

(2) Keeping the European vertical datum - NAP level - of UELN95/98 at Epoch 2000

(3) Reduction of data - Tidal system

(4) Observation of vertical movements of UELN points against the conventional value W_{0E}

(5) Determination of a W_{0E} at Epoch 2000, fixing it and observe the relationship to a W_0 of a IVRS

V. EVRS 2007 Conventions Definition

The European Vertical Reference System (EVRS) 2007 is a kinematical height reference system. The EVRS definitions fulfils the following four conventions:

1. **The vertical datum is defined as the equipotential surface for which the Earth gravity field potential is constant:**

$$W_0 = W_{0E} = const.$$

And is in the level of the Noormals Amsterdam Peil.

2. The unit of length of the EVRS is the meter (SI). The unit of time is second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modelling;

3. The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points P and the potential of the EVRS conventional zero level W_0. The potential difference ΔW_P is also designated as geopotential number c_P:

$$-\Delta W_P = c_P = W_{0E} - W_P \,.$$

Normal heights are equivalent to geopotential numbers.

4. The EVRS is a zero tidal system, in agreement with the IAG Resolution No 16 adopted in Hamburg in 1983.

VI. EVRS 2007 Conventions Realization

(1) EVRS Datum

Keeping the vertical datum European NAP level of UELN95/98 at Epoch 2000 by fitting the UELN07 (free) adjustment to the UELN95/98 solution by identical points:

$$\sum_{i=1}^{n} (c_{P2007} - c_{P95/98}) = 0$$

The EVRS 2007 geopotential number of each datum point realize the EVRS datum.

Selection of a couple (10 – 15) of identical points for which it can assumed, that they are stable marked and located in the stable part of the European part plate and connected by precise measurements.

(2) EVRS vertical components - geopotential numbers and normal heights

(a) New adjustment of the UELN network using all current available levelling and gravity observations reducing to the epoch 2000

$$c_P = -\Delta W_P = W_{0E} - W_P .$$

(b) GNSS levelling

using GNSS vertical components in ITRF/ETRS89 – GRS80 and a EVRS aligned European Geoid EGG07

$$c_P = -\Delta W_P = W_{0E} - U_P - T_{P\,EGG}$$

Computation of Normal Heights

Both the geopotential numbers and the normal heights of EVRS 2007 were handed over to the participating countries. Nevertheless, each country can convert the geopotential numbers in any height system for its own use. The formula for the normal heights H_n is

$$H_n = \frac{C_P}{\bar{\gamma}}$$

where is the average value of the normal gravity along the normal plumb line between the ellipsoid and the telluroid and between the quasigeoid and the earth surface respectively.

The average value of the normal gravity along the normal plumb line is determined by

$$\bar{\gamma} \approx \gamma_m = \gamma_0 - \frac{0.3086\,\text{mgal/m} \cdot h}{2} + \frac{0.072 \cdot 10^{-6}\,\text{mgal/m}^2 \cdot h^2}{2}$$

with the Gravity Formula 1980 and latitude in ETRS89.

(3) EVRS Data Reduction

Time varying observations should to be reduced to Epoch 2000 and to zero tidal system.

If the UELN 2007 levelling observations have no tidal corrections, the EVRS 2007 geopotential numbers are approximately given in mean tidal system.

The EVRS vertical components are delivered in zero tidal system and mean tidal system.

(4) EVRS Time Evolution

Observation of vertical movements of EVRS against a conventional value W_{0E} by

time series observations of the ECGN as carrier network of the European Vertical Reference Frame and its datum.

Under the condition,

$$v_{hi} = v_{Hi}$$

the velocities of the physical heights H can be derived from time series of the the ITRFxx heights h:

$$H_P(t) = H_P^0 + \dot{h}_P^0 (t - t^0)$$

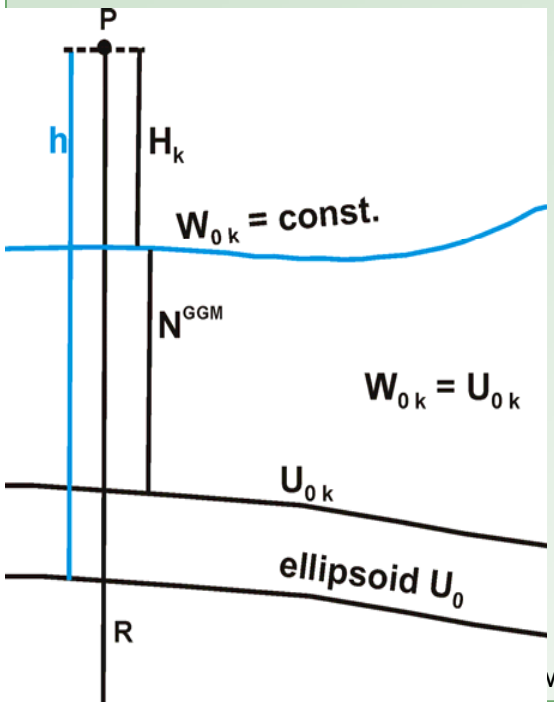
(5) EVRS Alignment to GVRS/IVRS

General case for realization and unification: combination of GNSS[1)] or GNSS/levelling[2)] with GGM

Transformation in a global VRF by determination of the level $W_{0\,k}$ of a regional VRF k:

$$W_P = W_{0\,k} - c_{P_k}$$

$$W_P = U_P + T_{P\,GGM} \quad with \quad U_P = U_0 + \partial U_0/\partial h \cdot h$$



$$1)\ W_P = U_0 + \partial U_0/\partial h \cdot h_{ITRF} + T_{P\,GGM}$$

$$2)\ W_{0\,k,i} = U_0 + \partial U_0/\partial h_i \cdot h_{i,ITRF} + T_{P_i\,GGM} + c_{P_{k,i}}$$

$$= U_0 - \gamma_0 (h_{i,ITRF} - H_{k,i} - N_{i,GGM}) \ app.$$

$$W_{0\,k} = mean\ W_{0\,k,i} \qquad \textbf{h and N shall be global}$$

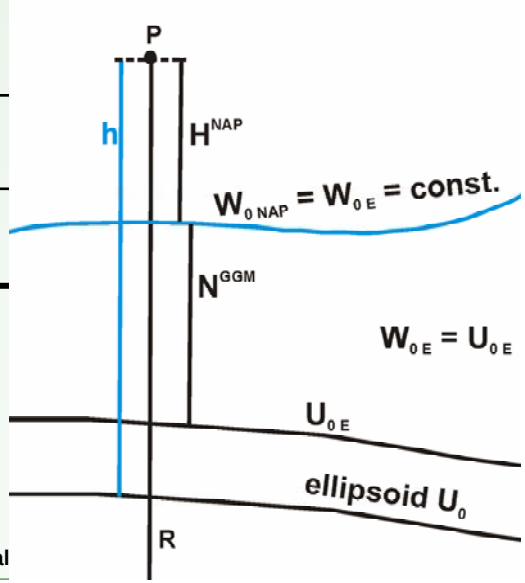
$$\Delta W_{0\,k} = W_{0\,k} - W_0$$

GPS/levelling heights compered with GGM´s

Area	No of points	Reference ellipsoid	GGM	RMS	bias*	bias* IERS 2003
EUVN	96	GRS 80	EGM96	0.43	- 0.51	- 0.11
EUVN	96	a = 6378136.3 m	EGM 96	0.43	+ 0.19	
EUVN	96	GRS 80	EGG97	0.19	+ 0.02	
EUVN	96	GRS 80	CG01C	0.28	- 0.61	- 0.21
EUVN (H. Denker)	96	GRS 80	CG03CEG (GRS 80)	0.12	0.40	- 0.13
Germany	680	GRS 80	EGM 96	0.29	- 0.62	
Germany	680	GRS 80	EGG 97	0.10	+ 0.07	
Germany	680	GRS 80	GCG05	0.02	+ 0.01	

*bias = $h^{ETRS} - H^{NAP} - N^{GGM}$

$$W_{0E} = 6\,263\,6857.28\ m^2\ s^{-2}$$



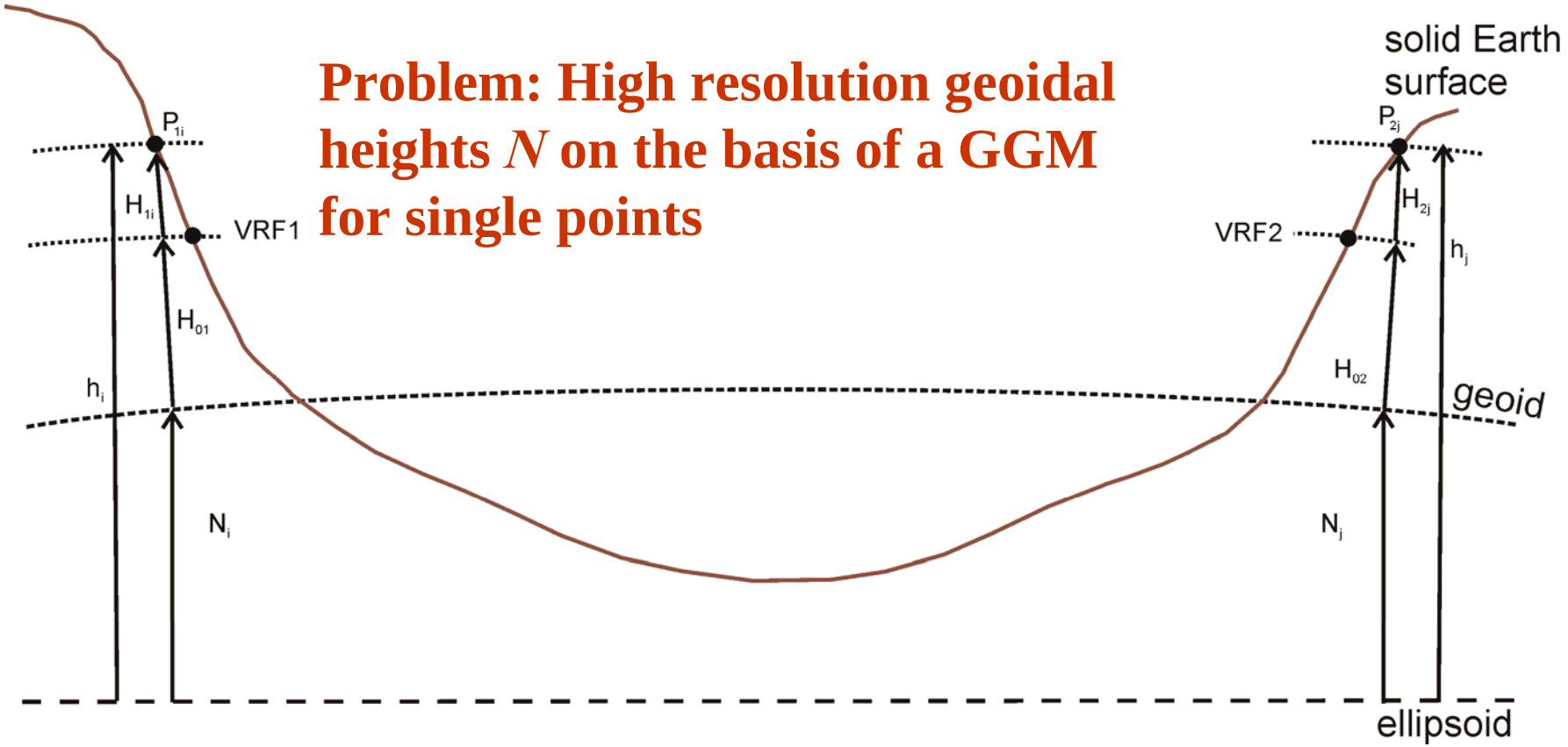
Unification by global a GGM and GNSS/levelling

$$H_{0,VRF} = h_{i,ITRF} - H_{i,\,VRF} - N_{i,GGM},$$ **h and N shall global**

VRF realization in single points:

$$H_{VRF} = h_{ITRF} - N_{GGM}$$

Problem: High resolution geoidal heights *N* on the basis of a GGM for single points



ECGN – CG01C

$$bias = h^{ETRS} - H^{NAP} - N^{CG01C} = U_0 - U_{0E}$$

GRAZ	-0.913
PFAN	-1.072
GOPE	-0.805
FR04	-0.987
WTZR	-0.389
LT02	-0.623
WSRT	-0.585
ES05	-0.649
ZIMM	-1.096
SODA	-0.801
JOEN	-0.736
GB08	-0.855

Statistic

number points: 12		(96 ECGN)
bias:	- 0.79	(- 0.61)
rms:	0.20	(0.28)
min:	- 1.10	
max:	- 0.39	

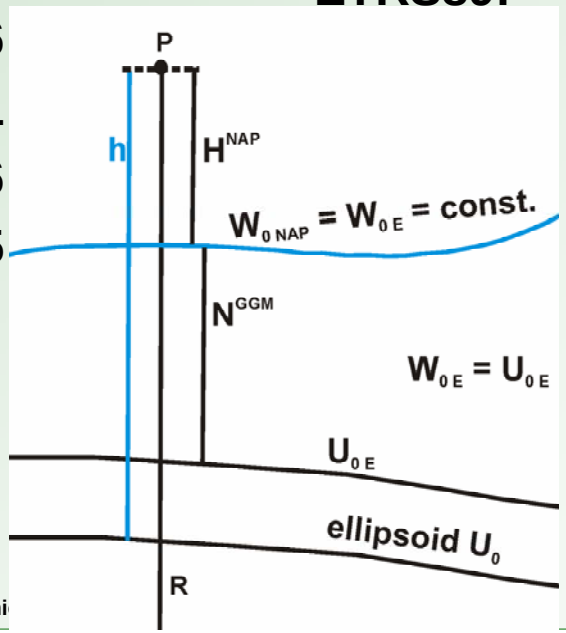
Parameters

ETRS89: a = 6378 137 m

a = 6378 136.46 m

GM = 3 986 004.415 10^8 m^3 s^{-2}

(W0 = 6 263 6856.0 m^2 s^{-2})



V. Summary and Outlook, Next Steps

- New UELN adjustment 2007, all participating countries are asked to contribute up-to-date data
- Fixing the EVRS2000 datum (NAP)
- Using the ECGN for EVRS time evolution
- Using IAG EGG 2007 solution (on basis of a IAG GGM)
- Alignment to IVRS

V. Summary and Outlook, Next Steps

(1) Selection of identical levelling points

(UELN-DC together with participating countries) *Sept. 2006*

(2) Selection of ECGN/EVRS datum points and determination of all measure elements

(Responsible agencies) *Dec. 2006*

(3) New adjustment of the UELN

(UELN-DC) *Feb. 2007*

(4) Time series analysis of ECGN stations

Beginning Jan. 2007

(5) Full parameter determination with EGG07 and IVRS realization

Sep. 2007

GPS/levelling heights compered with GGM´s

Area	No of points	Reference ellipsoid	GGM	RMS	bias*	bias* IERS 2003
EUVN	96	GRS 80	EGM96	0.43	- 0.51	- 0.11
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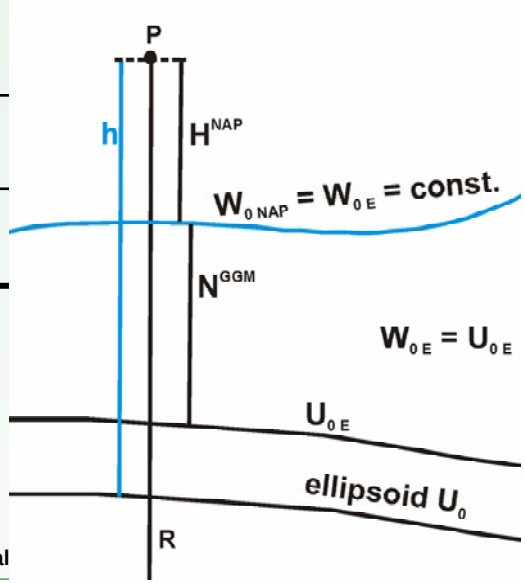
*bias = $h^{ETRS} - H^{NAP} - N^{GGM}$

$$W_{0E} = 6\,263\,6857.28\ m^2\ s^{-2}$$



ECGN – CG01C

$$bias = h^{ETRS} - H^{NAP} - N^{CG01C} = U_0 - U_{0E}$$

GRAZ	-0.913
PFAN	-1.072
GOPE	-0.805
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ZIMM	-1.096
SODA	-0.801
JOEN	-0.736
GB08	-0.855

Statistic

number points: 12		(96 ECGN)
bias:	- 0.79	(- 0.61)
rms:	0.20	(0.28)
min:	- 1.10	
max:	- 0.39	

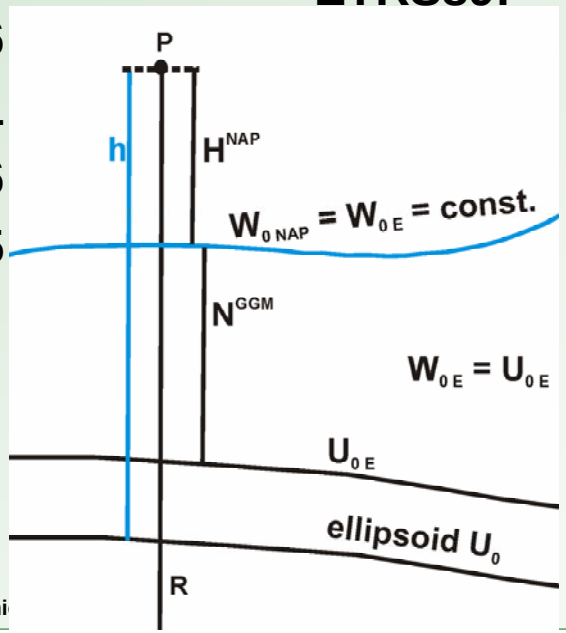
Parameters

ETRS89: a = 6378 137 m

a = 6378 136.46 m

GM = 3 986 004.415 10^8 m^3 s^{-2}

(W0 = 6 263 6856.0 m^2 s^{-2})



Sets of conventional parameters (of level ellipsoids)

ellipsoid	Semi-major axis a in m	flattening f^{-1}	Geocentric gravitational constant GM in $10^8 m^3 s^{-2}$	U_0/W_0 in $m^2 \cdot s^{-2}$	γ_e in $m \cdot s^{-2}$
Int. Ell. 1930 (Hayford)	6 378 388	297	3 986 329		
GRS 67	6 378 160	298.247	3 986 030		
GRS 80	**6 378 137**	**298.257222101**	**3 986 005**	**6 263 6860.850**	**9.78032 677**
WGS 84	6 378 137	298.25722356			
IUGG 91	6 378 136.3 0.5		3 986 004.41 0.01		
IERS 2003 Conventions (zero tide)	**6 378 136.6 0.1**	**298.25642 0.00001**	**3 986 004.418 0.008**	**6 263 6856.0 0.5**	**(9.78032 666)**
EGM96	6 378 136.3		3 986 004.415		
EIGEN CG01C (tide free)	6 378 136.46		3 986 004.415		

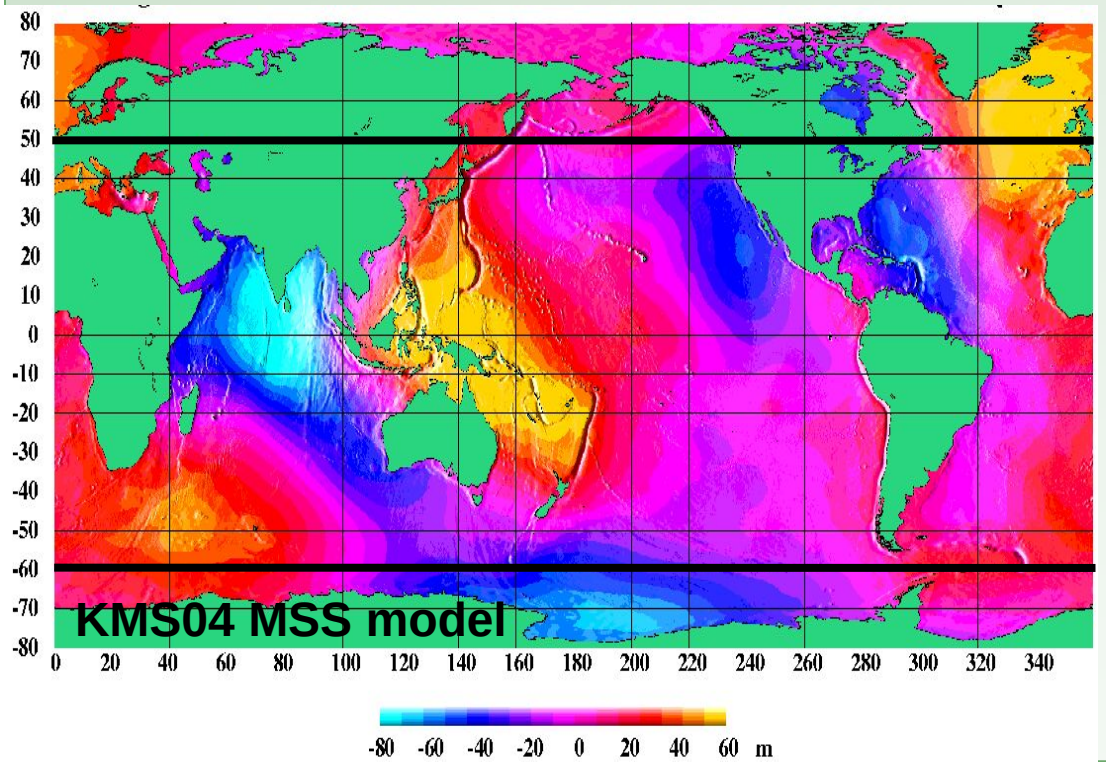
Angular velocity of the Earth rotation ω **7 292 115** 10^{-11} rad s^{-1}

Determination of W_0 of Mean Sea Surface

$$W_S = U_0 + \partial U_0/\partial h \cdot h_{S\,ALT} + T_{S\,GGM}$$

$$= \frac{GM}{r_S}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a}{r_S}\right)^n \sum_{m=0}^{n}\left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right]P_{nm(\cos\theta)}\right]$$

$$+ \frac{1}{2}\omega^2 r^2 \cos^2(90° - \theta).$$



KMS04 MSS model

$$W_{0S} = 1/S \cdot \iint_S W_S \, dS$$

With satellite altimeter observations (ALT) and a global gravity model (GGM)

- **in an agreed area of free oceans**
- **over a defined time period**
- **at a defined epoch.**

GVRS Realization - Time Evolution

Time series observations of an global integrated network of the Global Vertical Reference Frame and its datum

$$X_P(t) = X_P^0 + \dot{X}_P^0(t - t^0)$$

$$g_P(t) = g_P^0 + \dot{g}_P^0(t - t^0)$$

$$W_p(t) = W_p^0 + \dot{W}_p^0(t - t^0)$$

Under the condition

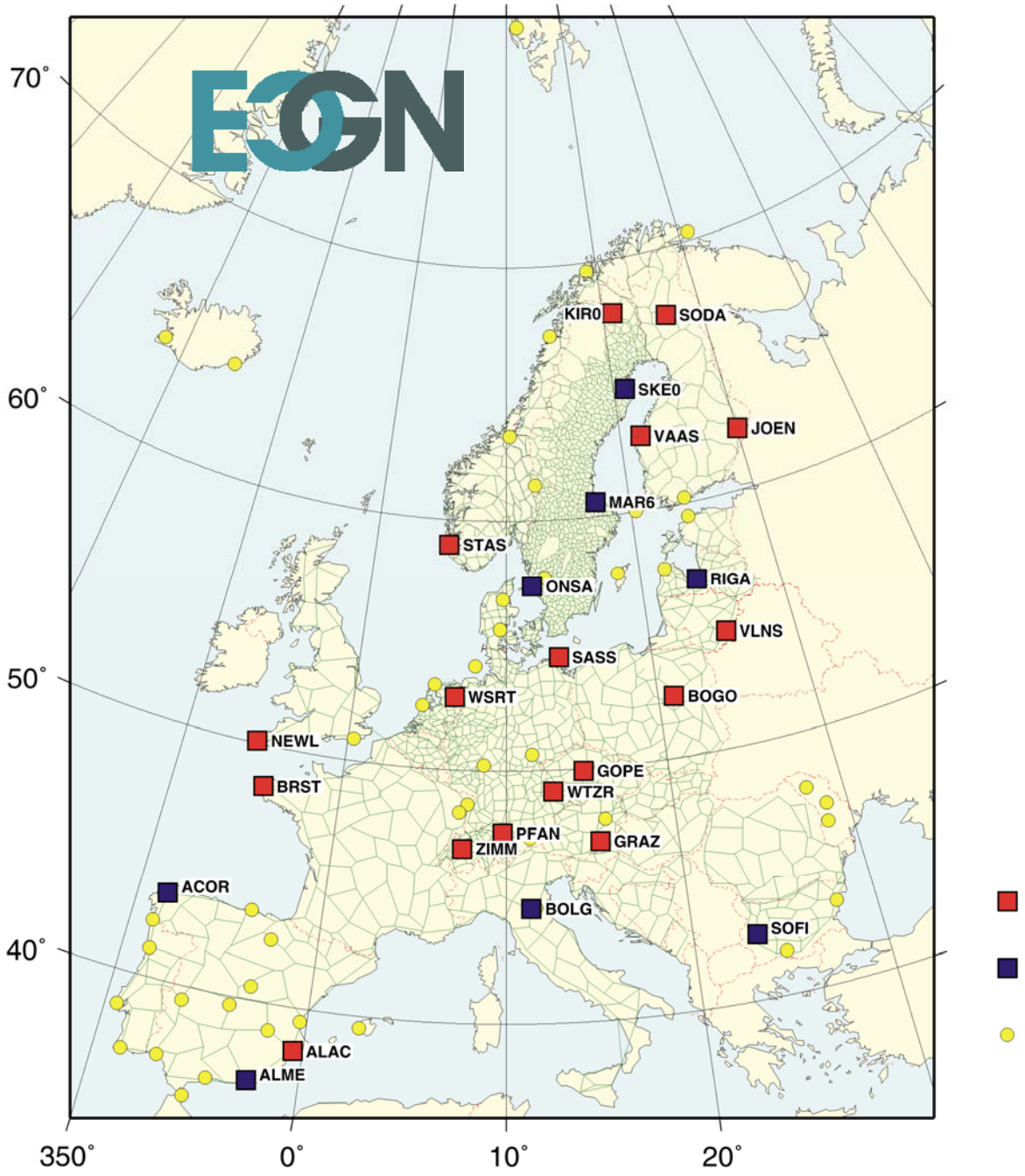
$$v_{hi} = v_{Hi}$$

the velocities of the physical heights H can be derived from time series of the the ITRFxx heights h:

$$H_P(t) = H_P^0 + \dot{h}_P^0(t - t^0)$$

(6) Summary and Outlook

- **Realization of a WHS for GGOS bases on combination of ITRFxx, tide gauge and gravity networks techniques with a GGM**
- **Combinations need conventions for parameter, models and procedures**
- **The mean sea surface has to be observed against a conventional W_0 value**
- **IAG ICP1.2 Vertical Reference Frames will propose 2007 conventions for definition and realization of a GVRS**



Selected ECGN stations for EVRS2007 time evolution control

Stations with GNSS, levelling, AG

Desirable additional stations

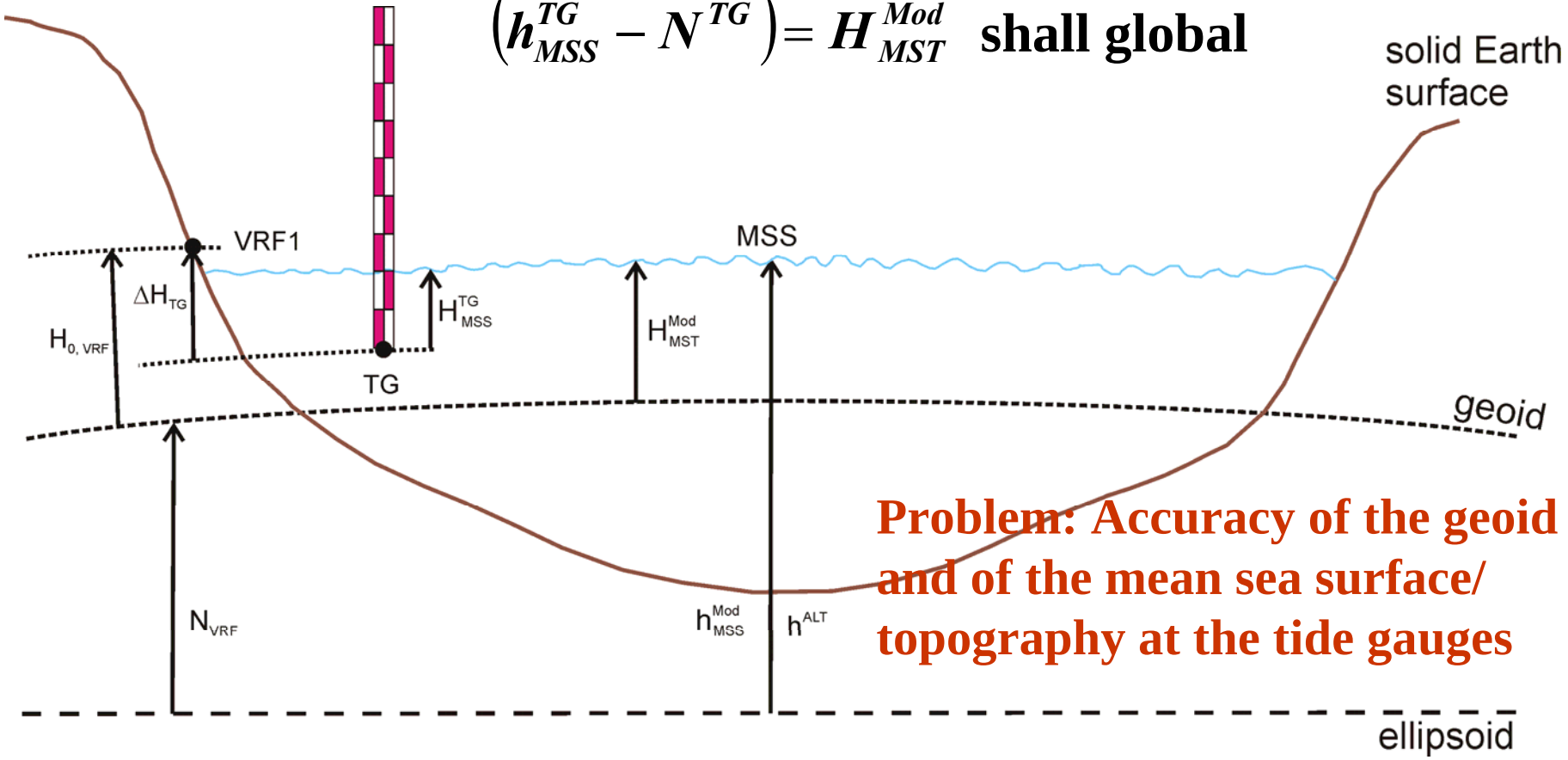
ECGN stations with missing elements

iii. Unification by tide gauge observations and mean sea surface model or altimeter observations

$$H_{0,VRF} = \left(h_{MSS}^{TG} - N^{TG}\right) - H_{MSS}^{TG} + \Delta H_{TG}$$

$$= H_{MST}^{Mod} - H_{MSS}^{TG} + \Delta H_{TG}$$

$$\left(h_{MSS}^{TG} - N^{TG}\right) = H_{MST}^{Mod}$$ **shall global**



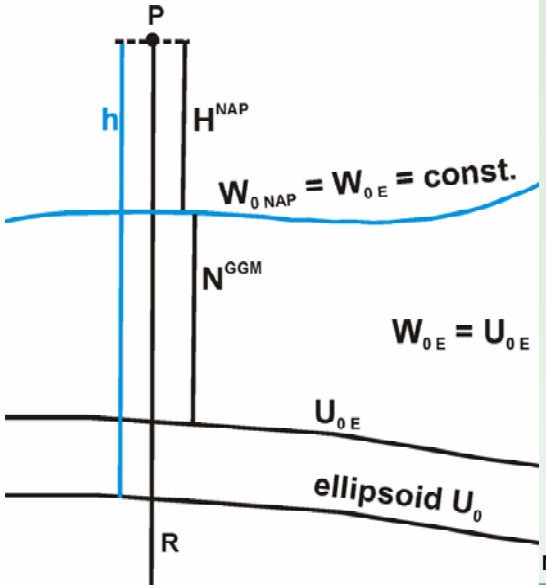
Problem: Accuracy of the geoid and of the mean sea surface/ topography at the tide gauges

EVRS Realization – IVRS Alignment

(5) Determination of a W_{0E} at Epoch 2000, fixing it and observe the relationship to a W_0 of a IVRS

by

GPS/levelling points of EUVN and ECGN and a European geoid bases on IVRS conventional GGM



$$W_P = W_{0E} - c_P$$
$$W_P = U_P + T_{P\,GGM}$$
$$\text{with} \quad U_P = U_0 + \partial U_0/\partial h \cdot h$$
$$W_P = U_0 + \partial U_0/\partial h \cdot h + T_{P\,GGM}$$

$$W_{0E} = U_0 + \partial U_0/\partial h \cdot h + T_{P\,GGM} + c_P$$
$$W_{0E} = mean\ W_{0Ei}$$

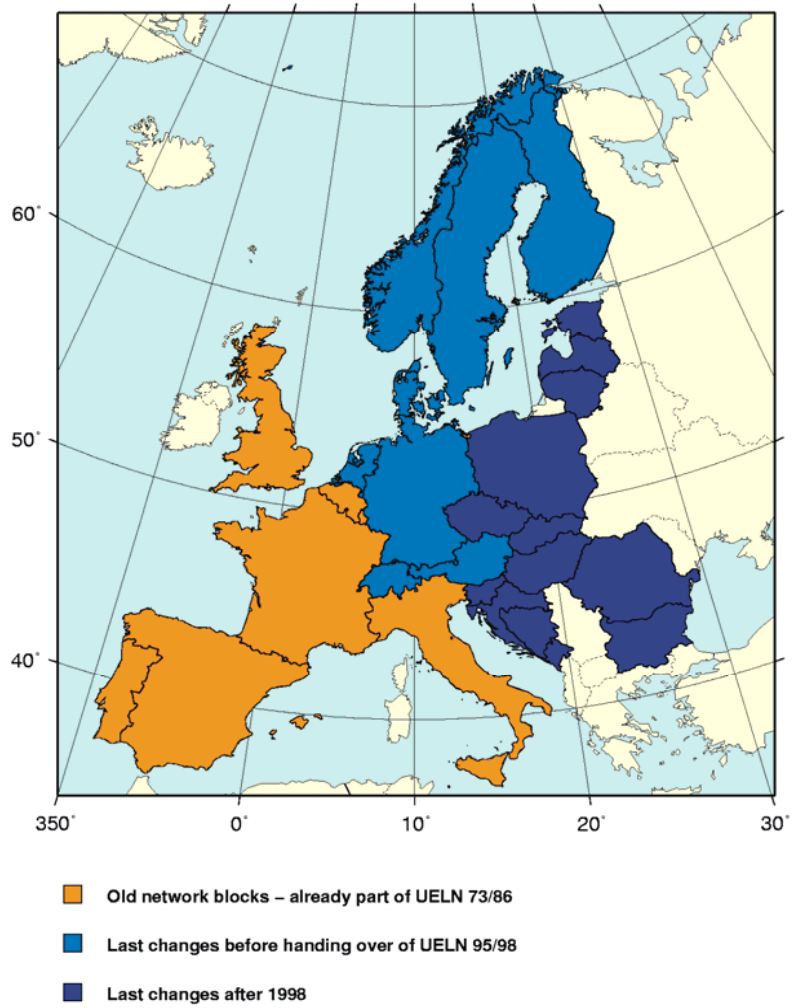
i. Common adjustments of existing levelling networks

United European Levelling Network (UELN) of

26 countries

• NAP

No. of measurements:	9542
Degrees of freedom:	2318
A-posteriori σ 1 km levelling distance in kgal·mm:	1.07
Mean value of σ of adjusted c_P (heights), in kgal · mm:	17.19
Average redundnce:	0.24



EVRS Realization – Data Harmonisation

(3) Reduction of data – Tidal System

	gravity g/Δg	geoid W/N	levelling height ΔH	altimetry h	mean sea level msl	position X/h
Mean tidal system **Mean/zero crust** (Stokes is not valid if masses outsite the Earth surface)	Δg_m	N_m	ΔH_m	Relation to N_m for oceanographic studies	h_{msl}	
Zero tidal system **Mean/zero crust** (Recommended by IAG Res. No. 16, 1983)	Δg_z	Stokes → N_z (EGG97)	ΔH_z c_p			
Tide-free system **Tide-free crust** (unobservable, far away from the real earth shape – there is no reason for the non tidal/tide free concept)	Δg_n	Stokes → N_n (EGM96)				X_n ITRFxx, ETRS89